ELEMENTS OF PRODUCTION PLANNING AND CONTROL

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PREFACE

Production planning and control has for many years been considered a somewhat glorified clerical occupation, concerned mainly with masses of forms that should be kept in circulation, but having little authority in formulating policies or making decisions: humdrum routine and no glamor. In recent years, however, it has become more and more apparent that production planning and control systems are one of the basic activities that determine the effectiveness of a production enterprise; that problems relating to utilization of production facilities involve operational analysis and policy evaluation of the highest degree; in short, that there is far more to production planning and control than a few cleverly designed forms. If properly taught and applied, this subject is perhaps one of the most exciting fields in industrial engineering.

This volume is an attempt to present some of the basic principles of production planning and control and to indicate in what way it is interwoven with other functions in the framework of production management. As the subject is of interest to engineers practicing in this field as well as to students both at undergraduate and graduate level, presentation of the material in a form suitable to all three parties is probably an impossible task. Certain chapters and sections are therefore marked with an asterisk to indicate that these can be omitted in the more elementary courses or by those interested in the subject from a more cursory viewpoint.

Thanks are due to all those firms who willingly supplied information and material, and whose assistance is acknowledged in the text, and also to my students, who listened, commented, and criticized. I am particularly grateful to my wife, who has been more than just a constant source of encouragement, but has made many invaluable suggestions. Finally, acknowledgement is due both to the Israel Institute of Technology, Haifa, and to Imperial College, London, for providing facilities that made this work possible.

London S.E.

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FUNCTIONS OF PRODUCTION PLANNING AND CONTROL

"The highest efficiency in production is obtained by manufacturing the required quantity of product, of the required quality, at the required time, by the best and cheapest method."[1] To attain this target, management employs production planning and control, the tool that coordinates all manufacturing activities.

Production consists of a sequence of operations that transform materials from a given to a desired form. The transformation may be done in one or in a combination of the following ways:

1. Transformation by disintegration, having essentially one ingredient as input and producing several outputs. This transformation is almost invariably accompanied by changes in the physical shape of the input, such as changes in the physical state or in the geometrical form. Examples: producing lumber in a sawmill, rolling steel bars from cast ingots, making components from standardized materials on machine tools, oil-cracking which yields several products, etc.

2. Transformation by integration or assembly, using several components as inputs and obtaining essentially one product as output. Examples: producing machines, furniture, household appliances, automobiles, radio and television sets, alloys, sulfuric acid, concrete, etc.

3. Transformation by service, where virtually no change in the object under consideration is perceptible but where certain operations are performed to change one of the parameters which define the object. This may include: operations for improving the tensile strength, density, crystallographic structure, wear, or other mechanical properties of the object; operations that change its locality or state by transportation or handling means; maintenance operations. Examples: sizing and coining in press work, servicing and light repairs of automobiles, loading and unloading of trucks, etc. Many pure service operations are

[1] Alford, L. P., and Bangs, J. R., *Production Handbook*, Ronald Press Co., 1952 (a later edition, edited by Carson, G. B., was published in 1958.)

not considered to be part of industry, but the planning and control of such operations is basically similar to those of industrial operations. By analogy one could say that "the highest efficiency in servicing is obtained by processing through the service station the required volume, offering the required quality, at the required time, by the best and cheapest method."

The four factors mentioned above—namely; quantity, quality, time, and price—encompass the production system, of which production planning and control is the brain. Three distinct stages comprise every activity in such a system: planning, operations, and control.

Planning begins with an analysis of the given data, on the basis of which a scheme for the utilization of the firm's resources can be outlined so that the desirable target may be most efficiently attained. The production plan sets subtargets for the various departments in terms of predetermined time periods, and these subtargets are so defined that in achieving them the over-all aim is realized.

Operations are performed in accordance with the details set in the production plan.

Control initiates and supervises operations with the aid of a control mechanism that feeds back information about the progress of the work. This mechanism is also responsible for subsequently adjusting, modifying, and redefining plans and targets, in order to ensure the attainment of the first goal.

Hence, production planning and control may be summarily defined as the direction and coordination of the firm's material and physical facilities toward the attainment of prespecified production goals, in the most efficient available way. In its capacity as the brain and the central nervous system of the production program, production planning and control is responsible for having available every part and assembly at the right time at the right place, in order to ascertain progress of operations according to a predetermined time and place schedule. Specifically, the functions of production planning and control (see Fig. 1–1) can be classified in ten categories.

Materials

Raw materials, as well as standard finished parts and semifinished products, must be available when required, to ensure that each production operation will start on time. Duties include the specification of materials (both with respect to dimensions and quality), quantities and availability, delivery dates, standardization and reduction of variety, procurement and inspection. This function also covers the procurement of semifinished products from subcontractors.

Methods

The purpose of this function is to analyze possible methods of manufacture and to try to define the best method compatible with a given set of circumstances and facilities. This analysis covers both the general study and selection

of production processes for the manufacture of components or assemblies and the detailed development and specifications of methods of application.

Such a study results in determining the sequence of operations and the division of the product into assemblies and subassemblies, modified by the limitations of existing layout and work flow.

Machines and equipment

Methods of manufacture have to be related to available production facilities, coupled with a detailed study of equipment replacement policy. Maintenance policy, procedure, and schedules are also functions connected with managerial responsibility for equipment, since the whole problem of breakdowns and reserves can be seriously reflected in halts in production. Tool management, as well as problems both of design and economy of jigs and fixtures, constitutes some of the major duties of production planning and control.

Routing

Once the over-all methods and sequence of operations have been laid down, each stage in production is broken down to define each operation in detail, after which the issue of production orders can be planned. Routing prescribes the flow of work in the plant and is related to considerations of layout, of temporary storage locations for raw materials and components, and of materials handling systems. Routing is a fundamental production function on which all subsequent planning is based.

Estimating

When production orders and detailed operation sheets are available with specifications of feeds, speeds, and use of auxiliary attachments and methods, the operation times can be worked out. This function involves the extensive use of operation analysis in conjunction with methods and routing, as well as work measurement, in order to set up performance standards. The human element figures prominently in work measurement because it is sensitive to systems of work ratings and wage incentive schemes. Hence it may consequently reflect in a wide scatter of operation times and in unduly large fluctuations and perhaps instabilities in time schedules.

Loading and scheduling

Machines have to be loaded according to their capability of performing the given task and according to their capacity. Machine loading is carried out in conjunction with routing, to ensure smooth work flow, and with estimating, to ensure that the prescribed method, feeds, and speeds are best utilized. Scheduling is perhaps the toughest job facing a production manager because it determines the utilization of equipment and manpower and hence the efficiency of the plant. Scheduling must ensure that operations are properly dovetailed, that

semifinished components arrive at their next station in time, that assembly work is not delayed, and that on the other hand the plant is not unnecessarily loaded both physically and financially with work in process, i.e., with semifinished components waiting for their next operation. This calls for a careful analysis of process capacities, so that flow rates along the various production lines can be suitably coordinated. In machine loading, appropriate allowances for setup of machines, process adjustments, and maintenance down time have to be made, and these allowances form a vital part of the data constantly used by the scheduling function.

Dispatching

This function is concerned with the execution of the planning functions. Dispatching is "the routine of setting productive activities in motion, through release of orders and instructions and in accordance with previously planned times and sequences as embodied in route sheets and loading schedules."[2] Dispatching authorizes the start of production operations by releasing materials, components, tools, fixtures, and instruction sheets to the operator, and ensures that material movement is carried out according to the planned routing sheets and to schedules.

Expediting

This control tool is the executive arm that keeps a close watch on the progress of the work. Expediting, or "follow-up" or "progress" as it is sometimes called, is a logical step after dispatching. Dispatching initiates the execution of production plans, whereas expediting maintains them and sees them through to their successful completion. This function has to keep close liaison with scheduling, in order to provide efficient feedback and prompt review of targets and schedules.

Inspection

Another major control function is that of inspection. Although the control of quality is often detached from the production planning and control department, its findings and criticisms are of supreme importance both in the execution of current plans and in the planning stage of future undertakings, when the limitations of processes, methods, and manpower are known. These limitations can form a basis for further investigations in evaluating, with the view to improving production methods or indicating the cost implications of quality at the design stage.

Evaluating

Perhaps the most neglected function, but an essential link between control and future planning, is that of evaluating. The executive tasks of dispatching and expediting are concerned with the immediate issues of production and with

[2] *Ibid.*

measures that will ascertain the fulfillment of set targets. Valuable information is gathered in this process, but the feedback mechanism is rather limited in nature and unless provision is made so that all this accumulated information can be properly digested and analyzed, valuable data may be irretrievably lost. This is where the evaluating function comes in: to provide a feedback mechanism on a longer term basis so that past experience can be evaluated with the view to improving utilization of methods and facilities. Many firms consider this function important enough to divorce part of it from production planning and control and to establish it as a separate department in its own right, in which wider aspects of production management can be studied, using modern tools of operations research. Whatever the scope of evaluating in the production planning and control department, this process is an integral part of the control function.

The ten functions were listed above in the order of their operation and are further discussed in Chapter 3. As shown in Fig. 1–1, they are related to three stages: preplanning, planning, and control.

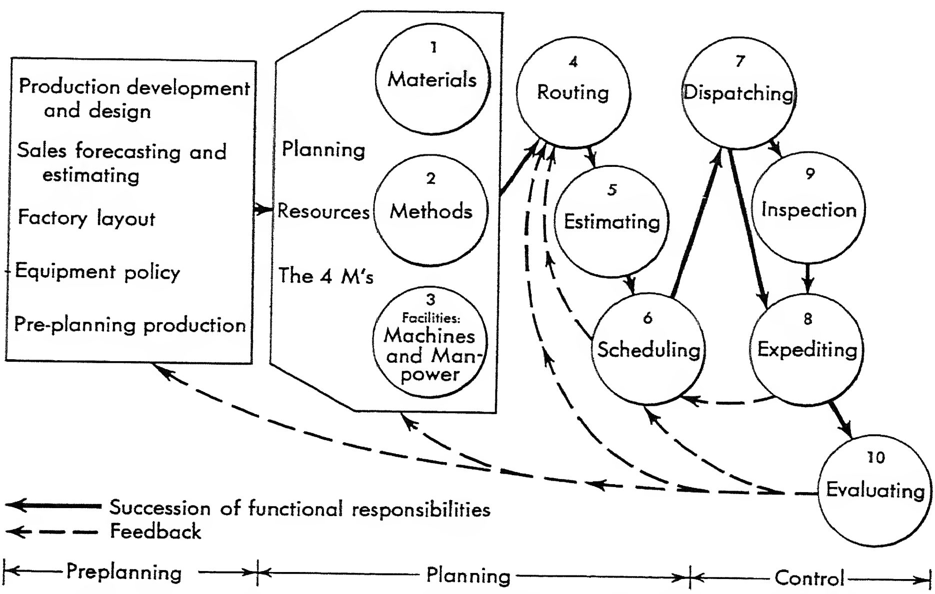

Figure 1–1. ***The ten functions of production planning and control cycle.***

Preplanning

This covers an analysis of data and outline of basic planning policy based on sales reports, market research, and product development and design. On the broad aspects of planning, this stage is concerned with problems of equipment policy and replacement, new processes and materials, layout, and work flow.

Preplanning production as a production planning and control responsibility

is also preoccupied with collecting data on the "4 M's," i.e., on materials, methods, machines, and manpower, mainly with respect to availability, scope, and capacity.

Planning

When the task has been specified, a thorough analysis of the "4 M's" is first undertaken to select the appropriate materials, methods, and facilities by means of which the work can be accomplished. As already mentioned, this analysis is followed by routing, estimating, and scheduling. The more detailed, realistic, and precise the planning, the greater conformity to schedules achieved during production, and subsequently the greater the efficiency of the plant. There are two aspects of planning: a short-term one, concerned with immediate production programs, and a long-term phase, where plans for the more distant future are considered and shaped. Prominent planning functions are those dealing with standardization and simplification of products, materials, and methods.

Control

This stage is effected by means of dispatching, inspection, and expediting. Control of inventories, control of scrap, analysis of work in process, and control of transportation are essential links of this stage. Finally, evaluation takes place to complete the production planning and control cycle. Professor Norbert Wiener has said of the social system that it "is an organization like the individual: that it is bound together by a system of communications: and that it has a dynamics, in which circular processes of a feedback nature play an important part."[3] If this is true of the social system, it is certainly true of the production system. Once the main policies have been defined by management, production planning and control is the director and coordinator of the plant production operations, having a similar function to that of a brain coordinating an animal's nervous system. The control functions have a very important role in providing the main sources of feedback information to ensure necessary corrective actions. Effective communication systems are prerequisites to efficient control and are therefore of great concern to production planning and control.

The ten functions of production planning and control were related in what might be regarded as a chronological order in the production procedure, which will be further discussed in Chapter 3. It is important to stress, however, that there is a very strong connection and interdependence between production planning and control and other industrial engineering functions, some of which are briefly described below.

Plant Layout

Layout not only affects the allocation of machines to perform given tasks, but it may also become an important factor at the design stage in selection of production processes. A rigid layout may hamper the integration of additional

[3] Norbert Wiener, *Cybernetics*, John Wiley & Sons, 1948.

equipment in a specific production center, either through lack of space or limited mobility of the equipment. This may lead to long lines of transportation, which increase the total production costs and the amount of work in process. On the other hand, when there is little choice between processes, machines, or sequences of operations, changes in plant layout must often be undertaken in the light of production planning and control requirements, in order to achieve a satisfactory work flow. Thus, production planning and control is affected by the restrictions imposed on the system by the layout, and at the same time it may greatly contribute through evaluation to modifications in layouts.

Simplification and Standardization

Production of different components, models, or products leads to a demand for different types of materials and methods of fabrication. At the various stages of manufacture, variety may therefore occur in materials, bought-out parts, manufactured components, minor and major assemblies, or finished products as well as in processes, methods of manufacture, tools, jigs and fixtures, machines, etc. Simplification and standardization are functions which aim at defining a limited variety of different types so that the basic requirements are satisfied and the efficiency of the plant is increased. Most aspects of simplification and standardization are the joint responsibility of several departments; e.g., the question of limiting the variety of finished products would involve the sales department, production departments, and the design office, while questions relating to simplification of materials would also include inventory control considerations and perhaps involve the research and development department. Some aspects of simplification and standardization are the major responsibility of the production planning and control department, such as problems relating to machines and methods.

Time and Motion Study

This field is closely allied to efficient utilization of manpower and to scheduling problems. Time and motion study consists of two fields of activity: operation analysis and work measurement.

1. Operation analysis or method study, which—as the name suggests—consists of evaluation, selection, and development of an efficient method to perform a given task. Operation analysis is concerned both with problems of limited scope (such as operator's work-place layout, an activity study of a gang of operators, or correlation of machine-operator activities) and over-all studies of the process, in which all aspects of routing, plant layout, and scheduling may play an important role.

2. Work measurement, which is concerned with establishing standard times for the various operations in the process for the estimating function in production planning. As already mentioned, no scheduling can even be attempted before some data on performance times become available.

From the foregoing remarks it should be appreciated that time and motion study is employed both at the planning and the control stages. Development of methods and information regarding the measurement of processing times can be obtained in two ways:

(i) By synthesis, based on past experience of similar circumstances, where the same processes were employed. Synthesis is an important tool at the planning stage.

(ii) By analysis of an existing production method and measurement of operation times, when the process is already in action. This obviously belongs to the control stage, and information gathered in this way provides a basis for replanning and readjusting of production schedules, when these are proved to be unrealistic, and for data required for future synthesis.

Although these two distinct functions of time and motion study are employed at different stages of production planning and control and for different purposes, they share the same philosophy, the same approach, the same techniques, and even if they can be divorced in time, they are essentially integral parts of the same field.

Inventory Control

The importance of materials availability at the various stages of production necessitates a mechanism of inventory control and stores organization. Inventories are a financial burden on the plant and management of stores may be very costly. Inventory control is sometimes a very complex function, as its policies are not dictated by internal needs and considerations alone but by external factors governing the purchasing of materials, such as vendors' offers and terms, market availability, transportation problems, and credit terms. These external factors may influence both quantities and delivery dates of materials and components and have to be taken into account by any inventory control mechanism.

Summary

Production planning and control is a management tool, employed for the direction of the manufacturing operations and their coordination with other activities of the firm. In the production system, which is primarily defined by the dimensions of quantity, quality, time and price, the functions of production planning and control comprise:

Materials (procurement, stock control, issue)
Methods (processes, operations and their sequence)
Machines (allocation and utilization)
Manpower (availability)
Routing (flow of work)
Estimating (operation times)
Scheduling (planning the production timetable)

Dispatching (authorizing the start of operations)
Expediting (follow-up)
Evaluating (assessing performance effectiveness)

The work of production planning and control is closely interwoven with other industrial engineering functions, mainly those of plant layout, equipment policies, time and motion study, simplification, and standardization.

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Problems

(The reader may find that reading the first four chapters may be helpful before attempting the following problems.)

1. Analyze the importance of each of the functions of production planning and control by discussing what would happen if each one in turn were deleted.
2. Scheduling is a planning function and expediting a control function. In a small firm it was proposed to allocate both responsibilities to one person. Would you approve of such a scheme?
3. Analyze the following statements on the place of work measurement in production planning and control:
   (i) Work measurement can be carried out only when the process is in operation, and not at the planning stage. Hence work measurement is a control function for the purpose of computing bonuses, but as data about operation times are not available at the planning stage, scheduling is a superfluous function.
   (ii) Scheduling is essential in order to facilitate the control of load distribution on the plant. Work measurement must therefore be a planning function, carried out on a "pilot plant" basis, all the operations and methods being tried out and measured quantitatively before the beginning of production.
   (iii) Unless work measurement is carried out under normal operating conditions, results are likely to be erroneous; hence measurement at the planning stage must be ruled out. On the other hand, scheduling must be performed before production starts. Therefore work measurement is not practical and is just a waste of time.
4. When asked to express his opinion, a foreman in charge of a machine tool shop said: "Why bother about production planning and control? Most of it consists of too much paper work, which has little bearing on reality. In our firm each foreman responsible for a section gets his instructions once a month together with drawings of the parts to be machined. It is then the foreman's responsibility

to do all the routing, estimating, and machine loading within his section. When each task is finished, the production department is notified. If the parts have to undergo additional operations in another section during the same month, an order is issued to transfer the parts to that section. If not, the parts wait until the next date for monthly allocation of tasks. The system is smooth and efficient —and it works".

Suppose you were assigned the task of analyzing the present system in order to determine its effectiveness. How would you set about doing it?

5. A transport department of a firm is located at its headquarters, which is situated three miles from the main production plant and about ten miles from the main stores, all three places lying approximately on a straight line. The transport department has to move personnel between all three centers, supply the plant with materials from the store, and remove finished products from the plant to the store; it also moves finished goods to the railroad station, which is situated about twenty miles either from the plant or from the store. These commitments may be classified as follows:
   (a) Ten trucks are required on Mondays and Thursdays to transport goods to the railroad station. The trucks finish unloading by lunch time.
   (b) Two truck loads are sent every day to move finished goods from the plant to the store.
   (c) Six truck loads are sent every second day to move materials from the store to the plant.
   (d) Personnel transport is carried out in passenger automobiles; orders are sent at random to the transport department, where all these automobiles are centralized.

   The problems that have to be analyzed are:
   (i) Should the transport department be centralized, with all the vehicle fleet and its maintenance garage situated at headquarters?
   (ii) What is the best way to schedule removal of materials and finished goods?
   (iii) Supposing that commitment (a) cannot be changed but commitments (b) and (c) can be conveniently distributed during the week (assuming a five-day week), how many trucks are required, if experience has shown that one truck is often marked down for repairs or maintenance?
   (iv) Should trailers be purchased to relieve the load on the trucks?
   (v) What procedures should be adopted to cater for passengers transportation under commitment (d)?

   How would you set about tackling these problems, what additional information do you require, how would you collect it, and what would you do with it when it is eventually available?

6. What functions of production planning and control can be exercised in constructing and controlling a timetable for a bus service along a specified route in a city?

MANUFACTURING SYSTEMS

Organization of manufacture and systems for its planning and control greatly depend on the type of plant in which they have to operate. The fundamental principles that guide the formulation of planning policy and its execution may be the same for all manufacturing concerns. But the emphasis on particular aspects of production management is a function of the specific requirements of the plant, and this emphasis is reflected in management approach to problems of inventory of raw materials and finished products, of machine selection and replacement, of machine setting and tooling, of scheduling methods, and of systems of follow-up and general control.

Three main factors may be said to determine the place of production planning and control in an organization:

1. The type of production; i.e., the quantities of finished products and the regularity of manufacture
2. Size of the plant
3. The type of industry; i.e., the field of specialization of the plant

Types of Production

There are eight types of production, which may be grouped under three headings, according to the quantities involved:

Job production

This is the manufacture of products to meet specific customer requirements of special orders. The quantity involved is small, usually "one off" or "several off," and is normally concerned with special projects, models, prototypes, special machinery or equipment to perform specialized and specific tasks, components or assemblies to provide replacement for parts in existing machinery, etc. Large turbo-generators, large engines, boilers, processing equipment, special electronic equipment, materials handling machines, shipbuilding, and many other manufacturing activities are of the job production group.

Three types of job production can be defined, according to the regularity of manufacture:

1. A small number of pieces produced only once

2. A small number of pieces produced intermittently when the need arises
3. A small number of pieces produced periodically at known time intervals

When the order is to be executed only once, there is little scope for improvement of production techniques by introducing intricate method studies, special tools, or jigs and fixtures, unless the technical requirements justify it. But, if the order is to be repeated, tooling and jigging as well as specially designed inspection gages should be carefully considered because the effect on production time may be appreciable.

Repeated orders for the same items usually do not require repeated planning, a fact that is reflected in the production costs of the product. Production control is also simplified in the case of repeated orders: The dispatchers and expeditors are familiar with the design, and from their past experience they can watch out for any expected difficulties in the course of production. The planning and control of schedules also becomes a simpler task when orders are repeated, especially at regular intervals, and a master schedule can be constructed in which production time is balanced against plant capacity. But such a state of affairs is rather rare. Usually the majority of job production orders are executed only once, and only a small percentage of them are repeated regularly or intermittently.

Scheduling is dependent on assessment of production times, and estimating (although it can be greatly improved by experience and skill of estimators) is based on judgment and is too often reduced to a rule-of-thumb affair. Scheduling must therefore be constantly amended to take account of reality, and this factor has a serious bearing on assessments of delivery dates. The output of the shop is mainly governed by plant capacity, and as soon as the load presented by incoming customers' orders exceeds this output, a queue of orders is formed. When immediate increase of plant capacity is impracticable, the length of the queue is a major factor governing the sales policy of such a plant, and a certain amount of discrimination in order selection may be essential.

Batch production

Batch production is the manufacture of a number of identical articles, either to meet a specific order or to satisfy continuous demand. When production of the batch is terminated, the plant and equipment are available for the production of similar or other products. As in job production, policies regarding tooling, fixtures, and other aids are dependent on the quantities involved. If the order is to be executed only once, there will be less justification for providing elaborate production aids than when the order is to be repeated.

In batch production, too, three types can be mentioned:

1. A batch produced only once
2. A batch produced repeatedly at irregular intervals, when the need arises
3. A batch produced periodically at known intervals, to satisfy continuous demand

Here, again, planning and control become more simplified as quantities increase and as manufacture becomes more regular. Two principal problems arise in batch production: the size of the batch and the scheduling of production.

The solution to these problems depends on whether production is governed by external orders only or whether the plant is producing for internal consumption. In the case of external orders, the batch size is normally determined by the customer to suit his specific circumstances. The plant, in this case a vendor, is mainly concerned with the effects of these orders on its production schedules and with the issues arising out of having to meet set delivery dates. When the plant produces to stock, both the batch size and scheduling problems are matters for internal management decisions. The problem of optimal batch sizes has to take into account the setup costs, which are involved before each production run, and the carrying costs incurred when the finished product is held in stock. The batch size determines the length of the production run and affects both the production schedule and batch size considerations of other products. These problems are further discussed in Chapters 10, 11, and 14.

Batch production is a very common feature in industry. Machine tool work, especially capstan and turret lathes, press work, forging and casting processes, some glass manufacturing, and chemical processes very often operate on a batch basis.

Continuous production

Continuous production is the specialized manufacture of identical articles on which the equipment is fully engaged. Continuous production is normally associated with large quantities and with a high rate of demand. While in the job and batch classes the rate of production normally exceeds the rate of demand, continuous production is justified only when its rate can be sustained by the market. Here, full advantage should be taken of repetitive operations in the design of production auxiliary aids, such as special tools, fixtures, positioners, feeders and materials handling systems, inspection devices, and weighing and packing equipment.

Two types of continuous production can be defined:

1. Mass production
2. Flow production

The difference between the two types is mainly in the kind of product and its relation to the plant. In mass production, a large number of identical articles is produced, but in spite of advanced mechanization and tooling, the equipment need not be specially designed for this type of article alone. Both plant and equipment are flexible enough to deal with other products involving the same production processes. If management decides that a certain line should be discontinued, the machinery can be switched over to produce another article, and such a change in policy will usually not involve major modifications in plant layout, although changes in tooling may be quite substantial. A shop of automatics

is an example associated with mass production. Although the automatics may be continuously engaged on the production of, say, a certain type of pinions, they can be switched over to production of screws or similar machine elements when the need arises. Another example is a highly mechanized press shop that can be utilized for the production of different components or products made of sheet metal, without having to introduce major changes in the shop layout.

In flow production, the plant, its equipment, and layout have been primarily designed to manufacture the product in question. Flexibility in the selection of products for manufacture is confined to minor modifications in layout or designs of models. Notable examples are automobiles, engines, household machinery, chemical plants, etc. A decision to switch over to a different kind of product may not only result in extensive tooling (this is often needed even when only the model is changed) but also in basic changes in layout and equipment policy, especially when special-purpose machines and complex materials handling systems are involved.

Production planning and control in continuous production is usually far simpler than in job or batch production. Extensive effort is required for detailed planning before production starts, but both scheduling and control need not usually be very elaborate. The output is either limited by available capacity or regulated within given limits to conform to production targets based on periodic sales forecasts.

There are many cases where plants are not confined to one particular type of production. Even very large plants engaged in manufacturing end products of the flow type resort very often to batch production of most of the components required for the assembly line. This situation arises from uneven production rates of different components, which cannot always be adjusted by engaging more machines or manpower on the "slow" items. Also, the rate of production of some "quick" items may exceed by far the rate of demand on the assembly line, so that different parts may have to be produced in succession on the same machines, leading to a clear case of batch production and inventory problems. Production planning and control in such plants may become rather involved because of the different types of production which are simultaneously employed in various departments.

Size of Plants

The size of plant has a relation both to the organization of the Production Planning and Control Department and to its procedure. The larger the plant, the more complicated its activities and the larger the number of employees; consequently the more complex becomes the organization of the department and the more necessary it is to draw demarcation lines to divide and define activities and responsibilities. In small plants less formality is required; i.e., more verbal instructions and exchange of information facilitates and lessens paper work.

By what factor is the size of plant measured? Is it the number of employees, the capital investment, the annual turnover? All three criteria are sometimes used, but for our purpose the first one is useful in illustrating the large number of small plants that exist both in America and in Britain. Analysis of industries in the two countries, showing the size of distribution of plants according to the percentage of employees, is given in Tables 2–1 and 2–2, from which it appears that the number of plants employing less than 500 is over 98 per cent in both countries.[1] (See also Fig. 2–1.) The predominance of small and medium size plants should therefore be borne in mind when studying the applications of production management principles in industry, particularly so when one is aware of the fact that many so-called large plants are actually a horizontal integration of several small plants, the general activities of which are being outlined at the head office.

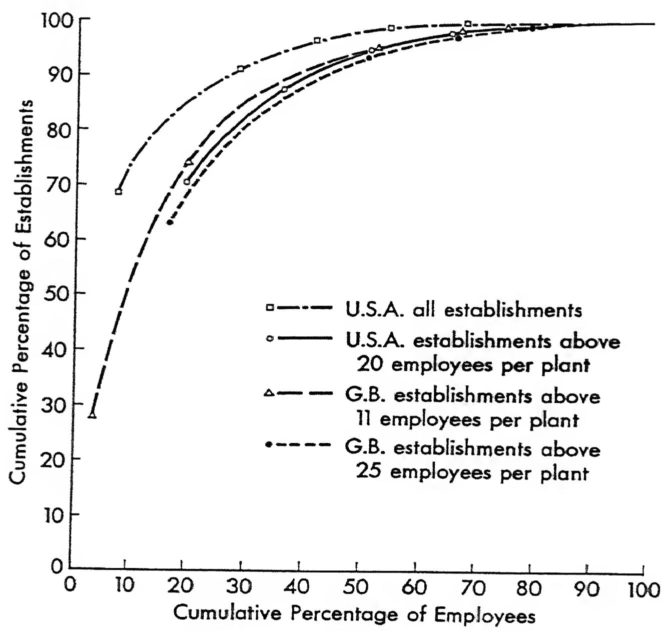

Figure 2–1. ***Distribution of employees in Britain and the United States.***

Numerous problems are connected with production planning and control of large enterprises. They vary in character and complexity with the size and organization of the plant, but most of them are related to the question: Should planning and control be centralized or decentralized? There are naturally many

[1] The percentage in the U.K. is 95, but only figures for plants with over 11 employees are available.

Table 2–1

U.S.A. Census of Manufactures: Selected Statistics by Size of Establishment for Major Industry Groups

Major Industry Group	Total All Establishments	Establishments with Average Employment of: 1 to 19 Employees	20 to 99 Employees	100 to 249 Employees	250 to 499 Employees	500 to 999 Employees	1,000 Employees or More
Manufacturing establishments incl. administrative and auxiliary units:							
Number of establishments	291,004	198,514	65,059	16,093	6,293	2,949	2,096
Per cent of establishments		68.2	22.4	5.52	2.16	1.01	0.72
All employees	16,125,550	1,210,132	2,889,559	2,500,858	2,190,032	2,043,597	5,289,694
Per cent of employees		7.51	17.9	15.6	13.6	12.7	32.8
Operating establishments only:							
Number of establishments	286,817	196,344	63,889	15,647	6,092	2,837	2,008
Per cent of establishments		68.4	22.3	5.45	2.13	0.99	0.70
All employees	15,651,294	1,195,447	2,834,998	2,430,255	2,118,438	1,963,907	5,106,571
Per cent of employees		7.58	18.1	15.6	13.6	12.5	32.6
Food and kindred products:							
Number of establishments	42,374	28,725	10,167	2,447	673	238	124
Per cent of establishments		67.8	24.0	5.77	1.59	0.56	0.27
All employees	1,647,204	189,966	452,058	374,784	229,084	162,382	238,793
Per cent of employees		11.5	27.4	22.8	13.9	9.86	14.5
Tobacco manufactures:							
Number of establishments	627	336	116	86	47	22	20
Per cent of establishments		53.6	18.5	13.7	7.49	3.51	3.18
All employees	94,863	1,547	6,085	13,769	17,339	15,415	40,697
Per cent of employees		1.63	6.41	14.5	18.3	16.2	42.9
Textile mill products:							
Number of establishments	8,070	3,208	2,620	1,154	611	328	149
Per cent of establishments		39.7	32.5	14.3	7.57	4.06	1.85
All employees	1,037,440	25,165	126,027	185,393	215,015	224,717	261,026
Per cent of employees		2.47	12.2	17.9	20.7	21.6	25.2

Apparel and related products:							
Number of establishments	31,372	18,092	10,691	1,920	502	141	26
Per cent of establishments		57.7	34.1	6.12	1.60	0.44	0.07
All employees	1,190,064	129,732	470,756	288,757	169,354	90,760	40,589
Per cent of employees		10.9	39.5	24.3	14.2	7.63	3.41
Lumber and products (except furniture):							
Number of establishments	41,484	35,097	5,255	851	215	57	9
Per cent of establishments		84.6	12.7	2.05	0.52	0.13	0.02
All employees	645,936	172,317	224,162	125,521	73,203	39,095	11,588
Per cent of employees		26.7	34.7	19.4	11.3	6.05	1.79
Furniture and fixtures:							
Number of establishments	10,273	7,261	2,282	500	159	52	19
Per cent of establishments		70.7	22.2	4.87	1.55	0.52	0.08
All employees	340,694	43,053	100,306	77,395	55,912	36,276	27,711
Per cent of employees		12.6	29.4	22.7	16.4	10.7	8.12
Paper and allied products:							
Number of establishments	5,004	1,827	1,761	857	373	142	44
Per cent of establishments		36.5	35.2	17.1	7.46	2.84	0.88
All employees	530,210	14,792	85,627	134,878	129,131	96,928	68,818
Per cent of employees		2.78	16.1	25.3	24.3	18.2	12.9
Printing and publishing industries:							
Number of establishments	32,531	26,476	4,750	837	262	117	89
Per cent of establishments		81.4	14.6	2.58	0.80	0.36	0.27
All employees	804,386	145,214	198,551	129,365	91,206	82,837	157,165
Per cent of employees		18.1	24.6	16.1	11.3	10.3	19.5
Chemicals and allied products:							
Number of establishments	11,075	7,115	2,818	648	255	116	123
Per cent of establishments		64.3	25.4	5.86	2.31	1.05	1.11
All employees	739,389	45,686	126,299	100,120	89,544	84,058	293,544
Per cent of employees		6.18	17.1	13.5	12.1	11.7	39.7
Petroleum and coal products:							
Number of establishments	1,381	700	333	172	90	46	40
Per cent of employees		50.7	24.2	12.5	6.52	3.33	2.95
All employees	215,843	5,316	16,179	25,892	32,996	32,762	102,680
Per cent of employees		2.46	7.50	12.0	15.3	15.2	47.6

Table 2-1—(Continued)

U.S.A. Census of Manufactures: Selected Statistics by Size of Establishment for Major Industry Groups

Major Industry Group	*Total All Establishments*	*Establishments with Average Employment of:* 1 to 19 Employees	*20 to 99 Employees*	*100 to 249 Employees*	*250 to 499 Employees*	*500 to 999 Employees*	*1,000 Employees or More*
Rubber products:							
Number of establishments	1,406	753	347	134	66	54	52
Per cent of establishments		53.6	24.7	9.53	4.68	3.84	3.70
All employees	246,526	4,868	16,435	20,994	22,717	38,681	142,795
Per cent of employees		1.97	6.67	8.52	9.22	15.7	57.9
Leather and leather products:							
Number of establishments	4,845	2,578	1,287	547	334	87	12
Per cent of establishments		53.3	26.5	11.3	6.90	1.80	0.12
All employees	356,578	16,856	61,947	87,524	116,963	55,671	17,584
Per cent of employees		4.72	17.4	24.6	32.8	15.6	4.93
Stone, clay, and glass products:							
Number of establishments	11,162	7,829	2,316	613	262	83	59
Per cent of establishments		70.2	20.8	5.48	2.34	0.74	0.53
All employees	491,814	47,492	102,988	97,657	89,207	56,645	97,813
Per cent of employees		9.65	20.9	19.9	18.1	11.5	19.9
Primary metal industries:							
Number of establishments	5,838	2,649	1,752	651	354	213	219
Per cent of establishments		45.4	30.1	11.2	6.06	3.65	3.76
All employees	1,117,059	19,644	82,256	105,126	126,848	149,513	633,614
Per cent of employees		1.76	7.37	9.43	11.4	13.4	56.7
Fabricated metal products:							
Number of establishments	22,516	15,168	5,332	1,219	460	242	95
Per cent of establishments		67.4	23.7	5.42	2.04	1.07	0.42
All employees	1,019,406	98,449	233,680	190,293	159,088	166,832	170,963
Per cent of employees		9.65	23.0	18.7	15.6	16.4	16.8

Machinery (except electrical):							
Number of establishments	25,601	18,079	5,213	1,168	551	317	273
Per cent of establishments		70.6	20.4	4.56	2.15	1.24	1.07
All employees	1,541,675	112,368	223,843	183,714	193,263	220,463	607,978
Per cent of employees		7.28	14.5	11.9	12.5	14.3	39.5
Electrical machinery:							
Number of establishments	5,758	2,921	1,487	589	339	215	207
Per cent of establishments		50.7	25.8	10.2	5.88	3.74	3.60
All employees	959,126	19,210	71,097	93,341	117,148	151,614	506,290
Per cent of employees		2.05	7.41	9.73	12.2	15.8	52.8
Transportation equipment:							
Number of establishments	5,348	3,031	1,211	402	206	186	312
Per cent of establishments		56.7	22.7	7.53	3.86	3.48	5.84
All employees	1,704,572	19,205	54,901	64,834	74,100	133,078	1,358,383
Per cent of employees		1.13	3.22	3.80	4.35	7.81	79.7
Instruments and related products:							
Number of establishments	3,142	2,157	610	185	84	56	50
Per cent of establishments		68.6	19.4	5.88	2.67	1.78	1.59
All employees	272,586	12,846	28,226	28,704	29,275	41,096	132,408
Per cent of employees		4.72	10.4	10.5	10.8	15.1	48.6
Miscellaneous manufactures:*							
Number of establishments	17,010	12,342	3,541	667	249	125	86
Per cent of establishments		72.7	20.6	3.92	1.46	0.74	0.50
All employees	695,917	71,775	153,575	102,194	87,045	85,084	196,132
Per cent of employees		10.3	22.0	14.7	12.5	12.2	28.2
Administrative and auxiliary:							
Number of establishments	4,187	2,170	1,170	446	201	112	88
Per cent of establishments		51.8	28.0	10.6	4.80	2.68	2.12
All employees	474,256	14,685	54,561	70,603	71,594	79,690	183,123
Per cent of employees		3.10	11.5	14.9	15.1	16.8	38.6

* Includes privately owned or operated establishments, or both, engaged primarily in manufacture of ordnance material and accessories.

Source: Statistical Abstract of the United States – Bureau of Census, Washington, 1958.

Table 2–2

Great Britain Manufactures: Selected Statistics by Size of Establishment, Major Industry Group

Major Industry Group	*Total All Establishments*	*Establishments with Average Employment of*					
		11–24 Employees*	*25–99 Employees*	*100–499 Employees*	*500–999 Employees*	*1,000–1,999 Employees*	*2,000 or More Employees*
Total number of establishments:	55,739	14,874	26,145	12,052	1,524	742	402
Total employees	7,735	258	1,326	2,492	1,048	1,021	1,590
Treatment of non-metalliferous mining products other than coal:							
Number of establishments	2,449	595	1,210	554	61	22	7
Number of employees	286	11	62	116	42	34	21
Chemicals and allied trades:							
Number of establishments	2,270	463	1,008	611	111	46	31
Number of employees	446	8	51	130	77	61	119
Metal manufacture:							
Number of establishments	1,845	314	769	526	124	58	54
Number of employees	519	6	40	121	87	80	185
Engineering, shipbuilding and electrical goods:							
Number of establishments	9,089	2,164	4,015	2,132	383	270	125
Number of employees	1,848	38	209	471	268	373	489
Vehicles:							
Number of establishments	5,930	2,052	2,785	782	119	89	103
Number of employees	1,027	35	132	154	86	125	495
Precision instruments and other metal goods:							
Number of establishments	5,705	1,570	2,825	1,104	149	46	11
Number of employees	581	27	143	220	102	61	28

Textiles							
Number of establishments	5,753	1,004	2,588	1,949	157	41	14
Number of employees	776	17	144	411	104	58	42
Clothing:							
Number of establishments	5,922	1,528	3,118	1,160	95	19	2
Number of employees	500	27	159	219	62	25	8
Food, drink and tobacco:							
Number of establishments	5,982	1,903	2,613	1,224	141	78	23
Number of employees	706	32	127	254	96	107	90
Paper and printing:							
Number of establishments	4,397	1,302	1,955	964	115	45	16
Number of employees	503	23	96	195	78	59	52
Other manufacturing industries							
Number of establishments	6,397	1,979	3,259	1,046	69	28	16
Number of employees	543	34	163	201	46	38	61

* The number of establishments with fewer than eleven employees is not known, but it is thought to be in the region of 150,000.

Source: Annual Abstract of Statistics, 1960. H.M.S.O. London.

advantages to centralization of methods planning, standardization, and simplification of products and materials and, in particular, purchasing and storing of raw materials and standard components. Centralized activities of this kind ensure the best use of facilities and past experience in an enterprise. Some activities of various manufacturing sections or subsections or subplants are so interrelated (such as problems of transportation, materials handling, layout, output, equipment and labor policies, investment, etc.) that centralized planning is absolutely vital, while in many other fields the least that is required is efficient coordination of policies and operations. On the other hand, too centralized planning and control is often cumbersome, detached from reality, and slow to react. It is obvious that operation between these two extremes should be selected to ensure efficient functioning of the enterprise and its plants.

Types of Industry

Another factor that has a bearing on the organization of production planning and control is the type of industry, i.e., the field in which the plant specializes. Industries can be classified into types by several methods: by availability of different kinds of labor in different geographical locations, by the demand for different grades of skills, and by factors relating to investment policy, but normally aspects that characterize the production itself are considered a convenient method for classification. In particular, aspects relating to three stages in production can be considered; namely, materials, processes, or end products.

Division by materials (e.g., iron, copper, aluminium, rubber, hydrocarbons) is not always practical because industries use different materials in the course of manufacture and only rarely can typical groups in this manner be obtained. Classification by end products is logical, since it immediately tells us something about the materials, methods, and perhaps skill that is involved, and each of the classes obtained in this manner has a common denominator comprising problems typical to all its members. This classification, however, is rather complicated, first because of the astronomical number of different products that could be named and secondly because many plants manufacture more than one product.

A simpler method is division by major production processes such as metal fabrication (or engineering industries) and chemical processes. Such grouping covers a wide range of end products, but problems in manufacture and organization are similar for plants of the same size in the group engaged on the same type of production. Even with such a broad classification it is not always possible to make a clear-cut division, and the plant is often labeled according to the main activity it is engaged on, though it may employ additional processes and raw materials that are by no means characteristic of the "type of industry" it is supposed to be part of. With such grouping in mind, the U.S. Census of Manufactures mentions 21 industries and the British Census of Production names 11. The industries classified in this way are shown in Tables 2-1 and 2-2.

Is there any correlation between types of industry and types of production?

In other words, is it possible to generalize as far as production planning and control principles and methods are concerned and thus produce "cookbook recipes" outlining precise techniques for various industries?

A recent survey of British industries, carried out by a joint committee of the Institute of Cost and Works Accountants and the Institution of Production Engineers, is given in Table 2-3.[2] A sample of 229 plants in 14 industries was studied and classified into types of production. One serious criticism may be leveled at the manner in which the sample was selected, namely, at the comparatively low percentage of small plants in the sample. Only 73 plants out of 229 had less than 500 employees each, i.e., about one-third of the entire sample, whereas the proportion of small plants in industry is far larger (see Table 2-2). With all its limitations, however, the sample may give some indication as to the nature of production employed.

Table 2-3

Relation between Type of Industry and Type of Production*

(A sample from British industry)

Type of Industry	*No. of Plants in Sample*	*No. with under 500 Employees*	*Largest No. of Employees*	*Smallest No. of Employees*	*Type of Production*		
					Job	*Batch*	*Continuous*
1. Bricks, ceramics, glassware	9	2	15,380	170	–	5	4
2. Chemical manufacture	9	3	1,150	50	–	2	7
3. Metal manufacture	6	1	7,770	376	–	1	5
4. Engineering	121	37	21,880	40	30	72	19
5. Vehicles and accessories	14	1	17,120	345	–	8	6
6. Metal goods, not elsewhere	18	7	11,904	60	2	13	3
7. Precision instruments	8	3	3,000	180	1	6	1
8. Textiles	12	3	4,000	175	–	7	5
9. Leather, leather goods	2	1	570	220	–	2	–
10. Clothing	8	2	2,000	40	–	8	–
11. Food, drink, and tobacco	5	2	1,800	150	–	1	4
12. Manufactures in wood and cork	1	1	350	350	–	1	–
13. Paper and printing	6	4	1,000	300	1	4	1
14. Rubber, toys, plastics, etc.	10	6	2,300	80	–	10	–
Total	229	73			34	140	55

* Production Control and Related Works Statistics, The Institution of Production Engineers, U.K., 1956.

With a few exceptions (leather, clothing, and rubber) it would seem that no industry is confined to one type of production. Continuous production appears to be mainly characteristic of the chemical, metal manufacture, food, drink, and

[2] The similarity in structure of United States and British industries would suggest that a similar state of affairs probably exists in United States industries.

tobacco industries. Batch production recurs in all industries; indeed it accounts for 61 per cent of the sample, and one might even have expected a larger percentage, had the sample been a more typical cross-section of industry. Some groups produce mainly or solely in batches, while job production occurs only in four industries. As batch production is far more complex as regards production planning and control than continuous production, and particularly in view of its important position in industry, its problems need very careful study in order to ensure the proper application of production management principles in the planning stage, mainly in the analysis of methods and scheduling.

Summary

Modes of manufacture, which may affect production planning and control systems, include three main relevant factors: types of production, size of plants, and types of industries (classified by major processes or products). There are three type-groups of production: job, batch, and continuous. Plant size has a very wide distribution, but the majority of plants are of small or medium size. Generally there is no connection between types of production and industries, but there are a few exceptions. Batch production seems to occur in all industries, while job production is mainly employed by engineering plants. Continuous production prevails in many industries but is mainly typical to those involving chemical processing.

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Statistical Abstract of the United States.

Problems

1. Give examples for each of the eight types of production.
2. (a) Do production planning and control systems for large plants and small plants have anything in common?
   (b) Should a small plant adopt a successful production planning and control system from a large plant? Why?
3. Assuming equal size plants employing similar processes and engaged on the same type of production can use precisely the same production planning and control system with all its details and trimmings, if you were asked to specify systems for all such classes of plants in industry, what would your reaction be?
4. Give an example of a large enterprise that consists of many small or medium size plants.

5. Use United States Census of Manufactures or Statistical Abstracts to construct Fig. 2–1 and find whether any considerable changes occurred after World War II.
6. A chain of supermarkets in a city is fed with goods from a central store where inspection of all merchandise received from suppliers is carried out. Inspection is performed in two stages: (a) acceptance inspection, to ensure that goods received are as per order; (b) inspection prior to issue to the supermarkets. This procedure applies to all types of goods without exception.

   For issuing and storage purposes the goods are classified in three categories:

   (i) Perishables, such as vegetables and fruit, which are brought to the central store every morning after dawn and supplied to the supermarkets in the morning before opening time

   (ii) Semiperishables, such as meat, which are supplied to the supermarkets every second day

   (iii) Nonperishables, such as canned food, which are supplied to the supermarkets once a week

   In your view, should these three categories be treated according to the same procedure? What alternatives can you suggest and how would you decide which to adopt?

   In what ways would the system of ordering goods by a supermarket manager differ from the system that should be adopted by the central store?

3

PRODUCTION PROCEDURE

The type of production and industry, and naturally the sales volume, prescribe the general framework of production management and dictate basic modes of internal procedures. In flow and mass production, with a constant load on the manufacturing departments, the procedures are found to be relatively simple. A suitable system of communication circuits and methods of control have to be set up in order to ensure that deviations from the planned schedule are recorded, as they occur, and that corrective actions are promptly taken. Ideally, this should become a self-regulating control mechanism. The procedures, however, become more complicated with an increase in the number of operations, number of parts involved, or the variety of products in the firm. These, coupled with fluctuations in demand and intermittent manufacturing, have a marked effect on the plant efficiency.

The Production Cycle

To appreciate the role of production planning and control as a vital tool in the complexities of production management, let us examine the framework within which the production order is initiated. The production procedure is described in Fig. 3–1, where the main flow-channels of instructions, information, and materials are shown. The cycle starts and ends with the customer:

1. The sales department studies the reception of products in the market and consumer reactions to new modifications and designs. Market research is also carried out regarding proposed new products.
2. The collected data are analyzed by the sales department, which prepares a sales forecast with a breakdown of products and models as a function of time periods. The detailed forecast is submitted to management.
3. A production budget is prepared by the financial department, in consultation with the manufacturing department. The proposed budget and the sales forecasts are closely scrutinized by management, and a decision is taken regarding the annual or semiannual quantity to be produced.
4. The engineering department is instructed to prepare drawings, parts lists,

and specifications, or to check and modify existing ones. The manufacturing budget is then adjusted accordingly.

5. The vice-president or the head of the department responsible for manufacturing is authorized to start production, and instructions are issued to the production planning and control department, specifying quantities, delivery schedules, etc.

6. The technical information is obtained from the engineering department (including drawings, parts lists, specifications, standards, etc.) and passed on to the planning section.

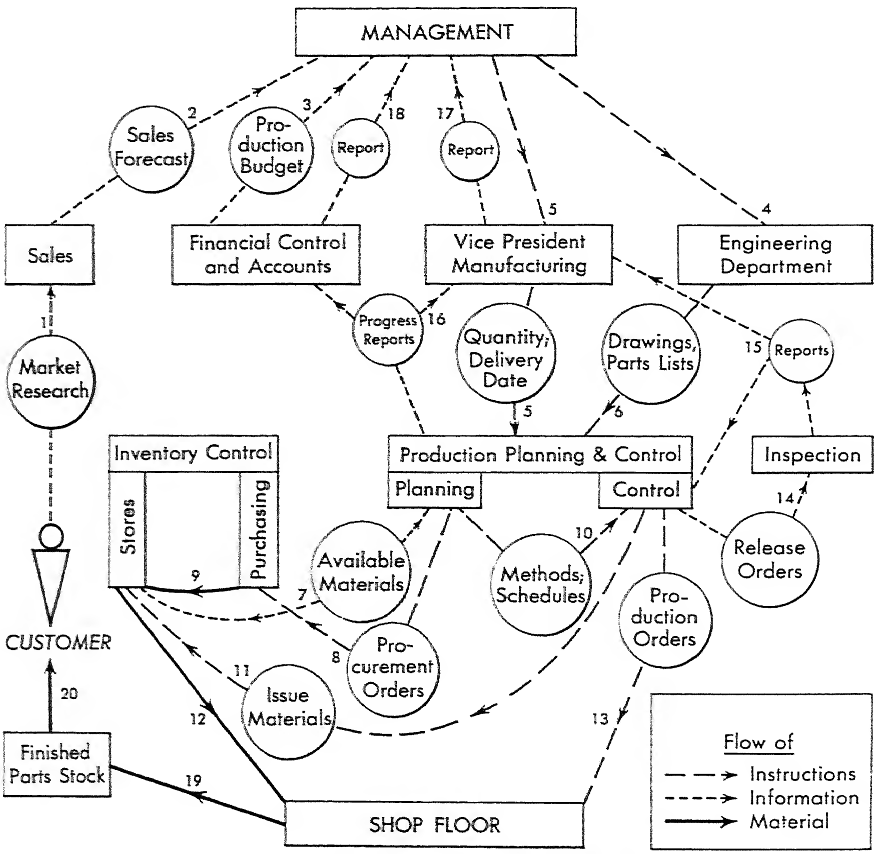

Figure 3–1. ***Production consumption cycle.***

7. One of the first functions of the production planning and control department is to be well informed about availability of materials and expected delivery dates of materials already ordered. Production planning is carried out and detailed schedules are prepared.

8. The inventory levels are checked to determine the orders for procurement of materials and standard parts that have to be issued. Parts and assemblies that are subcontracted are also ordered by the purchasing department.

9. The purchased materials and parts are inspected prior to acceptance and are stored until instructions are obtained to release them to the shops.

10. The production planning section supplies complete data on methods, machine loading, and utilization, as well as production schedules, to the control section for dispatching.

11. The control section releases orders for materials, tools, fixtures, etc.

12. Orders are issued to the shop.

13. Detailed production orders are dispatched to the shop by the production control section, specifying what, how, when, and where operations should be performed. The control functions are carried out throughout the manufacturing period, and progress is constantly compared with the planned schedules so that suitable modifications may be considered and incorporated when required. This necessitates a close and permanent contact between the control section and the manufacturing departments, to facilitate a constant flow of information and instructions.

14. Inspection orders are released. The purpose of quality control during the production processes is to ensure that the specifications as laid down are conformed with. Final inspection of the parts is carried out before the product leaves the shop and moves to the finished parts or products store.

15. Evaluation of the production operations is the main pillar of the control function and has to be carried out both during and after these operations. Inspection reports are one facet of evaluation, and they form the basis for corrective actions in the processes or methods, and sometimes even for modifications in the specifications of raw materials.

16. The production planning and control department reports on the progress of the work to the vice-president responsible for manufacturing. These reports are also studied by the financial control department. The control section also evaluates data obtained from the shops about operation times, idle time of men and machines, causes and effects of breakdowns, trends in the fluctuations of output, etc. Action initiated by the control section as a result of such reports has to be followed up, and its evaluation should also be reported to the vice-president.

17. Management receives interim and final reports from the vice-president manufacturing.

18. Management also receives a report from the financial department, after which a final evaluation can be made.

19. The finished product is transferred (after inspection) to stock.

20. Finally the product is sold to the customer, who, after comparing the product characteristics with those of its competitors and with his expectations, is ready to contribute his views and reactions to market researchers.

It is evident from this outline that the production procedure involves the cooperative and coordinated effort of all the departments of the enterprise. Even when the functions of each department are clearly specified and well understood, the departments cannot operate independently as disjointed limbs. They have to perform as parts of an integrated body, and the purpose of the procedure described above is to specify where and in what form their efforts are required, and what kind of flow information should be constantly maintained.

Coordination of Production Decisions

Numerous decisions have to be taken at the various stages of planning and control, and the task of decision making becomes rather complicated when the issues and variables involved are interconnected and interdependent. Every decision imposes restrictions on the other problems that have to be settled. Also, what may seem good for one purpose may be undesirable from other points of view, and each issue has to be constantly analyzed, keeping the over-all target in mind.

Some of the problems and decisions that are involved are summarized in Table 3–1. On each subject, management is presented with certain facts, and within this given framework the task is to adopt a course of action that will optimize the final result. To achieve even better results, the decision in some cases may include taking steps to modify this framework, even when great expenditure is involved. The subjects under consideration can be classified under three headings: the objective (i.e., the product that is to be manufactured), the subobjectives (components and assemblies from which the product is made), and the means (i.e., the required facilities to manufacture and subsequently assemble these components):

The product

From the preceding production procedure outline, and from Chapter 5 dealing with product development and design, it is clear that information about desirable product characteristics is a prerequisite to any production planning, although at various stages the production departments have to be consulted well before the design is finalized. Each of the items listed as "facts" is closely connected with production issues (such as selection of processes and materials, standardization considerations, equipment loading, and available capacity), and these may in turn affect certain facets of the design characteristics. The main departments which contribute to the basic data under the column "facts" are those of sales, research and development, engineering, and finance. The production planning function includes the problems listed in the column "planning," and the type of the decisions that have to be taken in conjunction with planning are self-evident from the headings in the list and are very briefly described below:

1. The annual or semiannual volume has to be divided into a number of batches, if the production is not of the continuous type. Even in continuous

production it is often found that components or assemblies have to be batch-produced. To be decided: what size batches and in what manner should they be spaced? (Responsibility of the control function in this respect would be to check the validity of the assumptions and data in the light of a postoperational cost analysis and to establish the cause or justification, or both, for discrepancies between these assumptions and the actual data.)

2. What does quality imply in terms of minimum requirements, as far as materials, processes, or workmanship are concerned? (Control function: Evaluate the design specifications after comparison of the quality obtained with the desirable quality.)

3. What possible materials, methods of manufacturing and processes should be considered?

4. How should the product be divided into assemblies? What is the best sequence of operations, and what production departments are involved?

5. How should the individual operations be integrated as a function of time, and what buffer stocks should be specified to ensure continuous flow in the shops? (Control function: How much work in process and why?)

6. Setting the schedules, both the over-all (master) and detailed. (Control function: Why are there queues and bottlenecks, and what is their effect quantitatively? What breakdowns and stoppages contribute to deviations from the schedules? Can they be avoided? If so, how?)

7. How should the product be inspected? What criteria determine whether it conforms to the specifications in its physical or chemical properties and its functional characteristics? Very often, in order to ensure objectivity and fairness, the actual decision about inspection methods is not taken by the production planning and control department but by the engineering or research and development departments. The manufacturing departments are consulted and kept informed about any procedure that is finally adopted. (The control function: Evaluation of inspection results to ensure that immediate corrective actions are taken and scrap is kept to a minimum. Also: What other causes contribute to scrap? How does it affect the scheduled quantities, and should replacement be ordered? How best could the scrap be utilized? What possibilities exist to develop by-products?)

The Components and Assemblies (or semifinished products)

Generally, in manufacturing, several semifinished products are involved in the final assembly or shaping of the product. In the metal, electrical, wood, plastic, and allied industries, these semifinished products are parts or subassemblies; in the chemical or food industries they are the various chemical compounds or ingredients that have to be used for the final reaction or mixing process. The given data about these semifinished products would include design details (or description of properties), functional requirements, and interchangeability specifications to allow the use of the same components in different models (or

Table 3–1

Summary of Planning and Control in the Production Procedure

	Facts	*Planning*	*Control*
I. The objective: *Product*	Sales reports and forecasts (annual or semiannual volume to be produced) Design specifications Product characteristics functional, durability, etc.) Relation to other products or models (in characteristics, price, demand, etc.)	How much to produce in a given period? Translation of product characteristics to quality specifications Analysis of the 3 M's (materials, methods, machines) Division into assemblies Sequence of operations Process charts Planning lead times and buffer stocks Planning master schedule and assembly schedule Delivery schedule and finished product stock How to inspect and inspection orders	Work in process Queues and bottlenecks Breakdowns and stoppages Inspection results Analysis of scrap and by-products Cost control How to evaluate results?
II. The subobjectives: *Components and Assemblies*	Design specifications Functional specifications Dimensional interchangeability requirements Tolerances	Breakdown of annual demand to batches Technological analysis of quality specifications Analysis of the 3 M's What to buy and what to make? Sequence of operations Process charts and routing Operation sheets Schedules for delivery and buffer stocks (including storage capacity) Inspection orders	Work in process Queues and bottlenecks Breakdowns and stoppages Expedite subcontractors Inspection results Quantity control Obsolescence study Analysis of scrap
III. The means (the 7 M's): 1. *Management*	Potentialities and skill Systems of communication and control Adaptability to changing situations	Procedures Redesign of communication and control systems Management training	Evaluation of results

2. *Men*	Labor force Skill Working conditions Place in organization structure	Allocation of operations and demarcation of responsibility Utilization of labor force and hiring of additional production personnel Working methods Training Incentives	Suitability of men to required skills Time and motion study Supervision methods Effects of training and study of learning curves Effect of incentives
3. *Money*	Available financial resources, including credits, etc.	Allocation of resources to products How to convert these resources to other facilities?	Financial control
4. *Matrix*	Limitations of buildings, grounds, etc. Layout	Design of transportation and handling systems Modifications to buildings, etc. Plant relayout	Evaluation of results
5. *Materials*	Specifications (dimensions; quality) Inventories of required materials Availability in the market	Standardization and simplification Determination of buffer stocks Requisitions for purchasing Requisitions for issue Planning acceptance test procedure	Dispatch of orders Expediting Inspection results Storage methods and maintenance in stores Utilization of materials and study of scrap Quantity control of inventories
6. *Methods*	Available processes	Selection of processes for specific operations Sequence of operations Defining methods for execution of work Planning movement of materials and operators Design of production centers and layout	Evaluate processes Time and motion study Evaluate effect of methods on efficiency

Table 3-1—(Continued)

Summary of Planning and Control in the Production Procedure

	Facts	*Planning*	*Control*
7. *Machines and Equipment*	Specifications Capacity Availability Maintenance and breakdown history	Design of tools, jigs, and fixtures Requisitions for purchase and issue of tools, jigs, etc. Estimate times Allocation of machines and priority procedure Machine loading Scheduling Standardization Plan machine replacement	Expediting tools, etc. Studying delays, breakdowns, etc. Machine interference study Machine efficiency Evaluation of schedules

chemical compounds or ingredients for various combinations in the mixing process). The main decisions involved at the planning stage are:

1. How much to produce or order of each component in specified time periods? The production of components may either be rigidly geared to the assembly line, so that the batch sizes of each component strictly adhere to the output of the assembly line at any given period, or production for several periods may be preferable if the economic analysis so justifies. A meticulous study of the cost function of each component is required to establish the effect of quantity. When the effect is negligible, only questions of convenience need be considered in scheduling. Flexibility in scheduling is sometimes so important that, even when the cost functions are sensitive to production volumes, the penalties associated with deviations from the rigidly calculated economic batch sizes must be evaluated before a final decision about quantities can be made. (The control function: study of fluctuations of variables that affect costs and measure of flexibility achieved by deviations from the theoretical optimal batch sizes; analysis of obsolescence of parts due to overproduction or frequent changes in designs, as an important feature of quantity control.)

2. What quality specifications are implied by the design characteristics? What materials or methods of manufacturing may be considered, knowing the technological and quality limitations of these materials and processes? Naturally these technical considerations must be sorted out before economic comparisons of various alternatives or questions of capacity can be tackled.

3. What components should be made by the firm and what should be bought or subcontracted? Questions of cost, availability, promptness, and dependability of supply, as well as quality and best utilization of plant capacity, have to be taken into account in a major decision of this kind. In some extreme cases plants buy all the required parts or assemblies, and their main activity lies in the final assembly line. At the other extreme some plants try to produce all their required components in an effort to be as self-sufficient as possible. This tendency for integration is particularly apparent in industries where dependence on certain components or processes is a crucial issue. In many cases it has brought integration to a stage where even raw materials and their extraction are controlled by the company, to ensure continuity of materials flow. Integrated growth may also be caused by an economic analysis, which becomes more favorable as quantities of production increase, and by quality considerations when it is felt that the firm does not get from the suppliers the quality or consistency in performance that it should.

Most companies fall between the two extremes, and very often the question of what to make and what to buy is not so much a fundamental policy issue as a continuous analysis of how best can the plant facilities be utilized. On the one hand, limitation of the variety of the plant activities leads to specialization and increase of effectiveness, but on the other it makes the plant more technologically dependent on the suppliers. Evidently a healthy balance must be struck between

the two. (Expediting subcontractors and a continuous analysis of cost and quality are within the responsibility of the control function.)

4. Questions of sequence of operations in relation to plant layout have to be settled, after which process charts, routing, and operations sheets can be prepared. When changes in the sequence are possible, effect of machine loading and scheduling has to be taken into account, and this interdependence of sequence and machine loading has to be resolved.

5. Scheduling operations and delivery dates, planning buffer stocks and storage space, and specifying a communication system require perhaps the most crucial decisions at the planning stage. (The control functions: studies of work in process, queues, bottlenecks, breakdowns, and other characteristics of the schedule, as already outlined in connection with the final product.)

6. Decisions about inspection of components resemble those that are taken with regard to the final product. Again, in most cases, the inspection procedure and methods are not laid down by the production planning and control department. (Control functions: Can the specifications, such as dimensional tolerances, strength, and composition, be met economically by the available processes?)

The Means

The facilities or factors that combine to make the product may be grouped under seven main headings, conveniently labeled the 7 M's. These, it should be noted, include the 4 M's (namely: materials, methods, machines and manpower) that are the more specific resources associated with the production planning and control department. While the other facilities (management, most personnel problems, money, matrix) are not within the framework of this department's responsibility, they naturally affect its decisions, which must always be taken with the general background in mind.

1. The skill and potentialities of management of all echelons is of prime importance, if a company is to attain its goals and an increasing rate of effectiveness. Planning includes the *design of procedures and systems of communications* that would ensure effective control. It also covers the field of management training, both of a short- and long-term nature.

2. The structure, working conditions, interrelations, and morale of the labor force are all factors that determine the potential capacity of the organization to perform its task.

(i) The planning of task assignment and utilization of manpower is among the responsibilities of the manufacturing department and the personnel manager, as well as problems of load fluctuations and their consequences, as far as hiring and firing of men is concerned. A constant rate of production employing a constant labor force is desirable on many counts. It facilitates better utilization of men and machines, it saves capital expenditure on equipment, it allows the operators to become more skilled in their jobs, and no time or money is repeatedly lost in training for the job until proficiency is gained. Furthermore

the feeling of security of the men can greatly contribute to their effectiveness. These considerations are particularly acute in an industry with a marked seasonal demand. In terms of full employment it was found that seasonal firing and hiring may lead to situations where the required skills are not available on the labor market, and some form of production smoothing is necessary if the demand at the height of the season is to be met. (Control functions: Are men employed on tasks for which their skill is suitable? Is supervision effective, and how can it be improved? How is effectiveness associated with the length of the production run, or what are the mechanics of the learning curve?)

(ii) The planning of working methods that will ensure better utilization of facilities, as well as make work more convenient and less strenuous, is a production planning and control department responsibility. (Control functions: time and motion study.)

(iii) The planning of working conditions, incentives, and training is a general planning function not usually connected with any particular production run. (Control functions: study of output per operator, study of absenteeism, evaluation of the effect of training on output and on adaptability of personnel to changes in tasks and methods.)

3. Finance planning functions (which are the responsibility of management and the financial department) cover the utilization of the firm's financial resources and include:

(i) How best should the financial resources be allocated in terms of the company's activities? How much should be invested in design and development, manufacture, sales, and promotion of each product or line? What policy should be adopted regarding expansion, buildings, machine replacement, basic research, and training? (Control function: How do the prospects of return on the capital invested compare with investment prospects elsewhere?)

(ii) Once the policy of allocation is determined, what are the best techniques and methods that should be used to convert financial resources to other facilities?

4. The matrix comprises the buildings, grounds, storage and manufacturing space, transportation systems, and the layout as a whole, and these components naturally have their limitations. Too often it is realized that they are far less flexible than one could wish for. Changes and modifications are usually a lengthy affair and must be planned with the total manufacturing load and future trends in mind, rather than for any specific product. Planning of relayout, materials handling systems, expansion schemes, and redesign of buildings are not within the scope of the production planning and control department.

Materials, methods and machines

As already indicated in Chapter 1, the effective utilization of materials, methods, and machines is the responsibility of the production planning and control department. Problems of allocation, balancing, and coordination are the

essence of production planning, and naturally provide the framework within which a multitude of possible alternatives have to be weighed. These are considered as items 5, 6, and 7 in the following paragraphs.

5. The breakdown of the production target volumes by products provides the basic data for specifications of *materials* as to quantity, quality, chemical, and physical and dimensional properties. It is a function of the purchasing department to provide information as to the availability and delivery dates of these materials.

(i) Study of standardization and simplification of materials (a joint responsibility of the production planning and engineering departments), to reduce the variety in the stores, enables the purchasing department to secure more favorable terms when larger quantities are ordered or to reduce the total safety stock that has to be carried, or to effect both advantages.

(ii) If ordering lead times (periods of time that elapse from the dates of orders to corresponding dates of deliveries) are too long, or when they are associated with uncertainties, alternative materials have to be studied.

(iii) A prime decision to be made in materials purchasing is the determination of quantities to be bought and level of safety stocks to be maintained. How should the materials be tested before acceptance? How and when should materials be issued to the shops? (Control functions: use of acceptance testing to rate vendors and to study fluctuations of quality characteristics; study of deterioration and obsolescence of materials in stock, and effectiveness of maintenance of materials and components while they are kept in the stores; dispatching and expediting procedures; quantity control of stock levels.)

6. *Methods* planning falls into three main categories.

(i) Selection of production processes to perform the operations and to determine the sequence of operations takes into account the technical, economical, and scheduling processes.

(ii) Process conditions, such as feeds, speeds, lubricants, temperature, pressure, flow, and operator's motions, must be contained in specifications. (Control function: time and motion study; study of optimal process conditions.)

(iii) Materials handling systems must be designed. (Control functions: Study of flow and bottlenecks and effect of handling on layout and work in process.)

7. In working out the capacity of *machines* and equipment, it is necessary to allow for setting-up times, maintenance, and possible breakdowns. The machine cards include details about previous commitments so that the available machine capacity can be calculated.

(i) Selection of the machine to do the job, planning how to use the machine, what other auxiliary equipment should be used (such as tools, jigs, and positioners), how to feed and how to eject the work from the machine, etc., has to be done in conjunction with methods planning and motion study. Requisitions for the purchase and issue of tools and other aids must also be planned. (Control function: Expediting.)

(ii) Scheduling is preceded by time estimation and machine loading. Scheduling could almost be described as the point of culmination of the planning functions, the stage where all the information and prior planning decisions have to match each other and build into one mosaic, in which allowances have been made for all the time and physical restrictions that have been imposed on the system. This is why in Fig. 1–1 the first feedback is shown after scheduling. If the building stones of the mosaic do not match, or if the over-all picture does not live up to the designer's expectations, previous decisions have to be reconsidered, modified, or completely reshaped. (Control functions: Study of delays, breakdowns, machine utilization, machine interference, and effects of deviations from the initial assumptions on the scheduling model.)

(iii) Planning machine replacement and standardization of the equipment is mainly concerned with the more distant future, although replacement or acquisition of additional equipment may sometimes be associated with specific production runs, and not with policy equipment as a whole.

Departmental Responsibilities

Planning and control of operations and effective utilization of facilities, as summarized in Table 3–1, play an important role at all stages of the production-consumption cycle described in Fig. 3–1. Some over-all planning and control functions are retained by management as its own tools, but most of the responsibility for planning, operation, and control is delegated to the various departments; the time span of this active responsibility is shown schematically in Fig. 3–2. When this active responsibility terminates, the respective departments may become actively engaged in the production cycle of another product, since the production-consumption cycles of the various products are displaced in time in relation to each other. Also, it should be appreciated that not every function mentioned in Table 3–1 is repeatedly exercised for every production-consumption cycle. When a product has to be remanufactured, most of the information and planning associated with the first production cycles of that product can be used in subsequent cycles, so that proceedings prior to manufacturing can be considerably cut down.

Some functions may also be undertaken jointly for several production cycles in the form of periodical planning or evaluation projects, such as layout studies, expansion, and equipment replacement analysis, and these may be carried out either concurrently with active participation in production-consumption cycles, or between them. Figure 3–2 does not include continuous functional responsibilities that are not actively associated with one particular production-consumption cycle but are related to over-all operational effects or general trends in the activities of the organization. These include training of personnel, development of managerial control systems, equipment and stores maintenance, inventory control, working conditions and social benefits to operators, etc.

One of the interesting, and perhaps vital, features of the inverted pyramid in

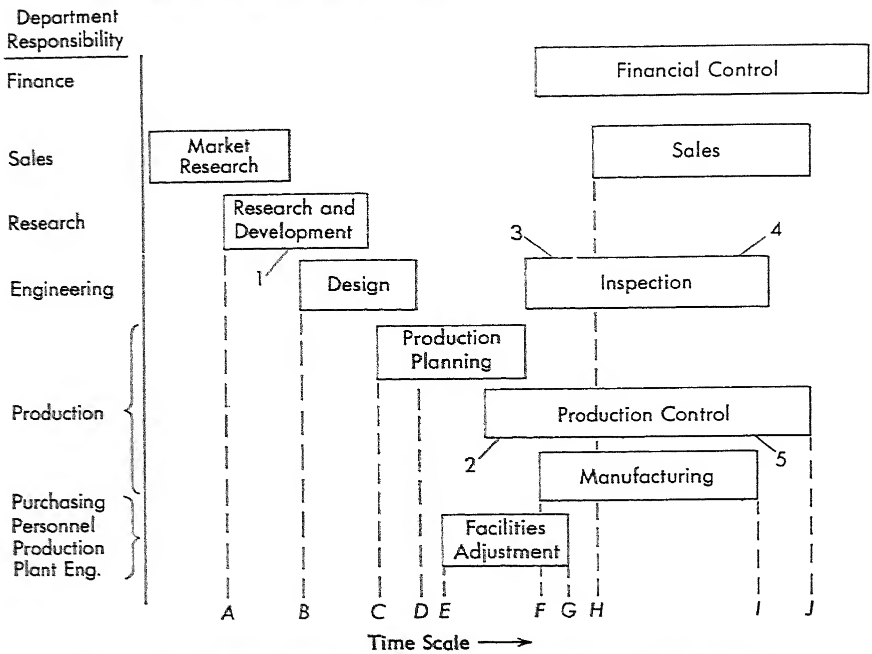

Figure 3–2. ***Time span of departmental active responsibilities.***

A—Research and development is started. Usually this is done after research gives some indication as to the prospects of the product in the market, but sometimes the initiative begins at the research and development department, and market research follows.

B—Order is released by management to start on detailed design of a new product or modification of an existing one.

C—Production planning and control department responsibility begins even before the design is completed.

H—Point where first batch is out of the plant and ready for sale.

A–H—Lead time for research before product becomes available.

B–D—Design period.

B–H—Lead time for design.

F–I—Manufacturing period, including transfer of finished products to the stores.

C–J—Period of active participation of the production planning and control department.

E–G—Period of time required for adjustment of facilities, such as purchasing of materials, tools or machinery, hiring and training of personnel for specific operations in the production cycle, changes in the layout, installation of material handling systems, provision for storage space, etc. This period normally starts soon after the production control section assumes responsibility and ends before the first batch is fully processed through the shops. The function of facilities adjustment belongs to the purchasing, personnel, production and plant engineering departments, each one performing the appropriate adjustments within its own precincts.

1—Research and development period, normally required only for new products.

2—Production planning and control overlap, since requisitions for purchase of materials and tools must be done in time.

3—Inspection of materials begins before manufacturing.

4—Inspection of finished product must necessarily be carried out after termination of the manufacturing process at point I.

5—Production control proceeds with the evaluation function after point I.

Fig. 3–2 is that there is always overlapping of active responsibilities, so that planning, operation, and control follow in a continuous fashion and no gap occurs at any time in the over-all cycle. The left side of the pyramid denotes the successive stages of planning, each stage obtaining processed data from the preceding stage. This accumulated flow downward, so to speak, facilitates the integrated production planning and other preparations to be performed, before the first batch of products is finished. The right side of the pyramid consists of the various departments that evaluate the progress of the operations and report to a higher authority. Here, too, actions of departments at certain stages depend on the analysis performed by the others, but progress and evaluation reports are almost independently made. This is why overlapping is far greater on the right side than on the left of the pyramid, which therefore looks slightly lopsided in shape.

Summary

The production-consumption cycle refers to the successive actions that take place from the time that demand for a product is indicated by market research until this product is purchased by the customer. These activities can be classified into planning, operations, and control functions. The specifications of the objectives (products, assemblies, parts) impose certain limitations on the methods of utilizing the available facilities of the firm, but even with these restrictions, many possibilities have to be explored, and this is the essence of the planning function. Planning should provide answers to the questions: what (the product), how (the method, process, sequence), where (the machine, department), when (the schedule), and who (the operator)? The control function is mainly concerned with the questions: why and how else? Planning and control are management functions delegated to various departments such as sales, research and development, engineering, production, personnel, and finance, each exercising responsibility in its own field. The overlapping of active responsibility ensures proper integration and continuity throughout the production-consumption cycle.

References

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Problems

1. Point out which department in an organization is normally responsible for each of the planning and control functions mentioned in Table 3–1.
2. How does money flow during the production-consumption cycle? Describe with the aid of a time chart the planning, operation, and control functions associated with the financial resources of the firm, and show at what stages they are converted to other facilities and assets.

3. "Management has a responsibility to itself." This maxim sometimes refers to the need of executive training. Discuss this statement.
4. List the means that the firm can employ to attain its targets. Can you indicate whether and how these facilities are interdependent? Should planning of each be carried out separately, or would integrated planning be more effective?
5. List the major facilities of a plant you have visited and discuss their limitations.
6. Explain how feedback information should affect the activity of each department shown in Fig. 3–2.
7. A wide variety of dishes are offered at lunch time in a restaurant; the dishes that appear on the menu may be classified into the following categories:
   (i) Those involving preparation time of 1 hour or more
   (ii) Those requiring about 10 to 15 minutes of preparation
   (iii) Those made from canned food, where the preparation time is practically negligible

   Dishes that have not been consumed in any one day are scrap and cannot be stored for use on the next day. Materials required for categories (i) and (ii) have to be ordered a day in advance, if delivery is required in the morning, while delivery of canned food is usually made in the afternoon. A certain amount of cold storage at the restaurant is available, and the management is prepared to expand this facility, if necessary.

   How would you use production planning and control procedures to:
   (a) Study customers' preferences and demand patterns?
   (b) Determine the number of dishes the restaurant should plan under each category to ensure maximum customer satisfaction and minimum scrap? (Are these two objectives compatible with each other?)
   (c) Outline a materials ordering system and plan materials storage capacity?
   (d) Exercise a control function to maintain a quick and effective waitresses' service?
8. A small manufacturer of hand drills started his business five years ago as an assembly shop. All the components were bought or subcontracted, and the assembly line (originally planned for an output of 25 drills a day) was producing only one model.

   The business has expanded considerably and now there are two assembly lines, one engaged in the production of 200 drills per day of Model I, while the second line produces Models II and III on a batch basis. Their quantities vary from week to week according to the sales department requirements. A winding shop is making all the motors for the hand drills and even obtains orders for motors from other plants, and a press shop is engaged on producing all the required pressings, which are no longer sub-contracted. The press shop is geared to the domestic requirements and does not entertain outside orders.

   These developments necessitate reformulation of production planning and control procedures. How would you set about carrying out this task?

4

ORGANIZATION

We have seen that production planning and control is an essential production management tool that assists the manager in charge of production to achieve his target efficiently, economically, and in the time allotted. The production planning and control department therefore figures very prominently in the structure of any organization. The definition of the department's duties and its scope is, however, by no means universal. The ten basic functions of production planning and control were discussed in Chapter 1, and these (with the exception of inspection, which is discussed in Chapter 19) normally constitute the minimum responsibilities the department is charged with. But we have already seen that there are many additional aspects of production management which closely concern production planning and control and each of its functions, for instance, plant layout, equipment policy, maintenance, work measurement, and even methods analysis. In some establishments, mainly of small and medium size, these additional aspects or some of them are included in the production planning and control department. In others, the department responsibility is confined to the production planning and control basic functions, while other fields of production management are grouped under a separate department, bearing the name of "industrial engineering department," "operations analysis department," "operations research department," "methods department," or "work study department," etc. None of these names adequately describes what the department is supposed to do, as definitions for these terms are not as yet universally accepted. The name "industrial engineering department," which is more in use than the others, is perhaps least suitable for the purpose because production planning and control itself is but a part of industrial engineering.

The demarcation of responsibilities in industry between the two departments varies considerably with the type of production, size of plant, the importance higher management attributes to certain functions, and the effects of personalities in the organization. A broad, though by no means rigid, division of industrial engineering functions is suggested in the accompanying chart.

INDUSTRIAL ENGINEERING FUNCTIONS

Functions concerned with immediate aspects of production	Functions concerned with evaluation of means and methods and with long term planning
Production Planning & Control	*Methods Engineering*

Materials: records, availability, procurement, storage, issue, control.
Methods: confined to choice from available facilities for manufacture of given products; tool and jig design.
Machines: specifications, availability, loading.
Routing, **Estimating**, **Scheduling**, **Dispatching**: all responsibilities as enumerated in Chapter 1.
Inspection: only concerned with inspection results, but not with its execution.
Expediting: all responsibilities.
Evaluation: concerned with immediate lessons derived from production control, but analysis of data for long-term planning is performed by the Methods Engineering Department.

Motion study: operations analysis (method study), micromotion study, work-place layouts.
Process evaluation: comparison of processes, new processes.
Machines: equipment policy, maintenance, and renewal.
Layout: flow of materials in the plant, location of machines and departments, materials handling systems, expansion plans.
Quality control: inspection, testing laboratories, cost of quality.
Standardization and simplification: of product, methods, machines, auxiliary equipment, recording systems, procedure.
Safety: instructions for safe handling of materials and operation of machines.
Incentive schemes: wage incentives, other incentives.
Suggestion schemes: means to encourage operators' contribution to improvement.

The purpose of a healthy organization structure is to provide:

1. A system for collecting and recording up-to-date facts
2. An efficient communication system, to facilitate flow of instructions downward and flow of information upward and sideward
3. A smooth and efficient procedure of operation
4. A demarcation of authority and responsibility, which clearly specifies the mechanism of facts, evaluation, and decision making

A typical organization chart is shown in Fig. 4–1. It should be emphasized, perhaps, that such charts are by no means universal and that there is a wide scatter of variations in organizational patterns as found in industry. The organizational structure grows and develops in relation to the past, present policies, and future plans of the firm. It is the outcome of an evolutionary process;

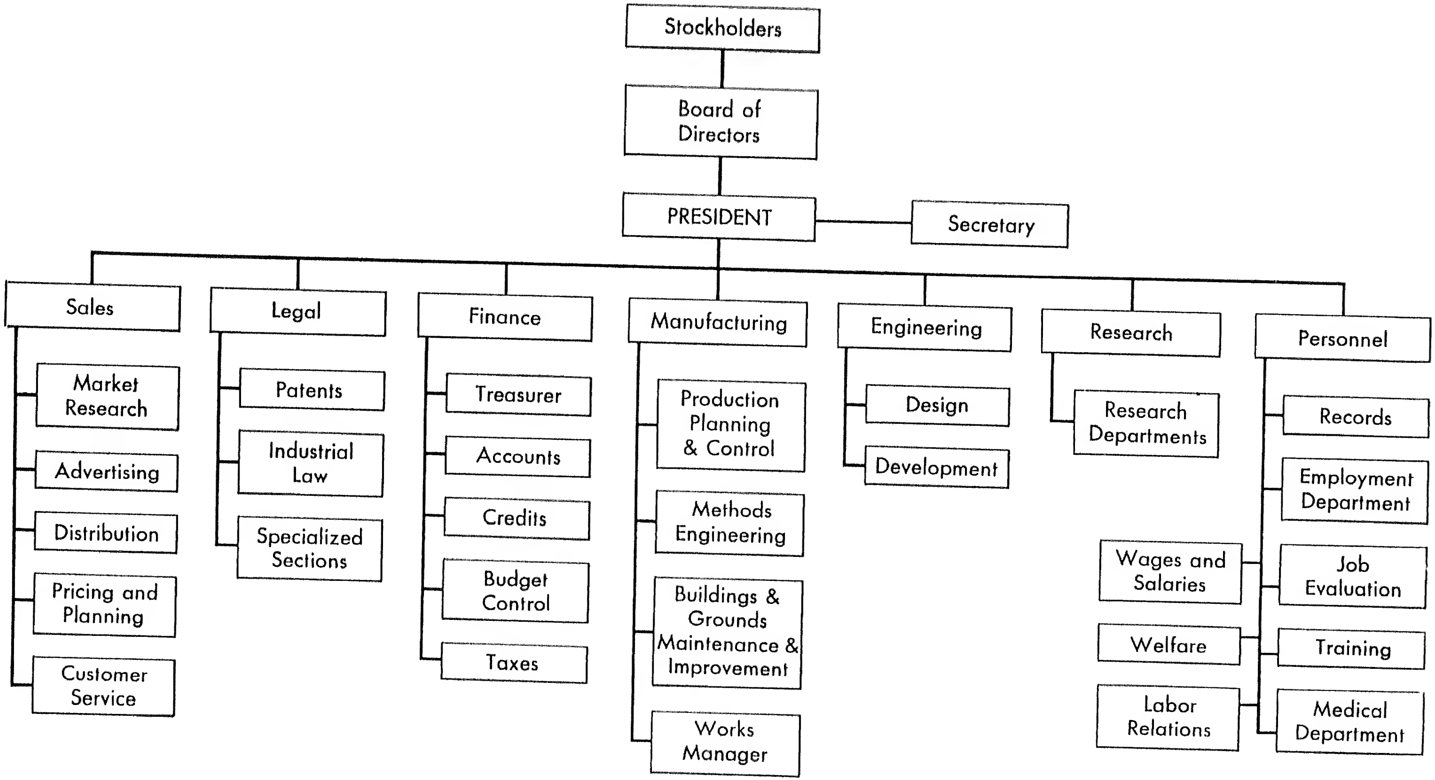

Figure 4–1. ***A typical organization chart for a manufacturing firm.***

it changes with circumstances and personalities; it can seldom be imposed at the outset and rigidly maintained thereafter. The chart in Fig. 4–1 shows a structure commonly found in medium size plants, where the production planning and control department is directly answerable to the vice-president in charge of manufacturing.

The internal organization of the production planning and control department normally follows the functional pattern described in Chapter 1. The department is headed by a senior production engineer, who is responsible for all the planning and control tasks connected with production and for the proper coordination of the various functions in order to ensure that the shops are provided with all the available instructions and facilities.

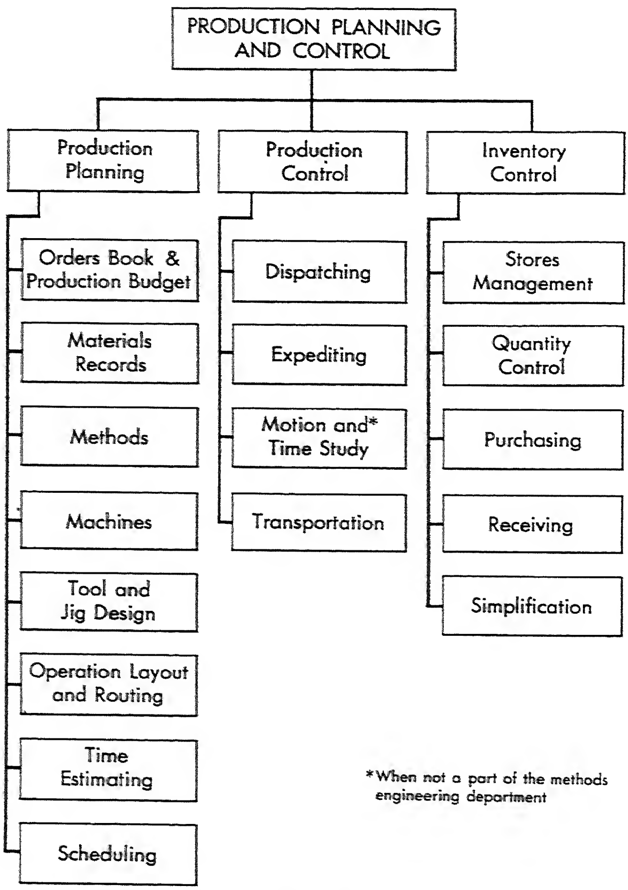

Figure 4–2. ***A conventional organization chart of the production planning and control department.***

Departmental Sectionalization

The planning and control department normally consists of three sections (see Fig. 4–2).

1. The planning section, the head of which is in charge of all the planning functions
2. The control section, the head of which is concerned with all control functions from the word "go"
3. The inventory section, which deals with all problems connected with materials and their requisition and storage

Production planning

This section includes eight distinct activities.

(i) *Production budget office*, where the incoming orders are obtained and recorded in the order book and where the budget requirements in connection with the execution of the order are worked out. Delivery dates are estimated after the planning of schedules is completed.

(ii) *Materials records office*, where planning engineers can obtain information about materials available in the stores, so that action can be taken to "freeze" or allocate the required amounts and to sanction the purchase of those materials which are not available. Allocation of materials in the stores is an important step at an early stage of the proceedings, as stocks of materials are liable to change before production is due to start, and if the required materials are issued in the interim period for other purposes, a chaotic situation may arise. Precisely for the same reason, records have to be kept up to date, to ensure that the picture they represent is realistic and reliable.

(iii) *Methods planning*, whose responsibility is to assess the potentialities of available processes and select the most suited for the production of each component of the product. Methods engineers also decide how the product should be divided into assemblies, the sequence of operations for each part, and the methods of assembling. Methods planning is an important job, as it lays the foundation for all subsequent planning and control activities. The methods engineer must be conversant not only with the multitude of manufacturing processes that are in use, their merits and limitations, and their technological and economical significance, but he must be also familiar with newly developed processes and new materials that are introduced into the market practically every year. Obviously these require that he knows fundamental motion-economy principles, in order to ensure that his prescribed methods are basically sound, workable, and efficient.

(iv) *Machines records*, from which information can be obtained on the following questions:

(a) How many machines are there? What is their accuracy, their range of speeds, feeds, etc.?

(b) What maintenance or over-all repair schedules do they have?
(c) What is the frequency of breakdowns (from past experience)? What alternatives are available in case of a breakdown?
(d) From past method studies, at what percentage of efficiency (actual working time ratio to over-all time available) do these machines normally operate?
(e) In case of multimachine supervision by one operator, what percentage of interference may be expected?
(f) What net production capacity do these machines have?
(g) What load has already been scheduled on these machines and hence what available time do they have?

(v) *Tool and jig design office*, where all auxiliary aids are planned and designed in meticulous detail. Tool engineering is a specialized trade. It requires a thorough knowledge of the production processes employed and of tool materials and their treatment. Understanding of the design of the product and its functional scope is needed for proper jig and fixture design, which can result in great economies of time and effort on the part of both workers and supervisors.

Tools, jigs, fixtures, mechanical handling systems, and other mechanical aids to production crystallize and finalize production methods to a certain extent, and must therefore be very carefully and painstakingly planned. This means, however, that design, manufacture, and inspection of tools and mechanical aids may result in a substantial lead time required at the planning stage. Mechanical aids, especially jigs and fixtures, can sometimes be re-used, either in whole or in part. In such cases not only the cost is reduced, but lead times also can be considerably shortened. This aspect of repeated use of production aids demands much thought and careful planning of their functional flexibility at the design stage. A large number of components, from which these aids are constructed, can in fact be standardized and constantly kept in stock so that they are readily available when a new fixture, positioner, etc., has to be designed or assembled.

Advanced standardization of mechanical aids requires great skill and ingenuity on the part of tool and jig designers, especially when the plant is engaged in manufacturing a variety of products on a batch production basis and when short lead times for planning are desirable in order to ascertain short delivery dates.

(vi) *Operation layout and routing office*, which is responsible for expressing the production plans in a form understandable to those who have to carry them out. Process charts are translated into route sheets and operation sheets, and the operations are described in great detail. All the tools, jigs, and fixtures that should be used for each operation are specified.

(vii) *Time estimating office*, where operation times are worked out from the given data in the operation sheets. These times include:

(a) Calculations of actual production times based on speeds, feeds, etc.

(b) Nonproductive times, which cover chucking or loading, setting, and unloading of the machines
(c) Times for additional tasks of the operators, during which the machine is not effectively employed
(d) Allowances for delays, stoppages, interference, personal fatigue, etc.

Here, too, standardization of times for tasks and allowances, based on experience and past records, is very helpful and time saving. Time estimators have to be conversant with the processes and methods employed, and they must be proficient in compiling standard time data and their application. We have already seen in Chapter 1 that time estimating is an essential link in production planning between routing and machine loading.

(viii) *Scheduling*, where machines are loaded against their available capacity and all the planning details and calculations are integrated into a final sequential pattern of target forecast, which sets the pace for production activities and their coordination. Data for commencement of each operation and for its completion can be specified and an assessment can be made on the plant over-all available capacity, delivery dates, effects of new orders on schedules, and on the length of the queue of production orders.

Production control

The functional responsibilities of production control have already been discussed in Chapter 1. As suggested by Fig. 4–2, the section covers the following:

(i) *Dispatching office*, which is responsible for the release of production orders. The dispatchers have to be acquainted not only with the job but also with the men in order to ensure that tasks are smoothly allocated and properly understood.

(ii) *Expediting center*, which has to maintain, with the aid of expeditors or progress men, an effective communication system between the shop floor on the one hand and the scheduling office on the other, and which regulates materials flow in the shop and secures conformity between plan and practice. This communication system can mark the success or failure of the schedule, and it therefore calls for alertness, skill, and a thorough knowledge of the details of the schedule and flow of materials.

(iii) *Motion and time study*, where working methods are studied and improved and work is measured. This section really belongs to the methods engineering department, but in industry it is sometimes found in various forms as part of the production planning and control department. Motion and time study includes:

(a) Recording of work methods (by process charts, layouts and string diagrams) and developing better methods
(b) Micromotion study for highly repetitive work (by motion picture cameras, chronocyclegraphs, analysis into elemental motions) and for developing simpler motions

(c) Measuring work, either for correlation with the estimating function or for setting standard times for existing operations or for newly developed ones
(d) Collecting, assessing, sorting, and standardizing time data for future reference in estimating
(e) Training operators to use new techniques and methods correctly and teaching them the principles of motion economy

(iv) *Transportation section*, which is responsible for the movement of men and materials within the plant and to and from it. It includes the movement of materials between stores, from the stores to the shop, and within the shop, and therefore transportation tasks have to be performed in close liaison with expediting. Other transportation responsibilities cover receiving of goods or materials and shipping of finished goods.

Inventory control

This department comprises five main sections:

(i) *Stores management*, which includes storekeeping, records keeping, maintenance of materials in store and issue of materials.

(ii) *Quantity control section*, which is responsible for keeping and studying records of inventories and prescribing methods for keeping stocks in control and specifying stock levels and batch sizes for ordering.

(iii) *Purchasing section*, which issues purchasing orders to vendors and follows up past orders. This section also maintains records of vendors, catalogs, technical and price information, and data about the vendors' reliability with respect to quality and delivery dates.

(iv) *Receiving section*, where goods are received and checked to ensure that they conform to the details given in the order with regard to specification and quantity. Specifications are checked by acceptance quality tests, which may include dimensional inspection and tests for composition, hardness, strength, electrical or heat conductivity, or other physical and chemical properties. Quantity checks include counting or measuring weights or volumes received as well as recording breakage, spillage, or scrap, so that appropriate claims can be made either to the vendors or to insurance companies.

(v) *Simplification section*, where problems relating to variety reduction are studied. Simplification and standardization is perhaps a fundamental function of stores management, but when many items are involved, especially when parts are both produced by the plant and ordered from subcontractors, problems of simplification and standardization very often justify a separate section to deal with them.

Variations on the conventional pattern of organization

Organization of the production planning and control department greatly depends on the type of establishment and its characteristic problems. Even in

similar establishments certain sections may vary in size and position in the organization, owing to different attitudes of management toward the importance and scope of some functions. Variations of the conventional pattern described in preceding paragraphs are mainly concerned with four sections.

Motion and time study

We have already mentioned that motion and time study should logically belong to the methods engineering department, i.e., in establishments where such a department exists. However, even when motion and time study is considered a part of the production planning and control department, it is still debatable whether it should be under the control section, as shown in Fig. 4–2, since motion and time study has both planning and control aspects. However,

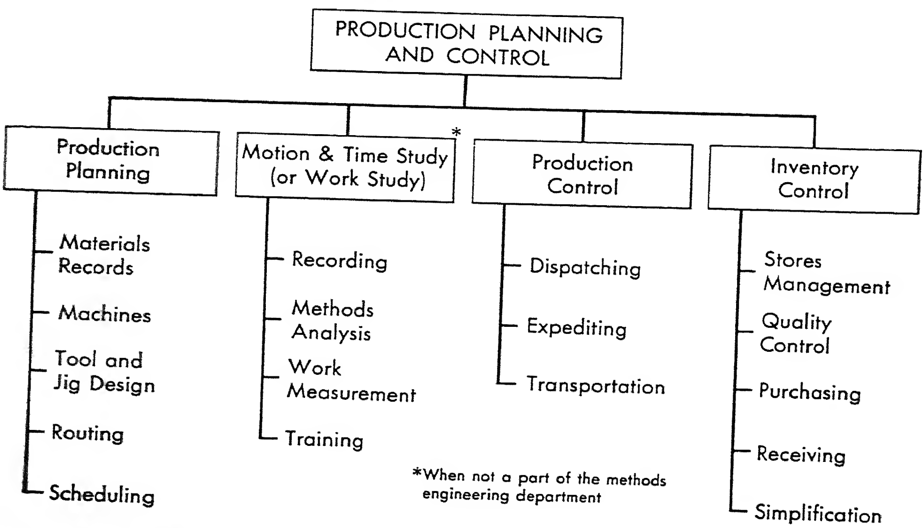

Figure 4–3. ***Variation on Fig. 4-2: Motion and time study as a separate section.***

the common viewpoint today is that motion and time study should be applied throughout the planning period, even at the design stage of the product. Motion and time study can therefore be quite commonly found as separate sections, incorporating both methods and estimating, as shown in Fig. 4–3, and may even include layout problems. In some large firms it goes beyond the conventional scope of motion and time study and incorporates analysis of over-all layouts, plant location, or optimization of load distribution between several plants or shops—problems that require operations research techniques.

Standards section

A separate section to deal with standardization and simplification (Fig. 4–4) of products, materials, equipment, and tools is particularly useful when the firm

consists of a number of plants. The standards section operates at the head office. It initiates meetings on standards topics in which designers, methods engineers, purchasing personnel, stores managers, and salesmen take part, and it issues reference sheets and standards volumes for the use of all concerned.

Transportation section

While internal transportation is closely associated with expediting, external transportation problems are often more detached in character. When external transportation involves additional major responsibilities, a separate transportation section can be organized on the lines shown in Fig. 4–4.

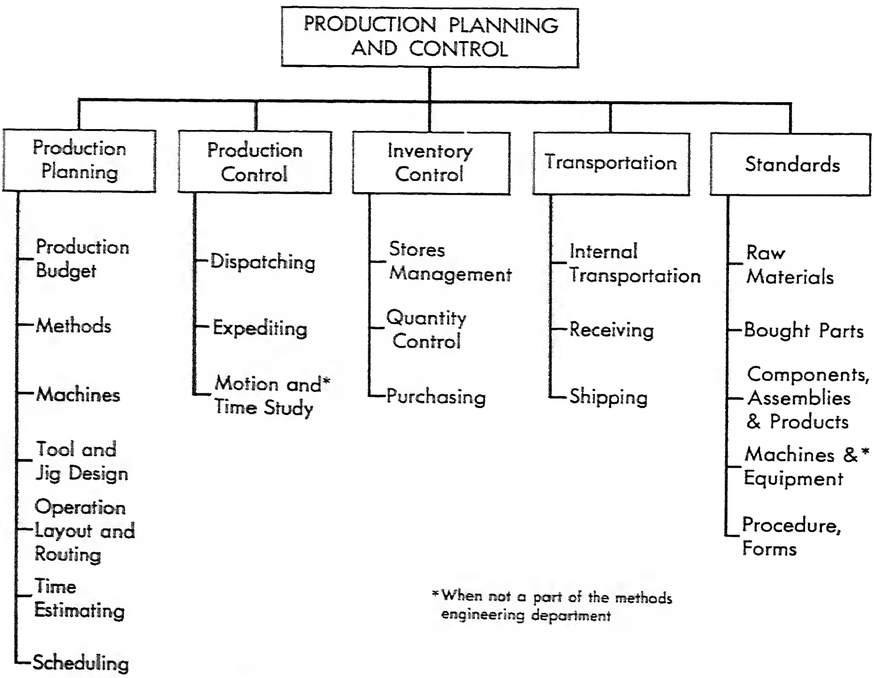

Figure 4–4. ***Variation on Fig. 4-3: Transportation and standards section in the organization chart.***

Central statistics office

In the conventional organization any data on available resources (materials, machines, men), capacities, loading, and scheduling are distributed among several sections, and any adjustments in the schedules, forecasting delivery dates, and processing of incoming orders may become complex problems in coordination. Organization and procedure can be greatly simplified when all records are

kept in a central statistics office, in which scheduling is also performed. The responsibilities of this office (see Fig. 4–5) include:

Records of available materials in stock
Records of available tools, jigs, fixtures, and other aids
Records of machines and their specifications and capacities
Machine loading cards, from which available time can be deduced
Scheduling
Production budget
Processing of incoming orders

The last two functions can be divorced from the central statistical office.

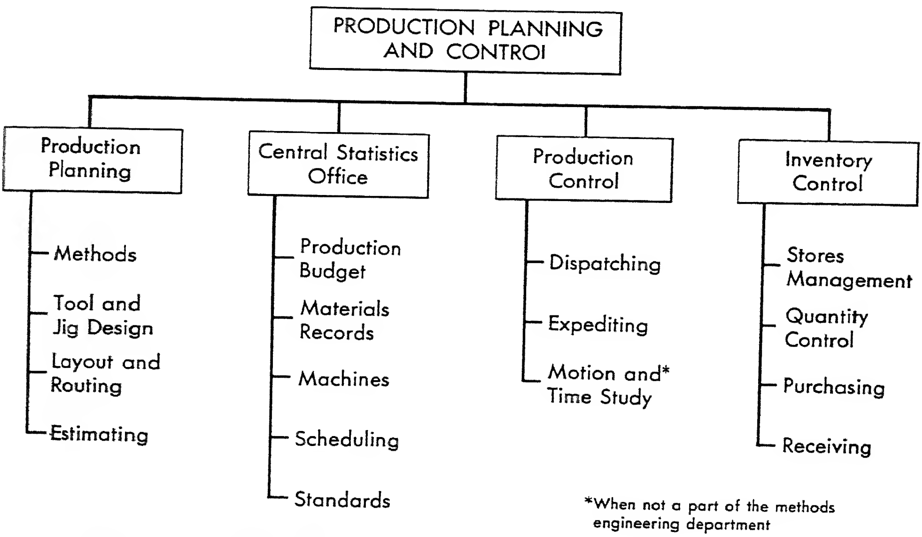

Figure 4–5. ***Variation on Fig. 4-2: The Central Statistics Office.***

The paper work at a central statistics office can be greatly mechanized (see Chapter 16), especially in larger establishments, and this is another advantage for the centralization of record keeping. In the production planning and control procedure, the CSO (central statistics office) has the following functions:

The CSO (or the production budget section) receives and records the orders.
It passes the orders to the methods section, where the material specifications are checked and approved.
The CSO checks availability of materials.
It notifies the stores about materials allocations for specific orders.
It requests the purchasing section to order materials not available in sufficient quantities (or available but already allocated for other orders).
It checks capacities and availability of machines and supplies the information to the planning section.

It obtains process charts, routing charts, and operation sheets with operation times, from the planning sections.
It performs the machines loading on the machine capacity cards.
It plans the schedules.
It specifies delivery dates and studies the orders queue.
It passes information and all instruction sheets and documents to the control section.
It adjusts and modifies the schedules and machine loading cards in conjunction with the production control section.

A well-organized CSO is a great asset to production planning and control. It coalesces the department activities. It facilitates the efficient execution of one basic function: that of collecting vital facts about the plant performance and potentialities, without which no intelligent analysis and planning can take place. It digests the enormous bulk of information about production that pours in and provides management with the basic data required for decision making.

Centralized and Decentralized Production Planning and Control

In multiplant establishments the organization of the production planning and control functions presents several difficult problems. On the one hand all the activities of production planning and control have to be coordinated in order to attain unity of purpose as expressed by top management policy. This is reflected in management planning with respect to budgeting, allocation of facilities, expansion, and plant renewal policy. It becomes particularly important when production schedules have to be coordinated, especially when the end product of one plant is fed to another. On the other hand, production planning and control functions are closely related to the production activities of each plant, and it is neither efficient nor practical to rely on "remote control".

These two aspects have essentially led to two different systems for organizing production planning and control in multiplant establishments: a centralized and a decentralized organization, as shown in Figs. 4–6 and 4–7. Apart from the natural desire of individual plants to be self-sufficient, and have a complete production planning and control organization of their own, versus the tendency on the part of some top managements to overcentralize in order to have everything "under control," there are obviously objective arguments pro and con each type of organization, and the arguments will depend on the particular case that is being considered. But even under circumstances of full plant autonomy, some form of centralized planning is necessary to provide a master production schedule and to set performance targets. Those functions of production control which are associated with everyday problems of dispatching and expediting (and even some aspects of evaluation) are vital in the plant, even in a centralized organization setup. These aspects are clearly shown in the organization charts in Figs. 4–6 and 4–7, and between these two, a working framework can be constructed to suit the particular needs of various multiplant establishments.

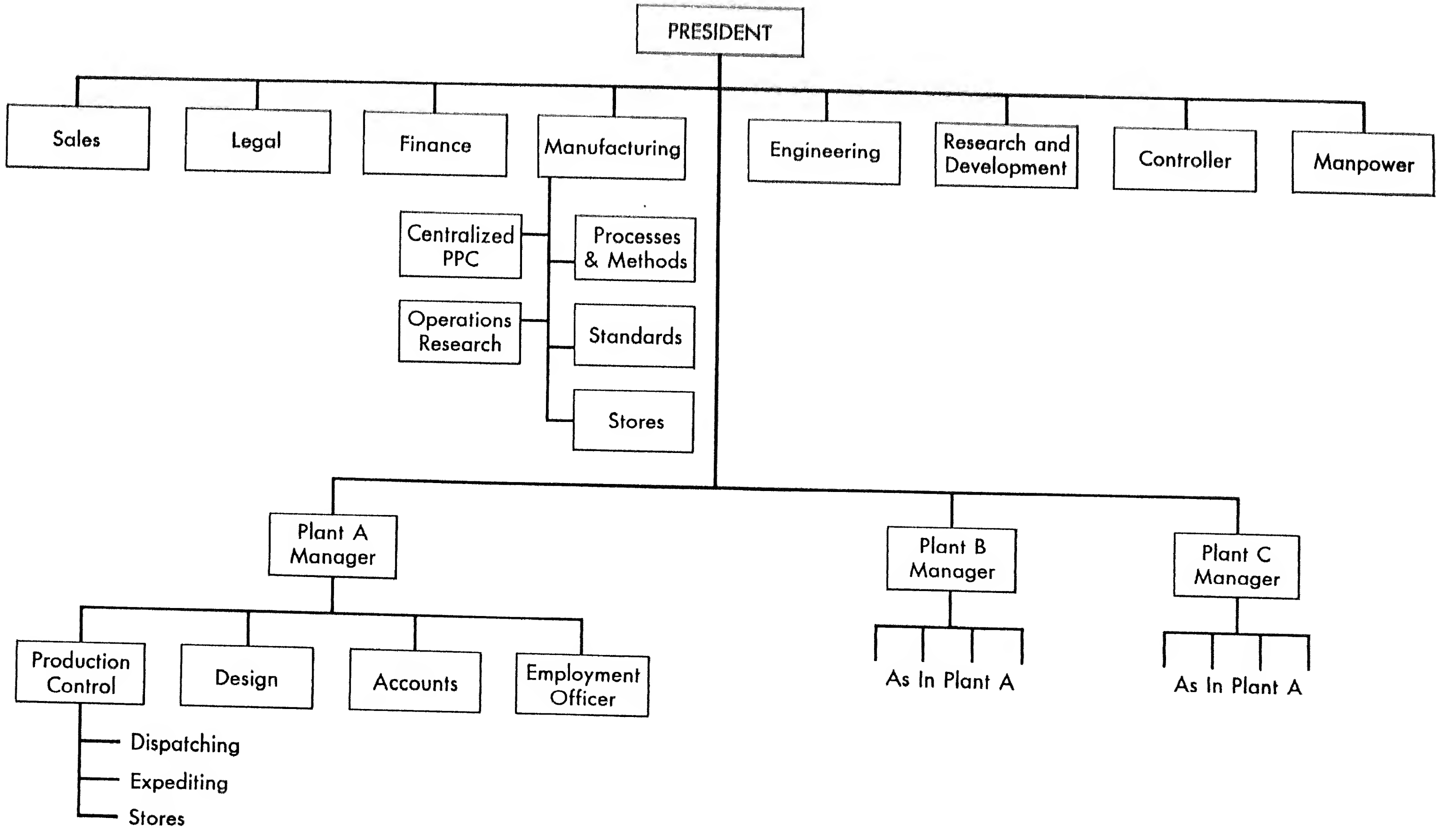

Figure 4–6. ***Centralized production planning in a multiplant organization.***

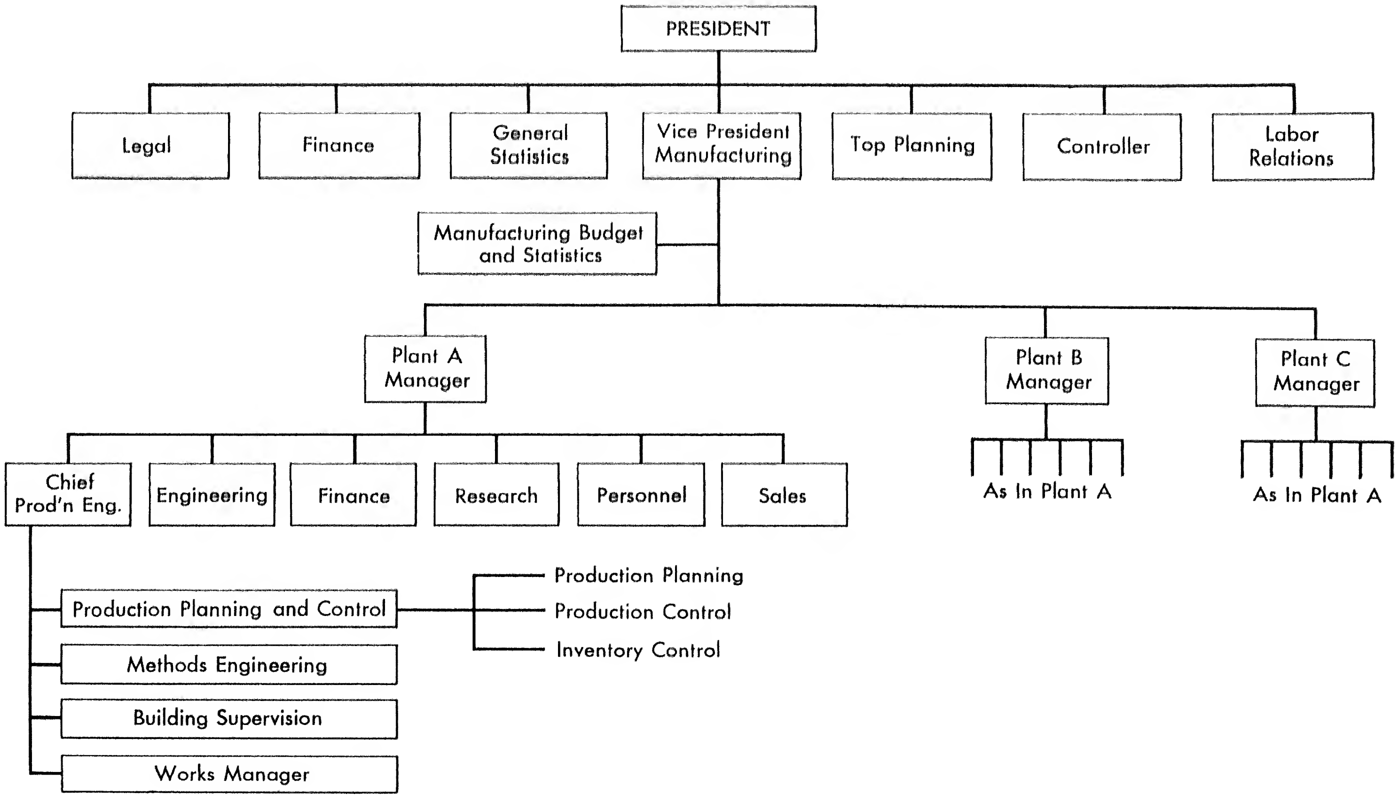

Figure 4–7. ***Decentralized production planning and control in a multiplant organization.***

Summary

Industrial engineering functions can be classified into two main categories: those concerned with immediate aspects of production operations (assigned to the production planning and control department) and those particularly associated with evaluation of methods of manufacture. Accordingly these functions are normally divided between two separate departments, the demarcation of responsibilities being greatly dependent on many factors, such as: the dynamics of process evaluation, the type of association that exists between time study and estimating and rate fixing, and the complexity of manipulating and processing data pertaining to production operations. Sometimes, therefore, the production planning and control department includes—in addition to the sections of production planning, production control, and inventory control which it normally consists of—such sections as motion and time study (for instance, when a separate methods engineering department does not exist in the plant), standards, transportation, and a central statistics office.

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Problems

1. Discuss the position of motion and time study in the organizational structure of a manufacturing firm.
2. Is it possible to divorce work measurement from method study? Is it desirable?
3. What difficulties would you envisage in an organization where inventory control is not answerable to production planning and control but to the finance division?
4. Problems of manpower, skills, supervision, and human relations were not included in the chart shown in Fig. 4–2. Should you infer from this that these subjects are of no interest to production planning and control engineers?
5. Suggestion schemes were shown as a responsibility of the methods engineering department. Suppose a modification of a certain tool is proposed by an operator, and bearing in mind that tool design is a responsibility of the production planning and control department, what procedure would you propose to ensure a proper flow of information and that the suggestion receives the attention it deserves?

6. What are the purposes of good organization? If these purposes are universal and can be achieved by a certain organizational structure, how can these facts be compatible with the statement that organizational structures are not universally applicable to all similar firms?
7. Discuss the functions of a standards section and its place in an organization chart.
8. A production planning and control department in a firm is organized on the lines suggested in Fig. 4–2, but the production engineer in charge contemplates the introduction of a CSO. He has asked you to perform the following tasks:
   (i) Prepare a memorandum explaining the merits of a CSO and showing how it is going to simplify the department procedure and speed up its operation.
   (ii) Outline how the changeover is going to take place and define the responsibilities of each section in the new organization.

5

PRODUCT DEVELOPMENT AND DESIGN

Product development and design are closely allied to the preliminary stages of production planning. When a new product is projected, the designer has to bear in mind the available resources of the plant and the possible implications of the plant having to acquire, modify, or substitute existing machines and equipment or subcontract various components to other suppliers. It is therefore obvious that product development and design is at the core of the development and growth of the manufacturing plant and its departments. This is why product design is one of the fundamental elements of management policy, and its features should be coordinated with the main targets management sets itself to achieve.

Company Policy

What is the product policy of an organization, and how does it affect the design of the product? Surveys in industry, by use of questionnaires and interrogation of key executives of manufacturing firms, have revealed that there is no such one policy for all concerns. One large chain of department stores aims at offering commodities to the public at a minimum price, whatever the quality. In fact one may be quite sure that articles bought at this firm cannot be obtained cheaper anywhere else or even at the same price. The company bases this policy on the assumption that the type of goods it offers need not have a very long life and that, if sold cheaply enough, the volume of sales is likely to be very large, so that even a very marginal profit per unit will lead to substantial gains. Other companies (for instance, in the aircraft industry and many precision instrument makers) define their aim as maximum quality, whatever the cost.

In some cases the need for safety is so paramount, that cost is of secondary importance. In other cases, precision and prestige play an important part, as in the case of a reputable manufacturer of automobiles, whose main business is now in the aero-engine field. The firm produces what may be considered the best automobile in the world, but the number produced per annum is rather small,

and in spite of the high price of the finished article, the firm cannot expect to yield a very high profit from this line of business alone. Another automobile firm, on the other hand, aims at large volume production of a low-priced car that will compete with more expensive models by having some of their successful and popular features and innovations. Most companies, however, say they aim at striking a satisfactory balance between very high quality and a reasonable price. Others go further and endeavor to improve this balance in favor of the customer, by improving quality and leaving the price unchanged or by improving production methods and offering the same quality at a reduced price.

In inflationary times when a decrease in price is not possible, the aim may be toward curbing the rate of increase in the product price to that below the general rate of increase in the cost of living or the cost of similar products. Other companies wish to increase the range of applications of their products, hoping in this way to attract wider consumer appeal and increase the volume of their sales.

Effect of competition on design

Whatever the proclaimed policy of the organization, it is in fact a definition of the tactics by means of which the company hopes to achieve a more fundamental aim; to be competitive. Being competitive implies that special features of the product are offered so that potential customers will be persuaded to exchange some of their money for the privilege of owning or using the product, even if by doing so they are deprived of purchasing power for other attractive commodities.

The fact can never be overstated that competition is not confined only to articles of the same class performing the same or similar functions. An industrialist with limited resources who wishes to change some of his equipment may be torn between the desire to buy a new arc welding or a shaping machine. By having to make a choice between these two, the industrialist has to make a decision in a situation where the arc welding and the shaping machines are in direct competition with each other. There are many aspects to this competitive situation over which the manufacturer has little influence (such as the technical condition of the existing equipment and the amount of maintenance it involves, or the expected increase in productivity of the proposed new equipment compared with the existing one, or some financial aspects; e.g., the industrialist preferring at this stage to invest in the cheaper of the proposed new items, etc.). However, competitiveness should be measured against the *general* background of consumption and not compared only with successes of direct competitors.

Long-range planning

The importance of product development and design for long-range planning by management is further emphasized by the period of time that elapses from

the inception of the idea for the new design until production starts. Some surveys in industry revealed remarkable figures for this "incubation" period:

Automobile bodies	2 years
Automobile engines	4–7 years
Radios and television sets	6–12 months
Specialized welding equipment	6 months
Telecommunications equipment	4 years
Aircraft	10–15 years
Household equipment	2 years
Metal-cutting equipment	4–5 years
Shipbuilding (depending on size of vessels; special shipbuilding designs may be developed over several years)	6–12 months (1–2 years in design and experimentation alone)
Fashion	Several weeks

In defense projects, where development and design are rather lengthy because of the complexity of the problems involved, some designs may become "obsolete" even before their production has begun, as new models are being hammered out on the drawing boards and in the testing laboratories.

Product Analysis

Many factors have to be analyzed in connection with development and design, factors varying in character and complexity, factors affiliated with different fields in production and industrial engineering. Some of these may be grouped as follows:

1. Marketing aspect
2. Product characteristics
   - (i) Functional aspect
   - (ii) Operational aspect
   - (iii) Durability and dependability aspects
   - (iv) Aesthetic aspect
3. Economic analysis
   - (i) The profit consideration
   - (ii) The effect of standardization, simplification, and specialization
   - (iii) The break-even analysis
4. Production aspect

All these factors are interrelated and each presents many issues that have to be carefully considered, as indicated by Fig. 5–1. Market research may guide product engineers in their work to improve existing products or to develop new ones. The design and its characteristics have to undergo an economic analysis and must be studied in the light of available production facilities and techniques. A costing analysis is naturally dependent on the sales volume; hence the suggested design has to be re-evaluated by market research so that a sales forecast

can be worked out. This expected sales volume provides the basis for a further study from the production methods aspect, and the economic analysis has to be rechecked and perhaps modified. Thus product development and design is an

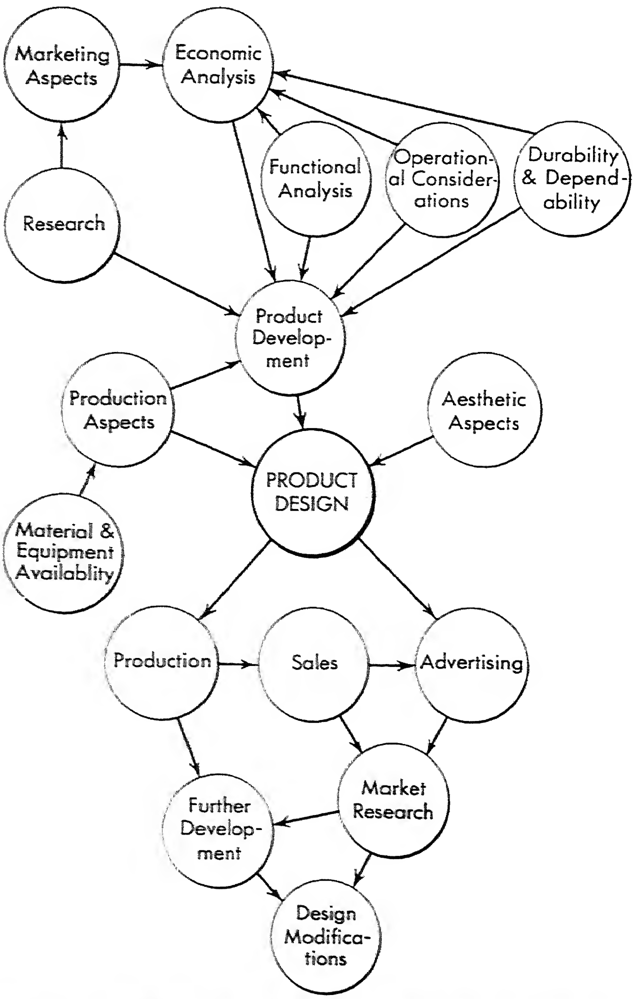


Figure 5–1. ***Some interrelations involved in product design.***

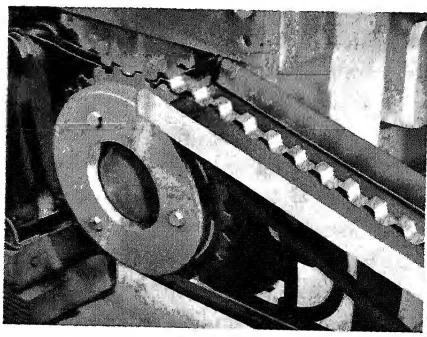
excellent example of interdependence of a multitude of factors that have to be reconciled and integrated into a final composition.

Marketing aspect

First it is necessary to establish that the proposed product will satisfy a demand in the market, that what it is supposed to do and the services it can offer are

both desirable and acceptable. If no consumption is envisaged, there is no point in proceeding with product design.

The demand for the product in the market may already exist, and its volume can then be assessed by consumer research and sales figures for identical or similar commodities. Demand can also be created with the introduction of a new product, either by filling in a gap in the market or by offering new properties, such as novelty, appearance, or some other specific merits (see examples in Figs. 5–2, 5–3). The volume of such a demand is more difficult to forecast. Market research is a useful tool in these cases, but experience and sound judgment are required to evaluate and apply the results of such research, and in some



(a) Manufacture of a corrugated strip by rollers

(b) Bending with heated pliers

(c) A lightweight beam that stands easily to stress

Figure 5–2. ***Honeycomb lightweight sandwich for aircraft construction*** (Courtesy Hawker Siddeley Group, Britain)

cases a certain amount of speculation is inevitable. We shall discuss some problems connected with market research in the next chapter.

The volume of demand is a function of a multitude of factors, some of which are closely related to local conditions and are sometimes difficult to define or measure. It is therefore essential for an enterprise to keep in touch with the market and "feel" its trends, especially when this market is remote and different in character from the local one. This is of particular importance to firms depending on export markets for the distribution of their products.

If we analyze, for example, the case of an American manufacturer of automobiles, we shall find that the percentage of output destined for export is rather small, and design policy would therefore be mainly dictated by American tastes and preferences. A British manufacturer, however, who sells a substantial proportion of automobiles outside Great Britain, has to watch carefully the trends in export markets in order to try and amalgamate the requirements and tastes of the various foreign and home markets in an acceptable design.

Another pertinent question related to product design is: Should the customer get what he wants or should he be offered what he is supposed to want? Basically this is an economic question. If management wants to achieve maximum satisfaction and sets itself as a target to supply the customer with what he wants, it may be faced with the possibility of having to produce an infinite variety of models to suit every taste. On the other hand, were management to ignore the customer's wishes or to maintain that he does not really know what he wants and should therefore be told what is good for him, the designer's task would become far simpler, but the sales department would have to face an unpredictable market.

In practice, product design is a result of some sort of compromise between infinite variety on one hand and the designer's concept of the ideal design on the other. In order to try selling this compromise to potential customers, management resorts to an advertising campaign the policy of which is dependent on the characteristics of the "compromised design" and on how far it conforms to, or differs from, the expressed desires of the market to which such a campaign is directed. Generally, the main objective of advertising is to expand the market, this being achieved by:

- Providing general information about the existence of the product.
- Providing technical information about its functional characteristics or utilitarian purposes.
- Drawing the customer's attention to those attributes of the product which he wants.
- Winning undecided customers by exhibiting possible attractions (such as color, design, novelty, and price) that may persuade him to prefer the product to one offered by competitors.
- Creating demand among a passive population of customers.
- Educating the customer, or telling him what he should want.

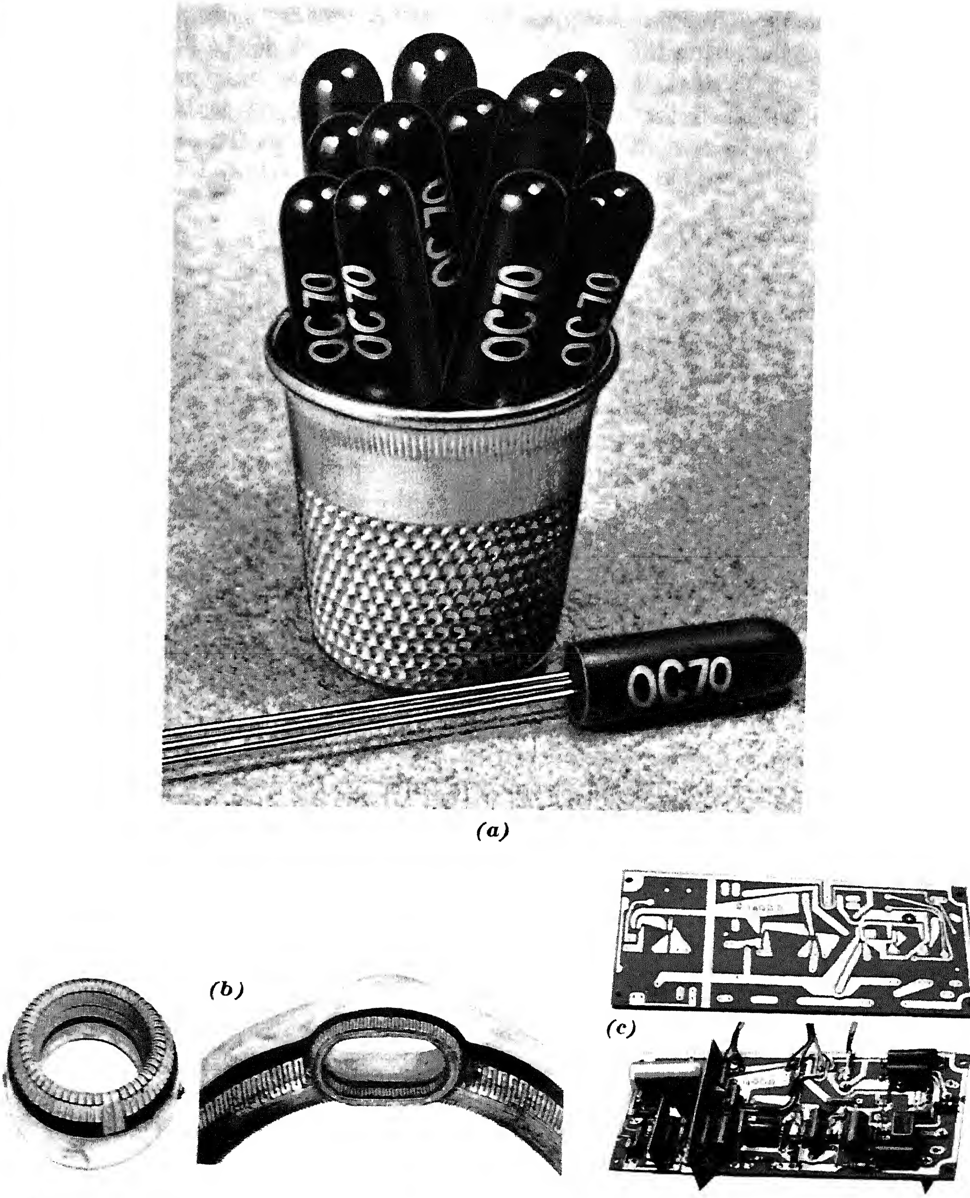


Figure 5–3. ***Examples of new products affecting approach to design.***
(a) A dozen transistors in an ordinary thimble illustrate the functional requirements of size. (Courtesy Mullard Ltd., London)
(b) De-icing of jet engine air intake, and
(c) Sound level meter (Dowe Instruments), illustrate use of printed circuits. (Courtesy Technograph Printed Circuits Ltd., London)

Apart from these direct techniques, management may have some additional aims, such as increasing the prestige of the firm as a whole, banking on the popularity of one product to strengthen or introduce another or to publicize one aspect of the firm's activity for the purpose of raising money or deviating attention from other activities, and so on. Once the design features of a product have been ascertained, appropriate advertising methods can be selected.

The product characteristics

Functional aspect

When the marketing possibilities have been explored, the functional scope of the product has to be carefully analyzed and properly defined. The definition of the objective itself rarely tells us very much about the functional scope envisaged. A washing machine, for example, has a clearly defined objective: to wash clothing. This does not state, however, how the washing should be carried out, whether the machine should be capable of heating the water prior to washing, whether rinsing or drying, or both, are to be done by the machine, and if so by what method. and what should the proportion be between automatic functioning and manual supervision. A functional analysis of this kind obviously affects the design of the machine, its complexity, its appearance, and its price.

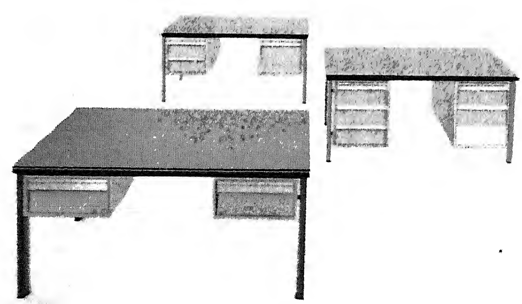
Sometimes functional aspects are detachable, and usage can be left to the customer's decision. A steam iron is a case in point. The additional function of dampening the cloth when required, prior to or during ironing, is incorporated in the steam iron, the main duty of which is to iron the cloth. The customer can decide whether and when to exploit this characteristic of the apparatus.

There is a trend to offer functional versatility of the product, thereby increasing the range of applications and sometimes combining several tools in one. A food mixer, for example, allows for a large number of attachments to be added for a variety of duties. Basically the mixer housing contains a power unit and a speed regulator, but it has to be designed so as to serve all the attachments, and the customer has to decide and define for himself the functional scope to be compatible with his needs, his taste, and his pocket. Household power-tool sets are designed on very much the same principle: The hand drill is the basic unit, and with attachments it can become a table drill, a lathe, a polisher, a hedge cutter, etc. Versatility of production machinery may quite often result in substantial savings in floor space and capital expenditure, and this may become one of the fundamental factors affecting design policy. Another example of versatility in design is shown in Fig. 5–4.

Operational aspect

After determining the functional aspect, the operational aspect has then to be considered. Not only must the product function properly, it must be easy to handle and simple to operate. Sometimes it has to be adaptable to various operational conditions, and very often it is subjected to varying degrees of skill

of potential operators. The designer's problem becomes all the more critical with the trend for increased versatility because this characteristic implies using basic attachments as elements for building suitable combinations for specific purposes. This requires a certain amount of operator intelligence and skill, which increases with the complexity of the machine. The scarcity of skill is a prohibitive limitation in this respect on the product designer.



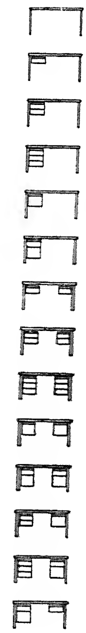


Figure 5–4. ***Desk combinations (simplification of design). Variety achieved through standardization: with a limited number of components (one piece table top without joined corners or seams, legs made of seamless steel tubes, and interchangeable drawers) the designer managed to offer 36 possible combinations while exhibiting a pleasant style and functional simplicity.***
(Courtesy N.V. Wed. J. Ahrend & Zoon—N. V. "Oda" Staalwerk—Amsterdam, Holland.
Designer: Friso Kramer, 1958)

The "get ready" stage before the operation proper and the "put away" time (including cleaning) should be carefully analyzed with respect to the expected skill of the operator. Too often one finds ingenious gadgets (for example, in the field of household equipment) that are capable of performing an operation in a fraction of the time normally required but which involve such complicated preparations or such lengthy cleaning and "put away" subsequent operations, that the ratio of net machine time to over-all machine time becomes absurdly small. The beneficial features attributed to the gadget in such cases are rather questionable.

Versatility of equipment should also be analyzed in this light. Especially when subsequent operations are to be carried out with the aid of different attachments, the designer should always bear in mind the time required for an operator to perform the changeover and should make certain that this time is in reasonable proportion to the operation time.

Durability and dependability

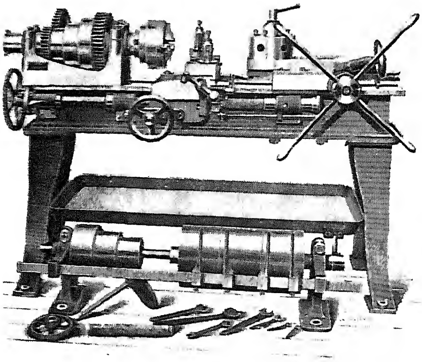
These are two factors closely related to the selection of materials and class of workmanship and hence to the design of the product and the economical analysis of its cost. Quality is not always a simple characteristic to define, but durability and dependability are two factors that often determine quality and have to be carefully considered by the designer. Durability is defined mainly by the length of the active life, or endurance, of the product under given working conditions, but a measure of the product capability to idle or withstand storage is also often considered in assessing durability. Durability need not always be associated with selection of good materials. The actual working life of a match or a rocket motor may be rather limited, but that does not mean that materials for these articles may be of low quality. An additional criterion, therefore, has to be considered, that of dependability, or the capability of the product to function when called upon to do its job. Returning to our matches, dependability may be related to the number of duds in a box, and while the manufacturer is eager to reduce this number to a minimum, he need not choose the very best raw materials to ensure that not even one match will fail. Dependability of rocket motors, however, may be more rigidly defined, and first class materials are chosen in spite of the short active life that is envisaged for them in some applications.

Another aspect of durability is that of maintenance and repair. The amount of repair and preventative maintenance required for some products is closely related to quality and design policy. This is of particular importance when the equipment is supposed to operate continuously and when any repair involves a loss of running time.

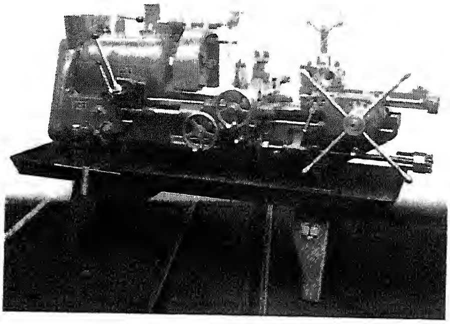
Problems of convenience and accessibility in operating the equipment have already been discussed, and the same remarks are valid for maintenance and repair. Easy accessibility is a fundamental principle in a sound design, and thorough knowledge on the part of the operational durability, dependability, and maintenance requirements of the product are absolutely essential to ensure a well-balanced design within the policy outlined by higher management.

Aesthetic aspect

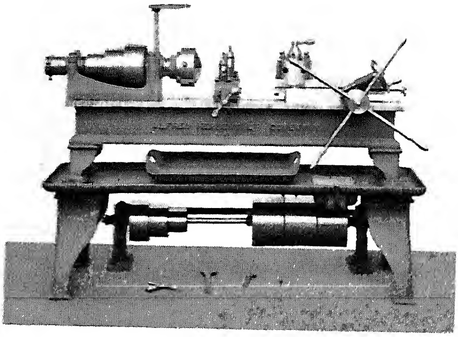
In what way does the appearance of a product affect its design? In most cases where the functional scope, durability, and dependability have already been defined, the aesthetics are mainly concerned with molding the final shape around the basic skeleton. This molding of shape may very often be severely limited in scope, and what finally emerges is sometimes termed a "functional shape." The view that functional shape is necessarily divorced from aesthetics, especially where engineering structures or equipment are concerned, is well exemplified by bridges, locomotives, or machines of the late nineteenth or early twentieth century (see, for example, Fig. 5–5). However, a study of the gradual changes in shape of these objects in the past few decades would convince us that there has been an increasing recognition of the role of aesthetics in design. This is perhaps



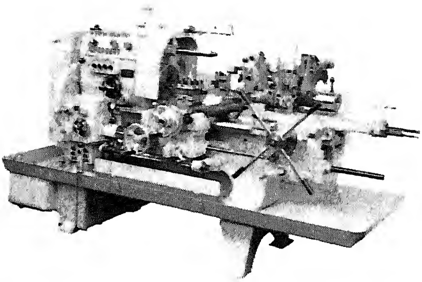
1894



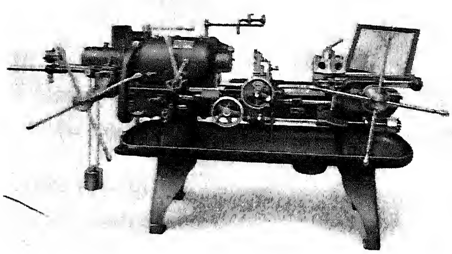
1930



1900



1957



1906

Figure 5–5. ***Development in the design of a Capstan Lathe.*** (Courtesy Alfred Herbert Ltd., Coventry, England)

partly due to man's aesthetic taste being reconciled to accepting these objects as an integral part of the landscape or everyday life, thereby leading to a modification of the original attitude that these "monstrosities" are hopelessly ugly and should be left alone.

Functional shape is a concept in its own right among designers. Those who believe in functional shape argue that compatibility of function with shape is logical and should therefore be accentuated and exploited, rather than covered up. A standard lamp is first and foremost a lamp and not a flying saucer, and there is nothing wrong with its looking like a lamp. This approach is referred to in Fig. 5–1, where the aesthetic aspects are dealt with at the design stage, after all the other aspects of the proposed product have been analyzed.

In some cases, however, molding of shape may have financial implications; for instance, when special materials have to be used or added to those basically required from the functional point of view or when additional processes are involved. Such cases will call for a careful cost analysis of the aesthetic aspects.

In extreme cases, aesthetics are the governing factor in design and completely dominate it. This is especially true for many consumer goods, such as automobiles and household equipment, or fashion goods. The functional scope, though more or less defined and accepted, may also be widened to accentuate the novelty of the new model. But the idea of the new design starts with the concept of its shape, from which the idea evolves and grows. The technical considerations have to be somehow fitted in at a later stage, this being in complete contrast to the conventional sequence shown in Fig. 5–1.

When styling is a dominant factor in product design, it is often used as a means to create demand. Changes in fashion and taste, evolution of form, and the introduction of new ideas quickly outdate previous designs. If the market is psychologically receptive and eager to discard former designs in favor of new ones, styling becomes a race against time, a race that determines the salability of the product.

Many tools can be utilized by the designer to bring out aesthetic characteristics. Some of these are:

1. Use of special materials, either for the parts of the housing or as additional decorations. Notable is the use of chromium strips, plastics, wood, glass, and fabrics for the purpose.

2. Use of color, either natural color of the material concerned or by use of paints, plating, spraying, or even lighting. Composition and contrast of colors is of great importance to the industrial designer in facilitating convenient operation and attractive appearance.

3. Texture supplements color, either by appropriate treatment of the given surfaces or coatings. Surface finish and requirements of brightness as determined by styling may in turn affect the production processes in the finishing stages.

4. Shape denoted by outer contours and similarity to familiar objects. Shape

can be exploited to accentuate particular features, to create a sense of spaciousness or illusions of size, richness, and dependability.

5. Line is used to break the form, also for the purpose of emphasizing parts of it, or to give a sense of continuity, graciousness, and stability.

6. Scaling the product, either to a blownup size or to a small size (modeling). This creates novelty and a sense of completeness. The success of styling of some popular small automobiles in Europe may be partly due to the designer's talent in creating a feeling of still having the full-size version, with all its features.

7. Packaging, especially for small items. Novelty and attractiveness of packaging is often transferred in the mind of the customer, attributing perhaps nonexistent values to the contents. In extreme cases packaging may assume an appreciable portion of the total production costs and become the center of the design project.

Aesthetic molding, especially when governed by the selection of material, color, texture, and sometimes even line, has great economic advantages, since great variety can be achieved at a comparatively low cost. The basic product remains the same, and variety is obtained by finishing processes alone. Henry Ford's maxim that the customer may choose any color he likes, provided it is black, is no longer valid. Modern production control techniques allow for a vast number of combinations of colors and textures to be offered with little difficulty.

Aesthetics have been fully recognized as an integral part of design, and no designer worth his mettle can afford to ignore their implications, their tools, and their benefits.

Economic analysis

As shown in Fig. 5–1, an economic analysis is the key to management decision in product design policy. Having obtained sufficient information about customers' requirements and market potentialities on the one hand and a detailed study about the functional, operational, and quality aspects of the proposed product on the other, the economic analysis can proceed by seeking an answer to the following questions:

What capital expenditure is required for manufacturing the new product?
What total production costs per piece are envisaged?
What is the reasonable margin of profit that can be expected?
Do the price (= total costs + profit) and the features of the product render it competitive in the market?
In what numbers is the product expected to be sold?

Here, again, the interdependence of variables should be strongly emphasized. Not one single question in this list can be isolated and solved independently of the others. The economic analysis is in fact a cyclic and repetitive procedure. Each question is weighted in the light of the answer and the data provided by

the previous question, and all the answers are checked when their turn comes again to be re-evaluated in the following cycles, until a state of equilibrium is reached and no further modifications to these answers are required.

Profit and competitiveness

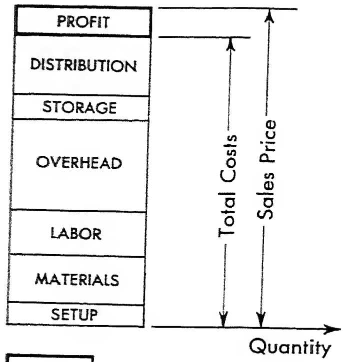
The measure of competitiveness of the product corresponds to the portion of the market it succeeds in capturing. This is largely dependent on the value the customer is prepared to put on the product, and on the ratio of this value to the price. As customer assessment of value is not universally uniform but subject to preference of features, performance, or taste, ratios of values to prices vary with customers. A state of equilibrium is formed in which the market is divided between different preferences. This equilibrium may change: If the ratio of value to price of the product becomes more favorable, when compared with other products, the product increases its portion of the market and becomes more competitive.

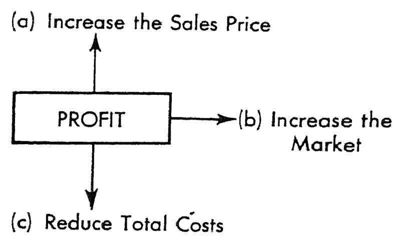
Such an equilibrium is shown in Fig. 5–6, where the total costs include setup, materials, overheads, storage, and distribution. The total profit is determined by the margin of profit per unit and by the sales volume. If the organization seeks to increase its profit, it can try one of the following methods (Fig. 5–6):

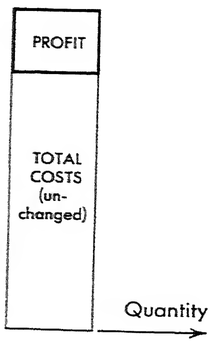
(a) *Increase the margin of profit per unit*, hence the sales price, but leave the total production to costs unchanged. If such a course would not affect the sales volume, the total profit would be proportional to the increase in the margin of profit per unit. Such an increase, however, can upset the market equilibrium unfavorably, in that both the ratio of customers' value of the product to its price will deteriorate and the products of competitors will become more attractive. The market may shrink, and the total profit, far from attaining the expected value, may in extreme cases fall below its original level.

(b) *Leave the total costs unchanged*, but try to improve the ratio of value to price and thus widen the market. This can be done (1) by producing a better or more attractive product at the same cost, (2) by launching an intense advertising campaign in order to boost the customer's assessment of the product value, or (3) by reducing the sales price at the expense of the margin of profit per unit, in the hope that the market will expand enough to increase total profit. Too marginal a profit per unit is, however, undesirable, as it allows little protection from possible fluctuations in the market, and even slight instabilities may turn a small profit into a sizeable loss.

(c) *Reduce the total production costs* and pass some of the benefit to customers in the form of reduced sales prices. If both the profit per piece and the size of the market increase, a substantial improvement in total profits will be achieved. This course calls for a continuous search after better methods, better processes, better materials and their utilization, and better management to reduce overheads. There are, however, some limitations to the rate of improvement one can attain, such as basic labor and material costs and limited resources or credit hampering expenditure on new equipment and machines. Minimum requirements of quality should also be studied and met, as a reduction in price at the



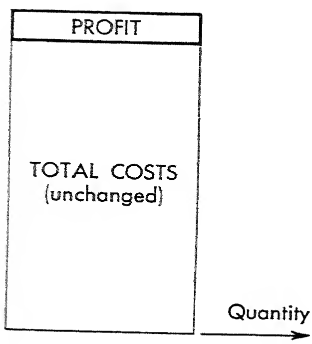




(a) Increase the sales price by increasing the profit per unit.

Limitations: (i) competition.
(ii) customer's willingness to pay.

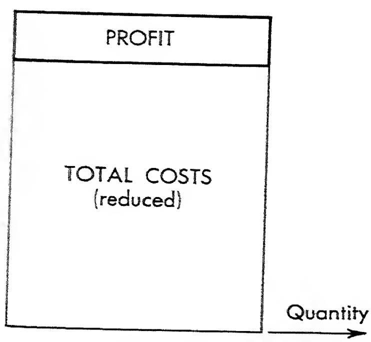
Danger: shrinkage of market leading to possible decline in total profit.



(b) Increase the market by reducing the profit per unit, etc., by advertising.

Limitation: competition.

Danger: too low a margin of profit per unit should be avoided due to possible instabilities in the market.



(c) Reduce total costs and pass some benefits to customers by reducing sales price.

Limitations: (i) expenditure on new equipment.
(ii) basic labor and materials costs.
(iii) minimum requirements of quality.

Figure 5–6. ***Methods for increasing total profit.***
(a) increase the sales price (b) increase the market (c) reduce total costs

expense of quality is easy enough; customer's assessment of the product value, however, deteriorates accordingly. But reducing production costs and thereby expanding the market, while sustaining accepted quality standards, offers a challenge to the production engineer. Probably the most characteristic feature of this process is that it is both dynamic and continuous, that each success is a further advance along the spiral of increasing productivity and standard of living (see Fig. 5–7).

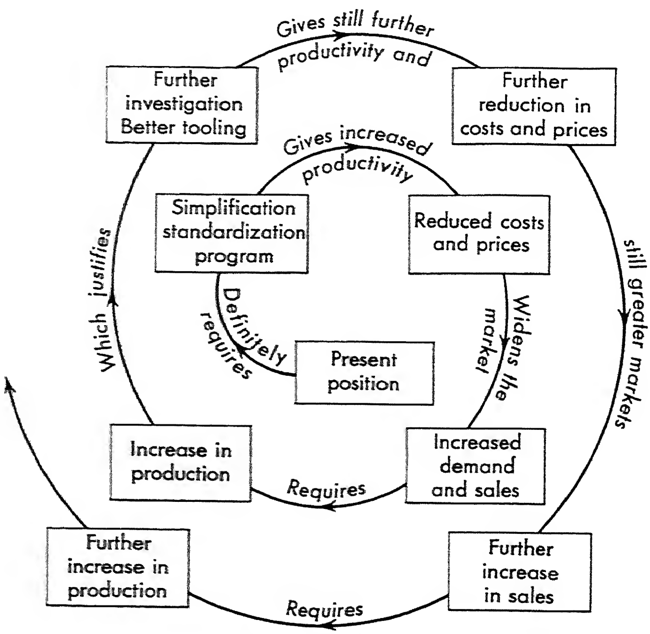


Figure 5–7. ***Spiral of increasing productivity and standard of living.*** (Courtesy the British Productivity Council, Report on Metalworking Machine Tools, 1953)

The three S's

The three S's refer to standardization, simplification, and specialization—three related subjects that are at the root of any economic analysis of product design. The three S's can be defined as follows:[1]

Standardization is the process of defining and applying the "conditions" necessary to ensure that a given range of requirements can normally be met with a minimum of variety and in a reproducible and economic manner on the basis of the best current technique.

[1] These definitions are quoted from the Lemon Committee on the Standardization of Engineering Products. (H.M.S.O., 1949).

Simplification is the process of reducing the number of types of products within a definite range.

Specialization is the process whereby particular firms concentrate on the manufacture of a limited number of products or types of products.

The three processes are usually linked together and develop as a logical sequence. From a wide range of requirements it is first necessary to sort out the essential features, define them, and then work out in a scientific manner the minimum variety required to meet these essentials. This is a process of standardization, and it is mainly an engineering process. Within a given range, whether covered by standards or not, a process of simplification can be carried out with the view of reducing the variety of products or materials that are produced or purchased. This is both an economic and an engineering process, and specialization is one of its natural outcomes.

Standardization

Standardization covers a wide field of activity, which may be described by the following main categories:

- Physical dimensions and tolerances of components within a defined range
- Rating of machines or equipment (in units of energy, temperature, current, speed, etc.)
- Specification of physical and chemical properties of materials
- Methods of testing characteristics or performance
- Methods of installation to comply with minimum precautionary measures and convenience of use

The first three categories relate to limitation of the number of sizes or grades and some aspects of quality, one of the important aims being interchangeability of components or assemblies. Adherence to standards of raw materials is one of the fundamentals of product design, since any deviation from the standards in this respect may cause a substantial increase in the cost of materials. Industry is rich with examples in which designers specify "special" materials whereas the standard grades can do just as well.

Standardization and interchangeability impose certain limitations on the designer and demand higher skill and effort in planning. It is easy enough when designing a new component to decide that no standard really meets the special requirements of the case in hand and that a special part has to be specified. What designers seem to forget is that one of the purposes of standards is to provide solutions to relieve them of the task of having to solve afresh some basic problems, and thereby allow them more time to concentrate on the broader aspects of the design.

Another prerequisite of interchangeability is the precision required of the manufacturing process in order to obtain production within the specified tolerances. This implies that production control has to be tightened so that any

deviations from the given standards will be immediately noticed and appropriate action can be taken to avoid the process getting out of control.

Standardization has, however, many advantages, some of which may be briefly listed below:

Reduction of material waste and obsolescence
Concentration of effort in manufacturing; hence, simplification and specialization
Reduction in inventories, both of materials, semifinished, and finished products
Reduction in bookkeeping and other paper work
Lowering the grades of skill required in manufacture and assembly
Reduction in price; hence expansion of the market
Reduction of repair and maintenance costs

Preferred Numbers

According to the American Standards Association[2], "preferred numbers" are defined as "series of numbers selected to be used for standardization purposes in preference to other numbers. Their use will lead to simplified practice and they should, therefore, be employed whenever possible for individual standard sizes and ratings, or for a series thereof," in applications relating to: important or characteristic linear dimensions (such as diameters and lengths) or specifications of areas, volumes, weights, and capacities; ratings of machinery and apparatus.

The problem of selecting preferred numbers was first tackled by Renard in 1870, and therefore the series is sometimes referred to as the Renard series. Renard was an officer in the French Army and was faced with the problem that 425 different sizes of cables were in use in his unit. He recognized that the process of standardization consisted of two problems: to define the number of sizes required in a given range, i.e., the number of terms in the series; and to determine the method by means of which these sizes should be preferred to others.

Renard suggested the use of a geometrical progression as a guide for selection, and this system has indeed been adopted in standardization to cover the given ranges satisfactorily.

Suppose a manufacturer wants to produce containers having between 10 and 100 gallons capacity. In selecting the type of series he can choose:

a 5-series, i.e., covering the range of 10 to 100 in 5 steps
or a 10-series, i.e., covering the range of 10 to 100 in 10 steps
or a 20-series, i.e., covering the range of 10 to 100 in 20 steps
or a 40-series, i.e., covering the range of 10 to 100 in 40 steps

The need for more terms in a series (having smaller steps than those obtained in the 40-series) is very rare in practice, but when such a need arises, it is possible to use the 80-series.

[2] ASA Standard Z17.1—1958, American Standards Association.

If the said manufacturer decides to adopt the 5-series, his capacity ratings according to the geometric progression series will be:

$$\left.\begin{array}{ll} \text{First size} & 10 \text{ (given)} \\ \text{Second size} & 10 \times q \\ \text{Third size} & 10 \times q^2 \\ \text{Fourth size} & 10 \times q^3 \\ \text{Fifth size} & 10 \times q^4 \\ \text{Sixth size} & 10 \times q^5 = 100 \text{ (given)} \end{array}\right\} \text{5 steps}$$

Hence, $$q^5 = 10$$

or $$q = \sqrt[5]{10} = 1.585$$

The calculated sizes would be

$$\left.\begin{array}{lll} \text{First size} & 10.00 & \text{i.e., select } 10 \\ \text{Second size} & 15.85 & \text{i.e., select } 16 \\ \text{Third size} & 25.12 & \text{i.e., select } 25 \\ \text{Fourth size} & 39.82 & \text{i.e., select } 40 \\ \text{Fifth size} & 63.11 & \text{i.e., select } 63 \\ \text{Sixth size} & 100.00 & \text{i.e., select } 100 \end{array}\right\} \text{5 steps}$$

Similarly, the step size of the other series can be determined for the

5-series	$q = \sqrt[5]{10} = 1.5849$	or a step increase by about 60%
10-series	$q = \sqrt[10]{10} = 1.2589$	or a step increase by about 25%
20-series	$q = \sqrt[20]{10} = 1.1220$	or a step increase by about 12%
40-series	$q = \sqrt[40]{10} = 1.0593$	or a step increase by about 6%

The basic preferred numbers for all these series, as suggested by the International System, are shown in Table 5–1, where the given rounded numbers do not depart from the theoretical calculations by more than 1.3 per cent. The 5-series is given in column *A*. To obtain the 10-series, column *E* should be added; hence the series would read: 10, 12.5, 16, 20, 25, etc. Similarly, for the 20-series read: 10, 11.2, 12.5, 14, 16, etc.

Table 5–1

Basic Preferred Numbers

(10 to 100)

Column	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
	10	10.6	11.2	11.8	12.5	13.2	14	15
	16	17	18	19	20	21.2	22.4	23.6
	25	26.5	28	30	31.5	33.5	35.5	37.5
	40	42.5	45	47.5	50	53	56	60
	63	67	71	75	80	85	90	95
	100							

5 series (60% steps), column *A*
10 series (25% steps), columns *A*, *E*
20 series (12% steps), columns *A*, *C*, *E*, *G*
40 series (6% steps), columns *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H*

Simplification

Simplification is a constant source of disagreement between the sales department and the production personnel. A production engineer prefers little variety, minimum setups, and long runs (Fig. 5–8). Simplification enables the production department to improve planning, achieve higher rates of production and machine

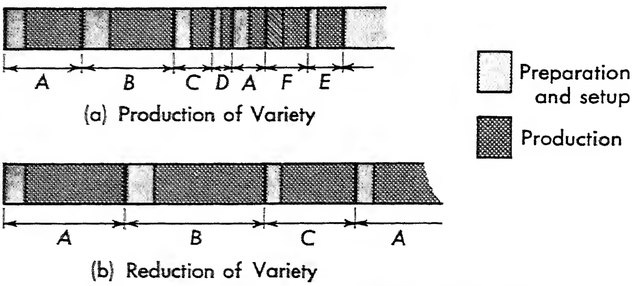


Figure 5–8. ***Effect of variety on scheduling.***

utilization, and simplify control procedures. The salesman, on the other hand, strives to satisfy the customer by giving him a choice or by offering him the nearest to what he wants. The pro's and con's of simplification are given in the accompanying listing.

Pro Simplification

Reduce inventories of materials and finished products
Reduce investment in plant and equipment
Save storage space
Simplify planning and production methods
Simplify inspection and control
Reduce required technical personnel
Reduce sales price (through production simplification and reduction of distribution costs); hence expand the market and the plant
Shorten or eliminate order queues

Pro Variety

Satisfy a wide range of demand
Enable better contact with the market to study its tastes and requirements
Avoid losing orders for more salable products because the customer directs all his orders to other vendors
Create demand

The last point in favor of variety deserves, perhaps, some further clarification. Some sales people claim that variety encourages consumption and that, especially where consumer goods are concerned, the psychological effect of plenty creates demand. Furthermore, market research by some firms seems to suggest that in some cases similar products tend to capture roughly the same portion of a given market. The prospects of increasing total demand on the one hand and the firm's portion of the market on the other, may have been the main causes for boosting variety to the extent found nowadays in industry. From the customer's

point of view this is a very unsatisfactory state of affairs. A flood of variety confuses the customer, who ceases in many cases to appreciate the fine differences between similar products and has either to make a haphazard choice or to invest effort, time, and study (and quite often money) to enable him to make an intelligent choice.

This is undesirable for the firm as well. Apart from missing all the advantages listed above when simplification is applied, an analysis of the market sometimes shows that variety has long passed the saturation point and that an increase in variety will not be even noticed in the market. Also, the division of the market between too large a number of products makes each portion so small that prices have to be kept at high levels to avoid losses. This problem is further discussed in the next chapter.

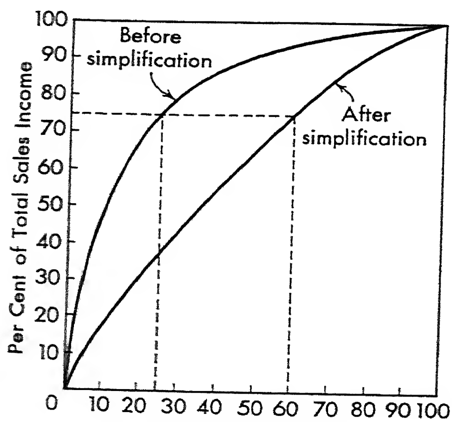


Figure 5–9. ***Analysis of sales by products.***

When a great variety exists, a sales analysis can be made to establish the salability of the products. When the accumulative sales income is plotted against the number of products offered for sale, it is very often revealed that a comparatively small number of products contributes very little in this respect (Fig. 5–9). This is sometimes referred to in industry as the "25% to 75%" relationship because in many cases it was found that 25 per cent of the products brought in 75 per cent of the income, although in some extreme cases studies revealed as small as 10 to 90 per cent relationships. This leads to unnecessary drain of the firm's efforts, which should be directed to promoting the more profitable products. A more desirable situation is when responsibility for income is more evenly distributed between products (i.e., when the curve is "flat" as is the lower one in Fig. 5–9), which is achieved through reduction of variety.

The break-even analysis

The effect of quantity on the profit contribution of the product is illustrated in Fig. 5–10, where the sales income is represented by the straight line bQ, in

which Q is the quantity sold and b is the income per unit. The costs to the firm consist of:

Fixed costs F, which are independent of the quantity produced and include executive salaries, depreciation of plant and equipment, etc.

Variable costs aQ, where a represents the constant total costs per unit, including materials, labor, and other direct costs that vary with the plant activity. The variable costs are shown in Fig. 5–10 by the straight line aQ.

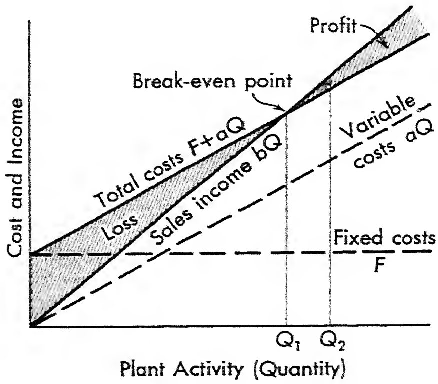


Figure 5–10. ***A break-even chart.***

The division into fixed and variable costs represents only an approximate interpretation of the total costs function and may not be valid for a very wide range of Q (this problem is further discussed in Chapter 20).

The total costs are given by the summation of fixed and variable costs $(F + aQ)$, and the point of intersection of this line with that of sales income is the break-even point (BEP) corresponding to a sales volume Q_1. Activity below Q_1 results in a loss; activity above Q_1 gives profit. At the point of intersection,

$$F + aQ_1 = bQ_1$$

hence

$$Q_1 = \frac{F}{b - a} \tag{5–1}$$

If a plant is operating at point Q_2, it is working with a margin of safety (denoted by Δ), which can be defined as follows:

$$\Delta = \frac{Q_2 - Q_1}{Q_1} = \frac{Q_2}{Q_1} - 1 \tag{5–2a}$$

and it can be shown that

$$\Delta = \frac{Z}{F} \tag{5–2b}$$

where Z is the profit of the plant. The desirable level of the plant activity can be expressed in terms of the safety margin or the profit as

$$Q_2 = Q_1(1 + \Delta) = Q_1\left(1 + \frac{Z}{F}\right) \tag{5–3}$$

The margin of safety is a measure of healthiness at the point of operation. When the margin is too small (i.e., when the product is manufactured near the break-even point), the plant is prone to market fluctuations.

Table 5–2

Sales Data

Month	*Quantity (units sold)*	*Sales Income* $\times 10^3$ (\$)	*Net Profit* $\times 10^3$ (\$)	*Total Cost* $\times 10^3$ (\$)
Jan.	409	40.9	− 4.5	45.4
Feb.	505	50.5	− 1.5	52.0
March	612	61.2	+ 4.0	57.2
April	751	75.1	+ 9.5	65.6
May	786	78.6	+ 9.0	69.6
June	802	80.2	+12.0	68.2
July	791	79.1	+12.5	66.6
Aug.	808	80.8	+12.0	68.8
Sept.	701	70.1	+ 7.5	62.6
Oct.	626	62.6	+ 5.0	57.6
Nov.	430	43.0	− 3.0	46.0
Dec.	710	71.0	+ 7.4	63.6
Total (annual)	7,931	793.1	+69.9	723.2
Monthly average	661	66.1	+ 5.83	60.3

Example

An analysis of annual sales is given in Table 5–2. The sales income and the total costs are plotted against quantity in Fig. 5–11, from which the fixed costs $F = \$21{,}000$ per month are obtained. The BEP (break-even point) occurs at

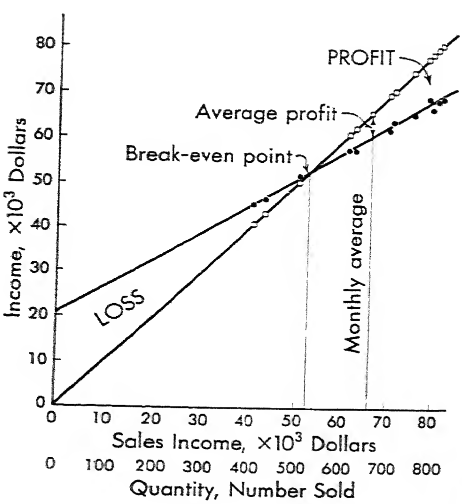


Figure 5–11. ***A monthly break-even chart*** (See Table 5–2)

$Q_1 = 520$ units, so that during three months the plant was working below this point. To find the coefficients a, b:

$$a = \frac{\text{(average total costs)} - \text{(fixed costs)}}{\text{average quantity}} = \frac{(60.3 - 21.0)10^3}{661} = 59.5 \text{ dollar/unit}$$

$$b = \frac{\text{average sales income}}{\text{average quantity}} = \frac{66.1 \times 10^3}{661} = 100 \text{ dollar/unit}$$

The margin of safety is

$$\Delta = \frac{Q_2 - Q_1}{Q_1} 100 = \frac{661 - 520}{520} 100 = 27.1\%$$

The abscissa may be marked either in quantity or in sales income (in which case the income line bQ has a slope of 45°, if ordinates have the same scale). The coefficients a, b were calculated for a quantity diagram, but if plant activity is measured in sales income, then

$$a = \frac{\text{(average total costs)} - \text{(fixed costs)}}{\text{average sales income}}$$

$$= \frac{(60.3 - 21.0)10^3}{66.1 \times 10^3} = 0.60 \text{ dollar/dollar}$$

$$b = 1 \qquad \text{(slope 45°)}$$

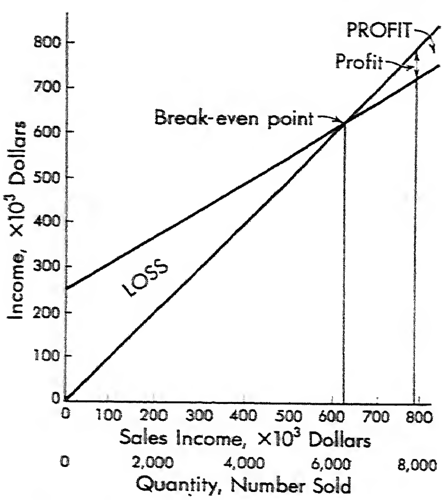


Figure 5–12. ***An annual break-even chart.***

The break-even chart may be either on a monthly (Fig. 5–11). or yearly (Fig. 5–12) basis. A low BEP is highly desirable because it increases the safety margin of the product. From Eq. 5–1 it is obvious that the BEP can be lowered by three methods (see also Fig. 5–13) as follows:

Reduce the fixed costs from F to F', thus lowering the BEP to

$$Q'_1 = Q_1 \frac{F'}{F}$$

Reduce the variable costs coefficient a to a'; hence

$$Q'_1 = Q_1 \frac{b - a}{b - a'}$$

Increase the slope of the income line from b to b', the new BEP being

$$Q'_1 = Q_1 \frac{b - a}{b' - a}$$

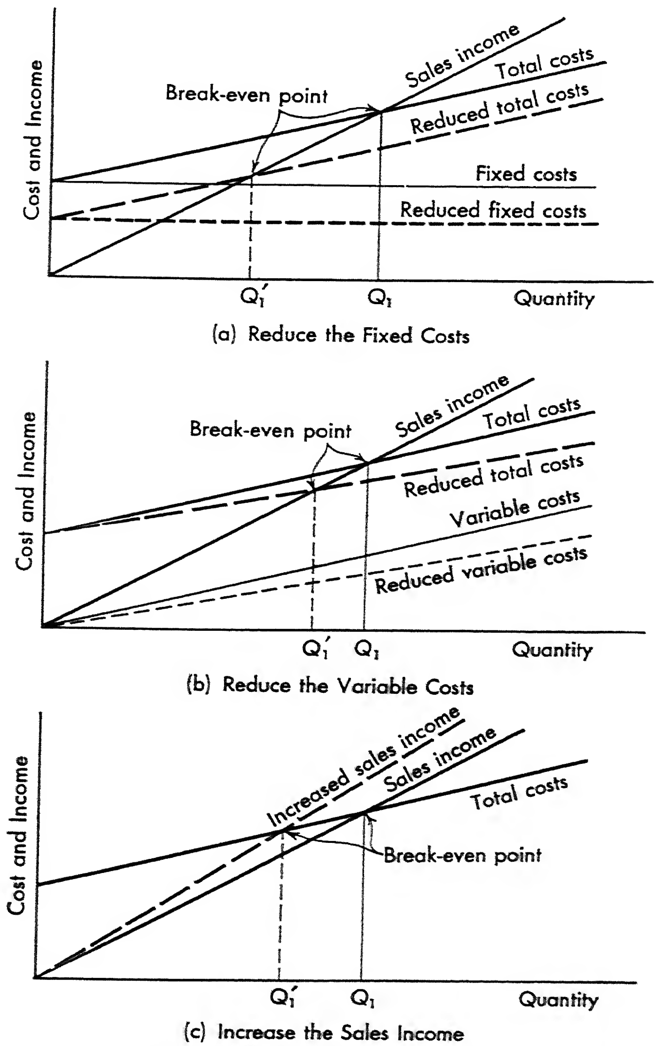


Figure 5–13. ***Methods for lowering the break-even point.***

A similar diagram to the break-even chart, called the *profit-volume chart* is shown in Fig. 5–14, where the fixed costs are marked as a negative quantity on the ordinate. The BEP is given by the intersection of the income line with the abscissa. Operation below the abscissa incurs a loss; operation above it, a profit.

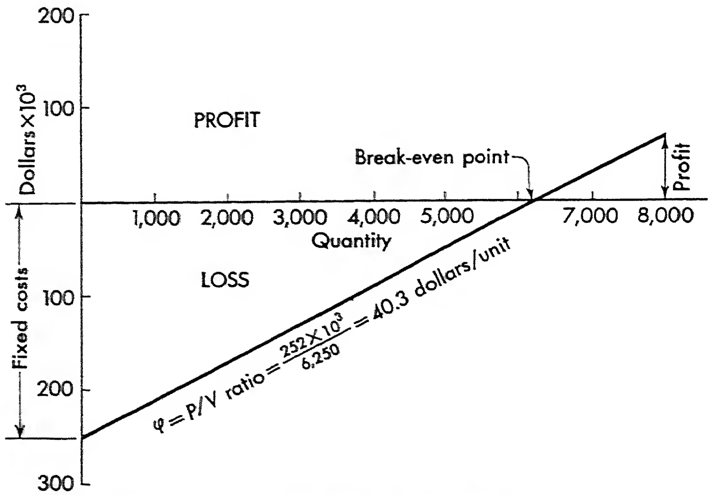


Figure 5–14. ***An annual profit-volume chart.***

The profitability of the product is indicated by the slope of the income line, called the P/V (Profit-Volume) ratio and denoted by φ:

$$\varphi = \frac{\text{fixed costs}}{\text{volume at BEP}} = \frac{(\text{profit}) + (\text{fixed costs})}{\text{volume}} = \frac{F}{Q_1} = b - a \qquad (5\text{–}4)$$

and the profit

$$Z = (b - a)Q - F = \varphi Q - F \qquad (5\text{–}5)$$

As in the break-even charts, volume or activity can be measured either in sales income or in quantity. In the example cited above,

$$\varphi = \frac{252 \times 10^3}{625 \times 10^3} = 0.40 \text{ dollar/dollar}$$

or

$$\varphi = \frac{252 \times 10^3}{6{,}250} = 40.3 \text{ dollar/unit}$$

A multiproduct profit-volume chart

In a multiproduct activity the profit-volume chart can be constructed as shown in Fig. 5–15, where three products A, B, and C are considered. First the fixed costs of A are marked downward on the ordinate. For the quantity produced

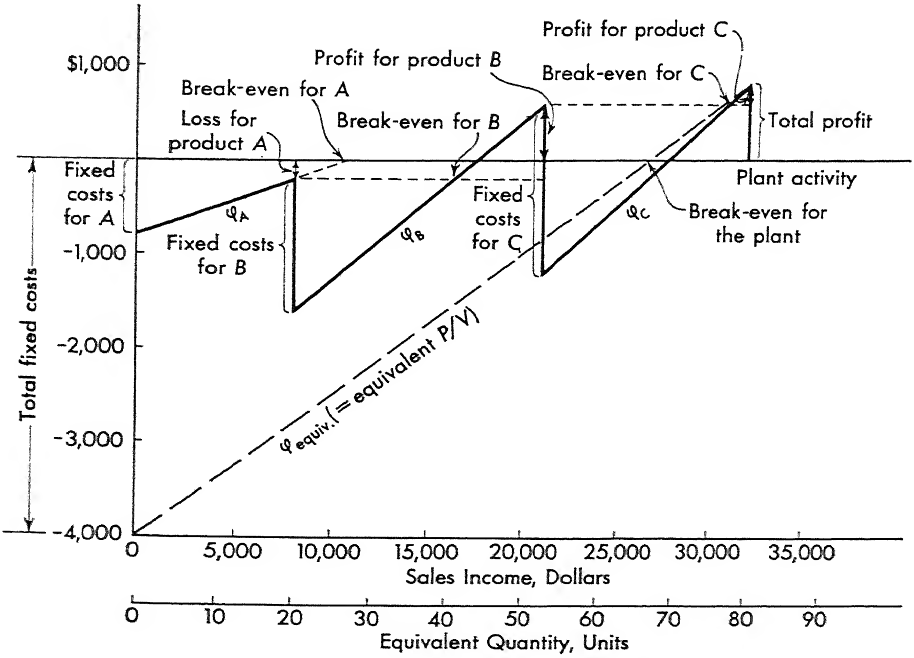


Figure 5–15. *A multiproduct profit-volume chart.*

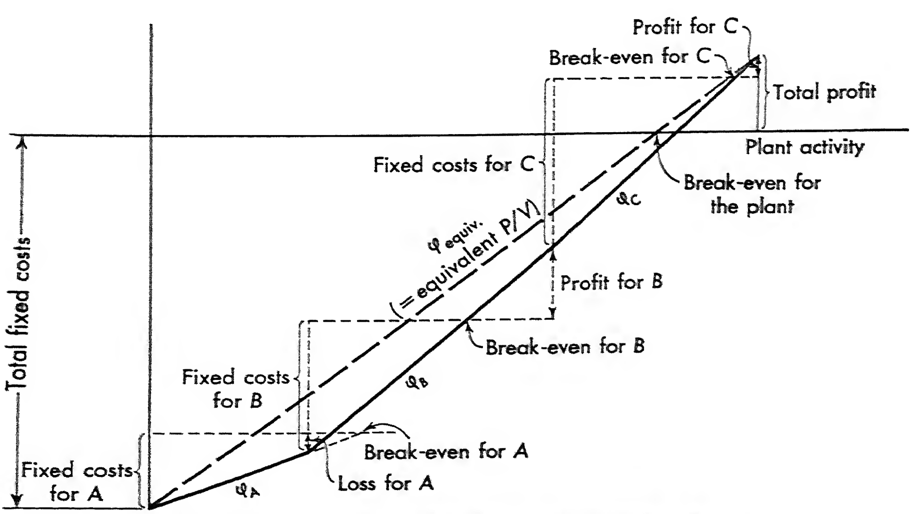


Figure 5–16. *A multiproduct profit-volume chart.*

of A, a loss (also marked negatively) occurs; hence the P/V ratio for A is established. The diagram is now repeated for the other products, and for each the BEP can be found. The accumulated profit of the plant is given by the ordinate of the final point on the broken P/V line and an equivalent P/V line, which is determined by the total profit and total fixed costs.

Another form of a multiproduct chart is given in Fig. 5–16, where the total fixed costs are marked prior to drawing the broken P/V line. In this chart, too, the individual and the equivalent BEP's can be found, but it also provides a ready visual comparison between the P/V ratios. In Fig. 5–16, the P/V for A and B are better (i.e., steeper) than the equivalent P/V, whereas the one for A is comparatively lower. This would indicate that, in order to increase the safety margin of the plant, management should:

1. Increase the P/V ratio of product A and thus improve the equivalent P/V ratio and lower the equivalent BEP;
2. Prefer products B, C by expanding production of these products either absolutely or even at the expense of product A, if this is possible.

From the break-even analysis we may conclude that an increase in production activity is always desirable (when $b > a$), since the profit becomes constantly larger when the margin of safety is increased. There are, however, two restrictions on such an expansion, namely: the limited capacity of the market to absorb and the limited capacity of the plant to produce. The main issue involved is the allocation of the limited facilities of the plant to the various products within specified restrictions. The problem may perhaps be best illustrated when two products are considered. Suppose a plant produces Q_A units of one product and Q_B of another, so that

$$Q_A + Q_B = Q$$

where Q is the total available plant capacity. How should this capacity be allocated to maximize profit?

In a two-dimensional coordinate system (Fig. 5–17) the condition $Q_A + Q_B = Q$ is represented by the line 1–2, which is the locus of all points of maximum activity. Any point inside the triangle 0–1–2 corresponds to operation below maximum capacity, a point on the ordinate 0–1 implies production of product A alone (the maximum possible volume being Q at point 1), a point on the abscissa 0–2 refers to production of product B alone (the possible maximum being at point 2). The profit function at point 1 as we move along 1–2 is

$$Q_A = Q; \qquad Q_B = 0$$

The profit when only product A is produced (using Eq. 5–5) is

$$Z_1 = \varphi_A Q - F_A$$

When one unit of product B is produced:

$$Q_A = Q - 1;$$
$$Q_B = 1 \qquad \text{(point 3)}$$

$$\therefore \qquad Z_3 = \varphi_A(Q - 1) + \varphi_B - (F_A + F_B) = Z_1 + \varphi_B - \varphi_A - F_B$$

Almost in every situation

$$\varphi_B - \varphi_A < F_B, \qquad \therefore \quad Z_3 < Z_1$$

At point 2:

$$Q_A = 0; \qquad Q_B = Q$$

hence $Z_2 = \varphi_B Q - F_B$.

Where only one unit of A is produced:

$$Q_A = 1; \qquad Q_B = Q - 1 \quad \text{(point 4)}$$
$$Z_4 = Z_2 + \varphi_A - \varphi_B - F_A < \text{(usually) } Z_2$$

And at any point between 3 and 4:

$$\begin{aligned} Z &= \varphi_A Q_A + \varphi_B Q_B - (F_A + F_B) \\ &= \varphi_A Q + (\varphi_B - \varphi_A) Q_B - (F_A + F_B) \end{aligned}$$

This is a linear function of Q_B, as shown in Fig. 5–17. The above analysis is based on the assumption that the total fixed costs are apportioned to F_A and F_B and that when one product is eliminated from the program, its fixed costs are eliminated too. Usually, in practice, this is not so, and residual fixed costs of the eliminated product have to be borne by the remaining products, in which case the values of Z_1 and Z_2 are somewhat lower than computed above.

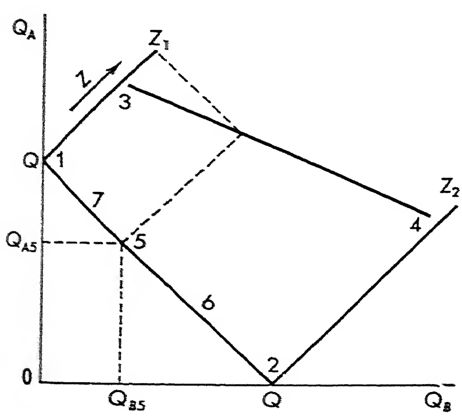


Figure 5–17. ***Profit function with two products A and B when total plant capacity is limited.***

It is apparent from Fig. 5–17 that the best solution is a corner point (this is further discussed in Chapter 12); therefore if management can afford to eliminate products, it should do so. By computing Z_1 and Z_2 and finding which is larger, it is simple to ascertain which product should be preferred. This analysis should indicate why simplification and specialization are so beneficial.

Another interesting fact illustrated in Fig. 5–17 is that product A on its own, though less profitable than product B, is still better for the organization than some combinations of A and B. In fact throughout the whole range of 1–5 no

such combination yields a profit as high as A does when produced alone. The condition of point 5 is given by

$$Z_5 = Z_1$$

$$\varphi_A Q + (\varphi_B - \varphi_A)Q_{B5} - (F_A + F_B) = \varphi_A Q - F_A$$

or

$$Q_{B5} = \frac{F_B}{\varphi_B - \varphi_A}$$

As already pointed out, elimination of products and specialization are not always possible. It may be that a certain amount of product A has to be produced (for instance, because their supply may be conditional to marketing product B), so that $Q_A \geqslant Q_{A6}$ (point 6, Fig. 5–17), or that the maximum quantity of product B that can be sold is restricted, i.e., $Q_B \leqslant Q_{B7}$ (point 7). The range 6–7 provides the flexibility that may be allowed within these restrictions, but from the above analysis the decision rule that emerges can be simply stated as follows:

1. When the range 6–7 falls between 1 and 5, eliminate product B and concentrate on A.
2. When the range 6–7 does not fall between 1 and 5, select the extreme point 6 on this range for maximum profit under the stated conditions (i.e., produce as little of A as possible).

When n products are manufactured, the total profit is

$$Z = \sum_{i=1}^{n} \varphi_i Q_i - \sum_{i=1}^{n} F_i$$

and the problem is to find the quantities Q_i that would yield maximum profit, where $\varphi_i \geqslant 0$ and $F_i \geqslant 0$. The total plant capacity is restricted, so that

$$\sum_{i=1}^{n} Q_i \leqslant Q$$

but the quantity for each product must be above a certain value specified by the sales department and endorsed by management policy; hence

$$Q_i \geqslant A_i \geqslant 0$$

Where no minimum quantity has to be met, $A = 0$. This is a linear programing problem, the profit function Z being a sum of linear terms. It is not proposed to relate in detail here the procedures by means of which an optimal solution may be sought, and the reader is advised to consult treatises on linear programing for this purpose.[3] If it is assumed that the fixed costs per product disappear when

[3] See also Chapter 12.

the product is deleted from the schedule, we must add another condition; namely,

$$F_i = 0 \qquad \text{if} \quad Q_i = 0$$

which implies that the profit function is discontinuous at the so-called corner points (a corner point is one where a variable $Q_i = 0$). One of the theorems in linear programing states that an optimal solution is obtained at a corner point, and this would suggest that a simplification program based on elimination of products is worth while (provided the plant capacity can still be fully employed on the product retained in the production schedule).

One additional remark about the construction of a multiproduct chart: as income per unit is probably different for different products, plant activity (the abscissa) should be given in sales income in order to preserve the same scale. If measurement of activity in quantities is desirable, equivalent rather than absolute quantities have to be used; the criterion of equivalency may be determined by ratios of incomes per units, as shown by the accompanying table, which refers to the example shown in Fig. 5–15.

Product	*Sales Income* ($)	*Quantity*	*Sales Income/ Unit*	*Equivalent Quantity (take product A as a basis)*
A	8,000	20	400	20
B	13,000	100	130	$100 \times \frac{130}{400} = 32.5$
C	11,000	40	275	$40 \times \frac{275}{400} = 27.5$
Total	32,000			80

The alternative scales for plant activity are shown in Fig. 5–15.

Special attention should be paid to the units of the abscissa, especially when the break-even study is governed by restrictions on the plant total capacity. When this capacity is limited, the allocation of facilities to the most promising products becomes the crucial problem of the analysis, and this study should be carried out in terms of the total available capacity. Suppose we had a plant producing (at full capacity) four products:

Product	*Quantity (annual)*
A	2,000 units
B	4,000 units
C	5,000 units
D	6,000 units

If we know that in production capacity one unit of product *A* is equivalent to two units of product *B*, one-half unit of product *C*, but only two-fifths of product

D, the over-all activity can be calculated in terms of any one of these products, as shown in the accompanying table.

Equivalent Quantity expressed in units of:

		Product A		*Product B*		*Product C*		*Product D*	
Product	*Quantity*	× *factor*	*units*	× *factor*	*units*	× *factor*	*units*	× *factor*	*units*
A	2,000	1	2,000	2	4,000	$\frac{1}{2}$	1,000	$\frac{2}{5}$	800
B	4,000	$\frac{1}{2}$	2,000	1	4,000	$\frac{1}{4}$	1,000	$\frac{1}{5}$	800
C	5,000	2	10,000	4	20,000	1	5,000	$\frac{4}{5}$	4,000
D	6,000	$2\frac{1}{2}$	15,000	5	30,000	$1\frac{1}{2}$	7,500	1	6,000
Total			29,000		58,000		14,500		11,600

Any product analysis in terms of the quantities as originally stated may be grossly misleading and a common denominator in the form of equivalent units is necessary. The total capacity is clearly defined in the table as 29,000 units of product *A*, or 58,000 units of product *B*, etc. This common scale, with the aid of the given conversion ratios, enables us to investigate the effects on the plant position by the expansion of one product at the expense of another.

In conjunction with multiproduct *P/V* charts, analysis of sales by products (Fig. 5–9) and analysis of profit by products should be carried out. The example given in Fig. 5–18 is clearly one that calls for action to reduce variety. The low total profit suggests that the plant is operating very near the BEP. It would also seem that products 2, 5, and 6 are the main causes for the low level of total profits. A *P/V* chart will show how far these products are below their respective BEP's, and if neither their volumes nor their *P/V* ratios can be substantially

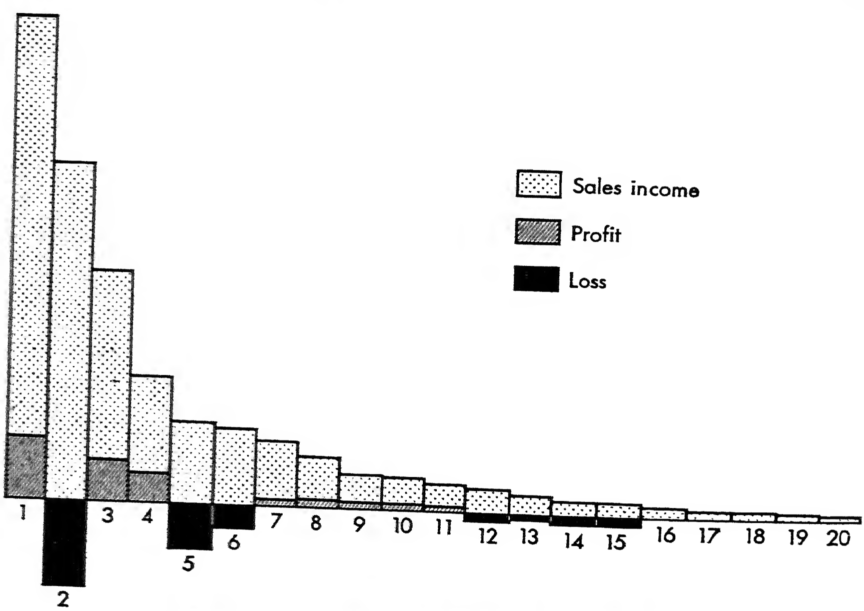


Figure 5–18. ***Analysis of profit by products.***

increased, management should seriously consider whether the elimination of these products from its schedule would be worth while. Furthermore, about half the number of products have very low sales incomes and profits (some of them even incur losses), and their inclusion in the production program should be questioned.

The Economics of a new design

When the launching of a new design or model is contemplated, a careful analysis of the economics of the proposed project has to be undertaken. The purpose of introducing a new model to the market may be twofold:

1. To increase the profit of the organization.
2. To avoid decline in sales of an existing model due to severe competition. Such a situation calls for incorporating novelty and new features in the company's products; even when no immediate increase in the profit is envisaged, it is aimed to achieve such an increase on a long-term basis.

The profit of an existing product is computed, using Eq. 5–5, as

$$Z_1 = \varphi_1 Q_1 - F \tag{5–5a}$$

where Q_1 is the number of pieces sold. If a new design is to be put into production, preparation costs incurred will include:

Design and engineering
Production planning
Tooling, jigs, and fixtures; resetting of machines, etc.
Purchase of special machines or equipment
Changes in layout

These preparation and "changeover" costs (symbolized as s) will have to be returned by the new design, so that the new profit should be

$$Z_2 = \varphi_2 Q_2 - F - s \tag{5–6}$$

It has been assumed here that the fixed costs are mainly dependent on the existing machinery of the organization and are therefore not likely to change very much. It is desirable that the new profit will be larger than, or at least equal to, the existing one or

$$Z_2 \geqslant Z_1$$

hence

$$Z_2 - Z_1 = \varphi_2 Q_2 - \varphi_1 Q_1 - s \geqslant 0$$

This condition tells us how many units of the new design ought to be sold in order to ensure that total profit does not decline:

$$Q_2 \geqslant \frac{s}{\varphi_2} + \frac{\varphi_1}{\varphi_2} Q_1$$

or

$$\frac{Q_2}{Q_1} \geqslant \left(1 + \frac{s}{Z_1 + F}\right) D \tag{5–7}$$

where D stands for the ratio

$$D = \frac{\varphi_1}{\varphi_2} = \frac{P/V \text{ ratio of old design}}{P/V \text{ ratio of new design}} \tag{5-8}$$

It is clear that unless the P/V ratio of the product can be greatly improved, the organization will have to sell more in order to justify the capital investment required for the introduction of the new design. If, for example, the P/V ratio of the new design remains at the same level as that of the old one,

$$D = 1$$

hence

$$\frac{Q_2}{Q_1} \geqslant \frac{s}{Z_1 + F} + 1$$

or

$$Q_2 > Q_1$$

This fact is illustrated in the profit-volume chart shown in Fig. 5-19. Line ① represents the existing product, yielding profit Z_1 when Q_1 units are sold.

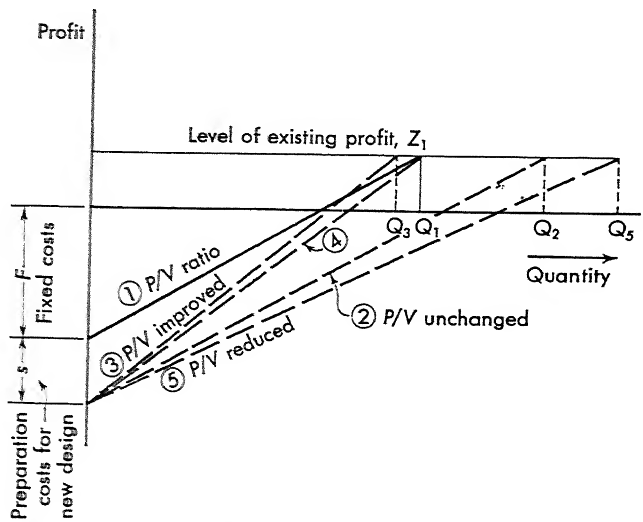


Figure 5-19. ***Effect of P/V ratio on the required market size of a new design.***

For a new design, the preparation costs are added to the fixed costs F. If the P/V ratio is unchanged (line ②), it is necessary to sell Q_2 units to obtain the same profit. From the similarity of triangles it is easy to see that in this case

$$\frac{Q_2}{Q_1} = \frac{s + Z_1 + F}{Z_1 + F}$$

which is what we obtain from Eq. 5-7 when $D = 1$. If an increase in the market

is not envisaged, the P/V ratio must be increased (line ③). It is possible to achieve the original profit at $Q_3 < Q_1$ if the P/V ratio is steep enough (line ③). However, even when the number of pieces sold is to remain unchanged ($Q_4 = Q_1$), it is necessary to have a higher P/V ratio than the existing one (line ④), while a decrease in the P/V ratio will increase appreciably the required sales volume (line ⑤).

Example

The annual fixed costs of a product are known to be \$200,000 and the annual net profit \$40,000, the average monthly sale being 820 units. A new design is contemplated, involving an expenditure for preparations amounting to \$80,000, to be returned in two years. It is expected that with new production methods the P/V ratio may be increased by 5 per cent. What should the annual sales figure for the new design be

(i) so that the same net profit will be realized;

(ii) so that in addition to this profit a yield of 10 per cent on the capital invested will be obtained?

Solution

(i) The ratio $D = 1.00/1.05 = 0.95$. The additional expenditure per year $s = 80{,}000/2 = \$40{,}000$.

$$\frac{Q_2}{Q_1} \geqslant \left(1 + \frac{s}{Z_1 + F}\right) D = \left(1 + \frac{40{,}000}{40{,}000 + 200{,}000}\right) 0.95 = 1.11$$

Annual sales required:

$$Q_2 \geqslant 1.11Q_1 = 1.11 \times 12 \times 820 = 10{,}820 \text{ units}$$

(ii) In the first year 10 per cent of the investment (i.e., \$8,000) has to be added to the profit, or $Z_2 = \$48{,}000$. The following expression for Q_2/Q_1 can be obtained by use of Eqs. 5–5a and 5–6.

$$\frac{Q_2}{Q_1} = \frac{s + Z_2 + F}{Z_1 + F} \times D \qquad (5\text{–}9)$$

Hence
$$\frac{Q_2}{Q_1} = \frac{40{,}000 + 48{,}000 + 200{,}000}{40{,}000 + 200{,}000} 0.95 = 1.14$$

or
$$Q_2 \geqslant 1.14 \times 12 \times 820 = 11{,}220 \text{ units}$$

Similarly, in the second year,

$$Z_2 = 40{,}000 + 4{,}000 = \$44{,}000$$

and
$$\frac{Q_2}{Q_1} = \frac{40{,}000 + 44{,}000 + 200{,}000}{40{,}000 + 200{,}000} 0.95 = 1.12$$

or
$$Q_2 \geqslant 1.12 \times 12 \times 820 = 11{,}020 \text{ units}$$

The case where the same profit should be realized by an unchanged volume of sales deserves special attention. It is often very difficult to forecast, let alone ascertain, an increased market for a new design, and considerations of "change-over" have to be based on the assumption that the sales volume will remain constant, i.e., $Q_1 = Q_2$. As already mentioned, this will require an improvement in the P/V ratio, which can be quantitatively determined by

$$\left(1 + \frac{s}{Z_1 + F}\right) D = 1$$

This relation is shown in Fig. 5–20. Any point on the curve refers to a resultant profit equal to the existing one, while for any point below the curve, an increased profit is implied.

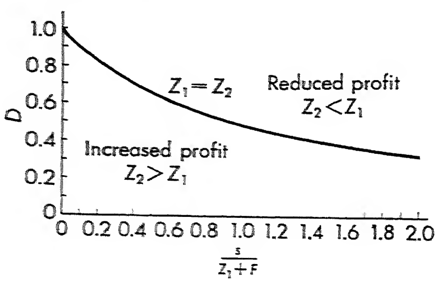


Figure 5–20. ***Required improvement of the P/V ratio when $Q_1 = Q_2$ to yield a profit $Z_2 > Z_1$.***

Example

In the preceding example, determine the required P/V ratio for the new design in order to yield at least $40,000 annual profit, assuming the market size remains unchanged.

Solution

$$\frac{s}{Z_1 + F} = 0.17$$

corresponding to (see Fig. 5–20)

$$D = \frac{\varphi_1}{\varphi_2} \leqslant 0.85$$

At present

$$\varphi_1 = \frac{F + Z_1}{Q_1}$$

$$= \frac{200{,}000 + 40{,}000}{820 \times 12}$$

$$= \$24.4 \text{ unit}$$

The required φ for the new design is

$$\varphi_2 = \frac{\varphi_1}{D} = \frac{24.4}{0.85} = 28.6 \text{ dollars/unit}$$

Production aspect

Last but not least in the list of factors influencing design is the production aspect. The product need not only be well planned on the drawing board, but its design must also be capable of being eventually translated into palpable fact. The designer must therefore face a multitude of practical production problems. "Design for production" has become a motto among designers. The following three aspects of production engineering have to be weighed:

1. Selection of processes that will be the most suitable and economical for the purpose. Such a selection will have to consider:

(i) The production quantities involved. Some processes are very expensive to operate unless used for a suitable production run.

(ii) Utilization of existing equipment. Such considerations may override acquisition of equipment for an ideally more suitable process.

(iii) Selection of jigs and fixtures and other production aids, the use of which may affect the design of components.

(iv) Sequence of operations and methods for subassembling and assembling.

(v) Limitation of skill. The selection of a process must be compatible with available skill and sometimes may be solely governed by it. Mechanized and pushbutton equipment is particularly suitable to nonskilled or semiskilled operators, but it is usually expensive to install and must be justified by long runs.

(vi) Application of new production processes. The designer has to consider not only conventional techniques but also the latest developments and research into newer production methods.

2. Utilization of materials and components with the view of:

(i) Selection of materials having appropriate specifications

(ii) Selection of method or design to reduce waste and scrap

(iii) Using standard components and assemblies

(iv) Having interchangeability of components and assemblies within the product.

3. Selection of appropriate workmanship and tolerances that satisfy quality requirements, but which are at the same time compatible with the precision and quality that can be attained through the available processes. Specification of quality may also affect the selection of processes.

These production aspects are analyzed in more detail in Chapter 8 and have to be considered in detail in the production planning stage. Their implications must, however, be understood by the designer right from the outset. Being an expert on production processes and methods is perhaps beyond the expected normal capacity of the designer, since the field is so wide that specialization in its various aspects is essential. Design and production must therefore be coordinated, and methods engineers have to be brought in at the design stage to contribute their part to the solution of the problems that arise from time to time.

Sometimes a project engineer is assigned the job of accompanying the product from the development stage to the production stage, in order to ensure continuity from one stage to the other and the integration of expert opinion at the right moment.

Summary

Product development and design is primarily governed by management decisions with respect to quality and pricing policy. A development program and a market survey can provide information as to market potentialities as well as functional, operational, dependability, and durability requirements and possibilities. Selection of the functional scope and application of standardization, simplification, and specialization principles are closely related to plant efficiency and to its net profit and must therefore be an integral part of management policy. The economics of a proposed new product or new model have to be analyzed in order to establish the market size that would justify production. Aesthetic considerations come normally at an advanced stage, but may sometimes be a dominant factor in design, especially with consumer goods. Finally, product development and design must be carried out with close liaison with the production departments, in order to ensure that the right materials and processes are utilized and that their implications are considered at a fairly early stage.

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Problems

1. From your own experience discuss examples where functional or operational features were the main cause for purchasing or discarding certain products.
2. A product involves \$6,000 per annum as fixed costs and yields \$3,500 profit. The sales income is \$16,000. Draw a profit-volume chart and find the profit-volume (P/V) ratio.
3. A plant manufactures products A, B, and C at the annual rate of 8,000, 6,000, and 4,000 pieces, respectively. Product A gives a P/V (profit-volume) ratio of \$1.75 per unit, but a loss of \$8,000 is incurred. Product B has \$32,000 fixed costs and has the same P/V (profit-volume) ratio as the whole plant. For product C the fixed costs are \$44,000. The net profit of the plant is \$15,000.
   (i) Draw a profit-volume chart as a function of plant output capacity when it is known that in terms of plant capacity 1 unit of $A = 1.2$ unit of $B = 0.5$ unit of C.
   (ii) It was suggested that the volume of product A should be increased to the break-even point to avoid a loss. This increase can be done at the expense of the volume for product B, while the volume of C remains unchanged. Find the net profit resulting from such a move.

(iii) Management policy demands a minimum annual volume of 3,000 pieces of each product. What production plan would you recommend with the present available capacity and what net profit would you expect?

4. A plant has a monthly sales income capacity of $96,000 and is producing two products, the data of which is as follows:

	Product A	*Product B*
Fixed costs, F	$16,000	$34,000
Break-even point	$43,000	$35,000
Profit	+ $ 8,000	− $ 3,000

In view of the high fixed costs and the loss incurred by product B, it was suggested to management that product B should be eliminated and production should be concentrated on product A. Analyze the situation and comment on this suggestion.

5. A plant was found to produce 25 products according to the following table:

Product	*Sales Income* ($)	*Profit* ($)	*Fixed Costs* ($)
1	308,000	32,800	62,000
2	209,000	15,200	41,000
3	83,000	− 8,200	19,000
4	74,000	5,200	16,000
5	41,000	8,200	8,000
6	35,000	7,800	7,000
7	21,000	− 4,000	4,000
8	13,000	− 1,500	2,600
9	11,000	− 1,100	2,200
10	6,000	500	1,200
11	5,000	400	1,000
12	5,000	400	1,000
13	4,000	200	800
14	2,900	300	600
15	1,800	300	500
16	1,700	− 500	500
17	1,600	− 400	500
18	1,000	200	500
19	1,000	200	500
20	1,000	100	500
21	800	100	300
22	800	− 100	300
23	800	− 100	300
24	800	− 100	300
25	800	300	300
Total	820,000		

Assuming the plant capacity is limited to the total sales income, explain how you would go about analyzing the data with the view to reducing the variety of products. What changes would you recommend? Compare the existing equivalent P/V ratio with the one obtained after the proposed simplification. What improvement can be envisaged in the margin of safety?

6. A product is manufactured at the break-even point. A committee has studied the situation and has recommended substituting the product by a new design, in spite of the fact that the additional setup costs will amount to 50 per cent of the fixed costs. The sales department has assured the committee that sales of the new product can be expected to be 20 per cent above the existing level. What are your views about the committee's report?

7. The break-even point of a product occurs at a sales income of \$120,000, but normally the sales income is \$180,000, the fixed costs being \$100,000. A new product involved additional costs of \$20,000, but the P/V ratio was improved by 20 per cent and sales income increased to \$240,000. What net profit did the new design yield?

8. If the P/V ratio remains unchanged when introducing a new product, explain how Fig. 5–20 can be used to assess the sales volume required, when the preparation costs, fixed costs, the existing profit, and the desirable profit are known.

9. Design for appearance is incompatible with design for durability. Discuss this statement.

6

SALES FORECASTING AND ESTIMATING

We have seen how the whole policy of product development and design is primarily dependent on the demands of the market, irrespective of whether this demand already exists or has to be created.

The effect of sales volume on the cost of production and the margin of safety at which the plant operates were also demonstrated in the previous chapter. If everything that is produced could be consumed by the market, the best policy would be to select that product which yields the highest profit-volume ratio or the one with the best prospects of doing so in the reasonably near future. Such a policy leads to increased specialization, with its obvious technical advantages. It facilitates concentrated effort in research and development in a narrow field, but it also makes the firm more and more dependent on one particular product. This dependence implies that a careful study of the market is warranted, to provide management with useful data on which to base its current development program, and to adjust production output and analyze the plant capacity in relation to total market demand.

When concentration of the plant facilities on the products with the highest profit-volume ratio is impracticable or impossible, owing to limited demand or to fundamental management policy considerations, the distribution and utilization of facilities (manpower, materials, machines, time, money), in producing different products or models depend on the relation between output and demand. If too much of one product is produced, the surplus has to be carried in stock for a long period. Moreover, available facilities are used up in overproduction so that opportunities to produce another product for which a demand exists will be lost.

All these contingencies emphasize the need for fundamental data in the form of sales forecasts, on which both product design and allocation of plant facilities can be based. The main purposes of sales forecasting and market research may be summarized as follows:

1. Data should provide the basis for decisions on production volumes; in

other words, the sales or demand forecast should provide enough data to facilitate the specification of production targets as a function of time.

2. Data should provide information about the relationships between demands for different products, so that a healthy balance of production can be worked out in terms of the quantities required of the various products—again as a function of time. It is obvious that such a balance will not only dictate the broad outline of the master production schedule but will also greatly affect the utilization of equipment and manpower, and hence the efficiency of the plant as a whole.

3. It follows that the sales forecast, by virtue of its providing the basis for the production schedule, is the starting point for budgeting too. Here, again, proper planning for the maximum utilization of the plant financial resources and for arrangement of credits can only be undertaken when the management decides on the volume of production after a thorough analysis of the sales forecast.

4. Inventory policy is also affected by the sales forecast, since problems connected with inventory are closely related to the production schedule and to budget policies.

5. Data are needed on which to base plans for the future of the plant as a whole. Is the demand for certain products likely to increase, and if so, by how much? Quantitative analysis of trends in demand are invaluable to the engineers occupied with problems of expansion and of major changes in production processes and methods.

6. Future trends are of great interest to those engaged in product development. If a certain group of products is facing a contracting market, owing to a multitude of possible reasons, should the company continue to invest in developing such products, or should it divert its resources and concentrate on others? These are considerations that may affect the more distant future of the company, and the purpose of market study is to provide some guidance in resolving these important issues.

7. Pricing policy may also be affected by market research. Pricing is related to the sales volume, but in addition to that, a scrutiny of the market provides information about the state of competition and affects pricing policy accordingly.

8. Finally, the results of market research may be useful not only for the purpose of sales control and for evaluating sales progress but also to plan the sales campaign as a whole, including the methods that should be adopted for distribution and sales promotion.

Thus, market research is an indispensable part of the planning function of the organization. The study of those factors in the market that contribute to the final sales forecast is so closely associated with sales activities that it is very often included as one of the functions of the sales department. In this chapter, however, we shall be mainly concerned with some of the techniques that contribute to a sales forecast, rather than with activities that are directed toward promoting sales. The reader interested in marketing should consult special treatises on the subject, some of which are mentioned at the end of this chapter.

Is Forecasting a Black Art?

Predicting the future is one of the most difficult problems which scientists have had to tackle. In a purely deterministic system, where an unequivocal relationship between cause and effect has been clearly established, it is possible to predict very accurately the course of events in the future, once the future pattern of causes is inferred from past behavior. Thus, in a set of experiments in classical physics, when a definite sequence of operations is performed, the outcome can be described in advance. Likewise, within a reasonable span of time the celestial bodies behave in a deterministic manner, and therefore their movements and positions in the future are predictable to a very high degree of accuracy.

Economic systems differ fundamentally from the above systems in two respects. First they depend on factors that have a very high degree of variability, so that any attempt to predict their behavior in deterministic terms has no justifiable basis. Secondly, the systems are highly complex in structure and do not lend themselves to description with the aid of a small number of major variables, as do many phenomena in classical physics. The interrelationships of variables within economic systems are so intricate that, even in deterministic terms, their effect on the behavior of the system is difficult to ascertain. Furthermore, economic systems are often nonstochastic in nature, and their future behavior depends on their development in the past. However, the effect of the past and present on the future varies greatly and the nature of this relationship is often not clear.

For all these reasons sales forecasting is not a precise business science, and the many attempts to simplify trends and behavior of forecasting methods, based on one or two variables, are therefore fundamentally invalid and prove to be unsuccessful. M. J. Moroney, a statistician, has written: "Economic forecasting, like weather forecasting in England, is only valid for the next six hours or so. Beyond that it is sheer guesswork." And J. Jewkes, the economist, has commented: "A major error of economists is the belief that they can predict the future. It cannot be too strongly emphasized that there is nothing in economic science which enables us to foretell events. Those who claim otherwise are dragging down their subject to the level of astrology." These are perhaps harsh remarks for the optimists, who tend to accept forecasting with a few slightly apologetic words. It is fair to say, however, that a comparison between sales forecasts and actual events in the past few decades reveals unsatisfactory discrepancies and indicates the inadequacy of existing knowledge of the subject. A familiar case in point is the forecasts that are expressed from time to time of national economic trends and the wide variations of opinions of various authorities. These different and sometimes conflicting views have one thing in common: Their authors try to simplify the system and mainly rely on a few selected variables, which they consider to be the dominant factors. By disagreeing on what the main factors are and by practically ignoring all the others, it is no wonder that different conclusions are reached.

Sales forecasts, however, are so vital to efficient production planning, that careful studies of a multitude of variables and whole systems, complex as they are, have to be undertaken. As those studies proceed and develop, the techniques for forecasting become more refined, more sensitive to detect interrelationship of variables and sometimes more accurate. Nevertheless there is little doubt that a great deal of further research into the behavior of economic systems will be required before sales forecasts become as accurate and as reliable a tool as the production engineer would like them to be.

What Restricts Consumption?

It was Adam Smith who stated that consumption is the sole end and purpose of all production. In the past century economists studied, evaluated, heatedly debated, and sometimes tried to modify this maxim, but the fact does remain that market research mainly revolves around the question: "What restricts consumption?" With an insatiable demand, there would be no need for sales forecasting, and production planning could be guided by technical considerations alone.

Hence, analysis of the factors that limit consumption can greatly contribute to the understanding of the market, its peculiarities, and its trends. These factors can be grouped under:

1. The product and its limitations
2. The consumer and his potential buying power
3. Competition, or values and utilities
4. Saturation
5. Distribution and promotion
6. State of business

The product and its limitations

This part of market research is of great importance to product development and design. The various aspects of product development were discussed in the preceding chapter, and an analysis of these may solve many important questions, such as: Does the product possess all the functional requirements that are desired by the customer? Would it be wise to expand its utility by modifying its design, and if so, how should this be done (design of attachments, the question of versatility, the problems of setting-up versus running times, etc.)? How should the design and specifications of the product affect the range of products to be produced, their sizes, the interchangeability of parts from one model to another, and the effects of variety on the production schedule?

What would the outcome of a simplification program be? Would the elimination of several products tend to enhance or to dampen the sales of other models or products of the company? Quantitative data are required to facilitate a logical analysis of simplification plans.

What is the market's attitude to the quality of materials used and the quality of workmanship in relation to its functional attributes and its price? What is the

effect of styling on the salability of the product, and to what extent is the periodic change of styling desirable? Would different packaging be more effective? What are the implications of maintenance, breakdowns, repairs, availability and prices of spare parts, and what conclusions should be drawn about the servicing policy of the company?

The consumer and his potential buying power

In order to understand the market and be in a position to prognosticate sales volumes, market research makes use of different kinds of collected data about the consumer.

First, who is the consumer—an individual, a family, a company? What numerical facts should be known about him? What likely changes in the composition of the population of consumers can be foreseen? Data about population structure, breakdown of wage earners, distribution according to ages and sex, distribution according to occupations and education, size and structure of families, rate of births and deaths and marriages, distribution of establishments according to size and main products—all these facts are readily obtainable from official statistical publications.

Secondly, facts about buying power are required. Naturally, distribution of wage earners according to income is of great interest. Also useful are statistics about trends in the national wealth, the state of employment, industrial indexes, and level of savings in relation to earnings. These figures are particularly helpful for the understanding of general trends in consumption.

Thirdly, the consumer's attitudes, opinions, and tastes have to be studied. How does he spend his money, his leisure? Here again, some existing statistics and surveys may enlighten us. These tell us, for instance, how much is spent on food, clothing, housing, furnishing, recreation, etc., as a function of personal incomes.[1] What are the causes, if these can be ascertained, for a spending structure of this kind? What groups of consumers deviate favorably from the average consumer, and would it be advisable to direct sales activities more toward them?

What do the consumers want to have, and how do their wishes compare with what they can get? This pertains to the basic question already posed in the preceding chapter: Should the organization try to give the customer what he wants or tell him what he should want? In many cases manufacturers choose to ignore the customers' wishes altogether, without properly evaluating the consequences. This is what President Eisenhower had to say on the subject in April, 1958:

I personally think our people are just a little bit disenchanted by a few items that have been chucked down their throats, and they are getting tired of them; and I think it would be a very good thing when the manufacturers wake up—and I am not

[1] Example: *Consumer Finances Survey*, published by the Federal Reserve System, and based on a sample of 3,000 spending units.

going to name names—and begin to give the things we want instead of things they think we want.

After-sales surveys also belong to this category. What does the customer think about the product? What features give him satisfaction, what aspects is he dissatisfied with, and for what reasons? From these surveys one can infer (i) about the reception of the product in the market, and (ii) about prospects of reselling to old customers.

Analysis of competition

Analysis of competition is essentially a study in values and utilities. The customer is constantly comparing the price of the product to the utility that he expects to get out of it. The word "utility" is used here in its broadest meaning and it should be understood to include the sense of prestige, pride of ownership, feeling of achievement, etc., apart from the satisfaction derived from the functional and economical attributes of the product.

Competition arises when several products or services have a claim to the consumer's limited spending potentialities. As he is unable to purchase everything that is offered but is obliged to remain within a certain budget, he proceeds to draw up a list of preferences. The competing products and services try to get on the customer's preference list and, broadly speaking, there are two ways of achieving this goal. The first is to lengthen the list, so that more items may be included in it. This may be done by increasing the available spending budget (raising the standard of living, credits, hire purchase) or by reducing prices. The second way is simply to push some other products off the list, and this is the essence of competition.

Competing for a share of the market

It now becomes obvious that competition is of two kinds: direct competition with similar products and indirect competition with other products or services. While the aim of the organization remains the same in the two cases (namely, to get on that preference list), the techniques of study and the strategy of sales promotion are basically different for the two cases.

In direct competition the ratio of utility to price of the competing products is submitted to the customer's sense of judgment, and he finally selects the one which to him seems to yield the highest ratio. This ratio, naturally, is not absolute. It depends on values attached to the various characteristics of the product (whether these values are attributed in a systematic manner or subconsciously), and these differ for different people and even for the same customer from time to time, depending on his reassessment of facts, situations, or objectives and sometimes even on his emotional reactions.

Products in direct competition are usually not identical. They have different features to offer, some of which may be more attractive to certain customers and unattractive to others. The distribution of demand becomes a function of the distribution of preferences and attributes of utilities, and each commodity

succeeds in capturing a certain proportion of the whole potential market accordingly.

The share of the market dominated by a product is a very useful index for judging the product position, and one of management's tasks associated with sales forecasting is an analysis of the needs, functional characteristics, attributes, and prices prevailing in the market, which contribute to this share. Management policy with respect to product development and design, appropriations for research and for modernization of plant equipment, and production methods and pricing policy as well as competitors' successes or failures are eventually reflected in the product share of the market. An efficient market analysis can point out to management the causes for fluctuations in the market share and the factors that contribute to the competitors becoming more consolidated, so that appropriate action can be taken. The facts about the competitors' position are not always easy to gather, as a great deal of information of this kind is guarded by companies as top secret. However, from a good market study a few accurate inferences can sometimes be made and later correlated with data that are eventually released, thus evaluating the methods of deductions and inferences. An example of a study in market sharing is given on page 115.

It was mentioned in the preceding chapter that, because the attribution of utilities is not absolute and since each product tends to capture a certain proportion of the market, some firms have been led to the erroneous conclusion that an addition of a competing product to the market is bound to reduce the share of existing products, and that it would therefore pay a firm to introduce such a product and thereby increase its total sales. This artificial increase in the variety can be achieved by marketing brands slightly different from the existing ones, or even by offering identical products under different names and packagings. The share for each product becomes smaller, but the total share of the firm (so it is argued) is increased.

There is no need for lengthy arguments to show how shortsighted such a policy is and that it can produce some very undesirable results. First, because the device of artificially increasing variety is open to the competitors as well, retaliation in this respect may soon restore the firm to its original position, or indeed even worsen it. Secondly, as we have seen in the preceding chapter, variety is bound to be reflected in increased production costs, and artificial variety may also involve increased costs for marketing and sales promotion. Thirdly, flooding the market with a multitude of similar brands may seriously injure the standing and reputation of the original product. Its personality, so to speak, may be lost in the crowd, and time and money invested in promoting its name may be irretrievably lost. Fourthly, the fundamental thesis that a new product is bound to capture a big enough share of the market to justify artificial increase in variety has not been proven. The effect of new product impact on the market is difficult to ascertain in advance; furthermore, one should distinguish between the short-term effect and the long-term consequences. An organization wishing

to increase its share of the market at the expense of its competitors should st
the reasons that induce customers to go to the competitors. A careful evaluat
of these data is a saner method on which to base management decisions.

Indirect competition with all products and services that have a claim to customer's limited purchasing power is a field of study that may throw sc light on market fluctuations and trends. This subject, however, contains ma intangibles and is far more difficult to evaluate quantitatively. A careful analy of the factors that promote the sales of indirect competitors is certai warranted, as well as a study of the customer's spending budget, his requi ments and tastes, and his environment and his community, as already discuss above.

Saturation

Demand is known to be affected by the stock that has already been absorb by the market in a given period of time. Beyond a certain level of consumptio demand tends to decrease as the supply increases. This is sometimes known the Law of Diminishing Utility, or simply the effect of a saturation proce The Law of Diminishing Utility states that the utility of a certain produ lessens in proportion to the quantity already purchased by the customer in given period. The more he gets of the product, the more his interest wanes acquiring some more, until a point is reached when the utility derived fro purchasing another unit is so small that the customer begins to prefer a differe product altogether. This is the point of saturation, which causes a product to relegated to a lower position on the customer's preference list or to be remov from it entirely. This has to be guarded against during two stages in selling:

Selling to satisfy demand

The Law of Diminishing Utility would suggest that when the demand for product begins to wane, owing to market saturation, changes in the produ characteristics are indicated. This may mean either switching over to a ne model or adding to the existing variety of products already offered by the com pany. Naturally, the problems of variety versus simplification, the changeov to a new model, and the timing of such an action have to be analyzed quant tatively, as shown in the preceding chapter.

Saturation is caused not only by the product under consideration but also b its rivals; hence all products in direct competition have to be taken into accour when analyzing saturation. Here, monopoly and patents may play an importar role in delaying or preventing competitors from coming on the scene and thereb securing a bigger share of the market before saturation is reached.

The effect of indirect competition, or the state of saturation of the market as whole, is also likely to be felt. The effect may be favorable when competin goods or services have been consumed to an extent that their demand begins t taper off. It may be unfavorable when a large-scale flooding of goods causes

general state of saturation and discourages buying as a whole. The problems of saturation caused by indirect competitors is a complex one, but a study of its causes and effects is often worthwhile.

Replacement

As the life of the product is limited, a demand for it will ensue when it fails to provide the service expected of it on account of wear. In the case of perishables and expendable goods, they may deteriorate and have to be scrapped after a period of time, even though they have not been used at all. As this can be interpreted to mean that long life restricts consumption, fundamental decisions regarding the quality of the product have to be taken. Should the product be made of inferior materials and poor workmanship in order to encourage early replacement? The case of nylon stockings is immediately brought to mind when one tries to compare the short life of a pair of nylon stockings today with that of the early specimens introduced to the market. Should, for example, a manufacturer of glassware decline and perhaps try to suppress inventions of unbreakable glass, in order to avoid early market saturation? Should decisions about inferior quality be deliberately taken, though the good name of the organization may be at stake? What effects would competition in the market have on the outcome of such a policy? How would it affect the prestige and status of the company's other products, and is it desirable to have a united policy in this respect for all the products? How would pricing be affected? With adequate data, a quantitative analysis, using operations research techniques, can be undertaken to determine what economic service life should best be aimed at. This "service life" can be translated into functional requirements and specifications of quality of material and workmanship and expressed in terms of production costs. Considering that this is one of the fundamental issues of the firm's long-term policy, which has far-reaching consequences on product development and design and on the firm's standing in the market, it is evident that replacement analysis is an intricate problem worthy of serious study.

Distribution and promotion methods

That distribution and promotion methods affect the sales volume is fairly obvious, and that is why market research (perhaps a more appropriate term here would be "marketing research") must include a proper evaluation of their effectiveness. Such an evaluation should cover an analysis of methods used in the past, appraisal of methods used by direct competitors, the possible application of new methods adopted from other fields, and the effect on the market of any change in the existing methods. This is quite a formidable task and a field in which it is difficult to predict with great accuracy. But again, an evaluation of this kind may be indispensable if better ways are sought under conditions of severe competition. Analyses of distribution and promotion methods may fall into three categories: advertising, effectiveness of distribution channels, and terms of sale.

Advertising

As mentioned in the preceding chapter, advertising policies depend on the type of product and its position in the market. The techniques of an advertising campaign would naturally be different for launching a new product than for maintaining an already well-established name. Different techniques also apply for a product with little competition in contrast with those for a market with keen competition; for a single product or for a range of products; for a market where a demand for this type of product exists; or when the customer must be educated to new uses, habits, or requirements. These fundamental considerations may also affect the amount of money spent on advertising. The Treasury Department estimated that in 1957 more than 7½ billion dollars were spent on advertising in the United States. On the average, corporations in the United States spend slightly more than 1 per cent of their receipts from sales on advertising, but there are great variations in different industries, as shown in Table 6–1.

Table 6–1

Expenditure for Advertising, 1957

(Some selected data)

Industry Group	*No. of Corporations*	*Advertising Expenditures*	
		Amounts Millions $	*Percentage of Sales and Gross Receipts*
Total	940,147	7,666	1.1
Agriculture, forestry and fishery	11,833	14	0.6
Construction	53,576	56	0.2
Manufacturing	138,566	4,447	1.4
Beverages	2,948	345	4.9
Food and kindred products	11,761	808	2.0
Tobacco manufactures	173	218	5.2
Textile mill products	5,293	97	0.7
Apparel and products made from fabrics	13,114	93	1.0
Furniture and fixtures	5,671	61	1.4
Paper and allied products	3,353	89	0.8
Printing, publishing and allied industries	16,368	69	0.6
Chemical and allied products	8,023	858	3.6
Petroleum and coal products	776	177	0.5
Rubber products	859	94	1.7
Primary metal industries	3,581	105	0.4
Fabricated metal products*	14,097	173	1.0
Machinery†	14,586	296	1.1
Electrical machinery and equipment	5,032	331	1.8
Wholesale	103,474	625	0.5
Retail	178,493	1,516	1.4
Finance	65,033	251	1.4
Services	90,597	303	1.7

* Except ordnance, machinery, and transportation equipment.

† Except transportation and electrical equipment.

Source: U.S. Treasury Department, Internal Revenue Service; Statistics of Income.

Effectiveness of distribution channels

There are four principal channels of distribution (Fig. 6–1). Nystrom and Frey[2] list these as:

1. Manufacturer direct to consumer.
   a. Manufacturer to household consumer by mail.
   b. Manufacturer to household consumer by door-to-door salesmen.
   c. Manufacturer to consumer by manufacturer's own retail store.
2. Manufacturer to retailer to consumer.
   a. Manufacturer to independent retailer, chain store, department store, or mail-order house to household consumer.
3. Manufacturer to wholesaler to retailer to consumer.
   a. Manufacturer to either full-service wholesaler or limited-function wholesaler to retailer to consumer.
4. Manufacturer to functional middleman to wholesaler to retailer (or, direct to retailer) to consumer.

Most industrial goods other than raw materials from the farm or mine are sold either direct to the industrial consumer or to one or two middlemen existing between the producer and the consumer. These two middlemen are the counterparts of the functional middleman and wholesaler in the marketing of consumer goods. Retailers as a rule do not sell in significant quantities to industrial consumers.

In connection with each of these channels, the manufacturer may use his own branches, either with or without stocks. Branch activities are at the wholesale level.

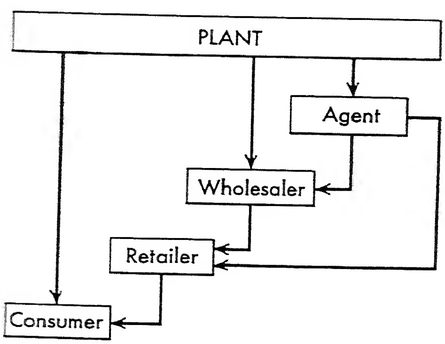

Figure 6–1. ***Basic channels of distribution for consumer goods.***

The manufacturer can either use one of these channels or a combination of them, and the question that has to be settled by marketing research is which method to adopt and how to measure its effectiveness. It may turn out that certain channels are better in some regions and worse in others. Or it may be that

[2] P. H. Nystrom and A. W. Frey (eds.): *Marketing Handbook* (The Ronald Press Co., 1948), p. 220.

selection of channels should mainly be governed by the type of product offered, in which case the question of unified distribution policy versus multidistribution policy should be carefully studied.

Another problem relating to distribution channels is that of allocation. How much should be allocated to each warehouse to minimize handling? How much of each brand should be channeled to each market, if a certain amount of interchangeability of models is possible? Problems in evaluation of channels and in allocation have been successfully tackled by operations research techniques, and for further study of these the reader should consult treatises on marketing.

Terms of sale

At what terms is it advisable to conclude a sale? This is obviously a fundamental question in planning distribution methods. It includes such issues as pricing policy and how it is affected by distribution channels and by market competition, hire-purchase policy and extent of credits, terms of discounts, problems of installation and servicing, training customers' operators, terms of guarantee, terms of supplying spare parts. All these may have a marked effect on sales efficiency on the one hand and on the firm's financial position on the other.

In evaluating sales promotion methods, the firm is anxious to know whether the best possible use is made of advertising appropriations and what effects might be expected if these appropriations were to be changed. This is an important issue on which management has to make a decision, and the question of evaluation is particularly acute when suitable advertising media have to be selected. The methods mostly used are mentioned in Table 6–2, which gives the estimated expenditure breakdown in the United States by advertising media for 1958. The important role played by newspapers in advertising is quite apparent (a third of the total). The direct approach by mail is also greatly favored (15.4 per cent), and advertising by television is becoming more and more popular (13.2 per cent in 1958 as compared with 3.0 per cent in 1950). A great deal of research that evaluates advertising effectiveness has been carried out in recent years, and the published results are invaluable when special cases are analyzed, both for the general background that is thereby readily obtainable and for correlation purposes.

State of business

Business as a whole is subject to fluctuations, usually referred to as "business cycles." There are periods of boom when sales are increasing and business is expanding rapidly. There are periods of recession, periods of shrinking markets, and contraction of business. These business fluctuations may account for more unstableness in the firm's position and for more deviations of actual sales from the forecast than those that can be attributed to factors within the firm's own sphere of influence.

Table 6–2

Estimated Expenditure in the United States by Advertising Media, 1958*

Medium	*Percentage of Total Expenditure*		
Total—National			61.5
Total—Local			38.5
Newspapers			
National	7.5		
Local	23.5		
		31.0	
Radio			
Network	0.6		
Spot	1.8		
Local	3.6		
		6.0	
Television			
Network	6.9		
Spot	3.9		
Local	2.4		
		13.2	
Magazines			
Weeklies	4.1		
Women's	1.5		
General	1.5		
Farm	0.3		
		7.4	
Farm papers		0.3	
Direct mail		15.4	
Business papers		5.1	
Outdoor			
National	1.3		
Local	0.6		
		1.9	
Miscellaneous			
National	11.6		
Local	8.1		
		19.7	
Total			100.0

* Taken from the Statistical Abstract of the United States, 1960.

Source: Compiled by McCann-Erickson, Inc., for Printers' Ink Publications, New York, N.Y., published in Printers' Ink.

Pricing policies

The question of pricing policy, for instance, has been mentioned as one of the basic factors affecting sales activities. But prices as a whole, it appears, are associated with business cycles, as can be seen from Fig. 6–2, which shows the change in consumer price index over a ten-year period. Apart from general trends

in prices, fluctuations of short-term duration are also clearly discernible, particularly in Fig. 6–3, which shows wholesale price indexes since 1926. Another interesting factor apparent from these curves is that different commodities are affected in different ways: Some are more sensitive and their prices fluctuate more violently than others; also, the times of peaks and troughs are not identical for all. In the absence of any available laws that describe business cycles and

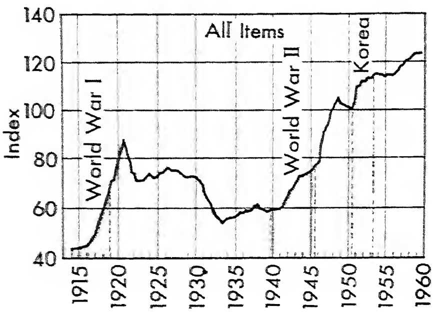

Figure 6–2. ***Consumer price index; 1947–1949 = 100.*** (Courtesy U.S. Department of Commerce, Bureau of the Census. Source: Department of Labor, Bureau of Labor Statistics)

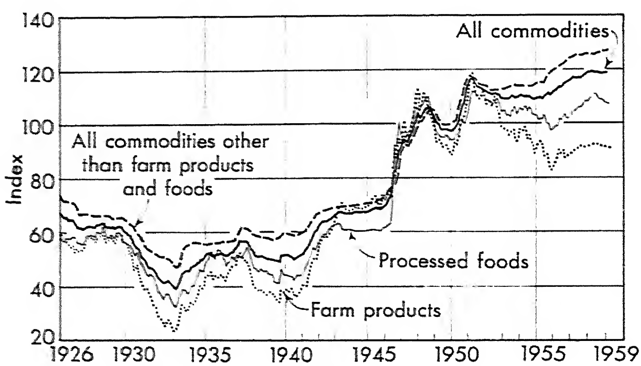

Figure 6–3. ***Wholesale price indexes; 1947–1949 = 100.*** (Courtesy U.S. Department of Commerce, Bureau of the Census)

assess their consequences quantitatively, the sales forecaster can only extrapolate from his estimates on the basis of general trends alone. The magnitude of the small fluctuations and their randomness would therefore indicate what accuracy is a reasonable objective in sales forecasting. An example of sales variations is given in Fig. 6–4, where indexes of department store sales in the United States are shown. It is interesting to note the effect of these variations on the level of stocks held by the department stores.

Business cycles

Extensive research has been carried out to establish the causes of business cycles, but a treatment of the subject is beyond the scope of this book. Suffice it

to say that the prevailing view among some economists is that business cycles are associated with inventories, or rather with the ratio of inventories to sales. Fig. 6–5 shows the variations of sales, inventories, and the inventory-sales ratio

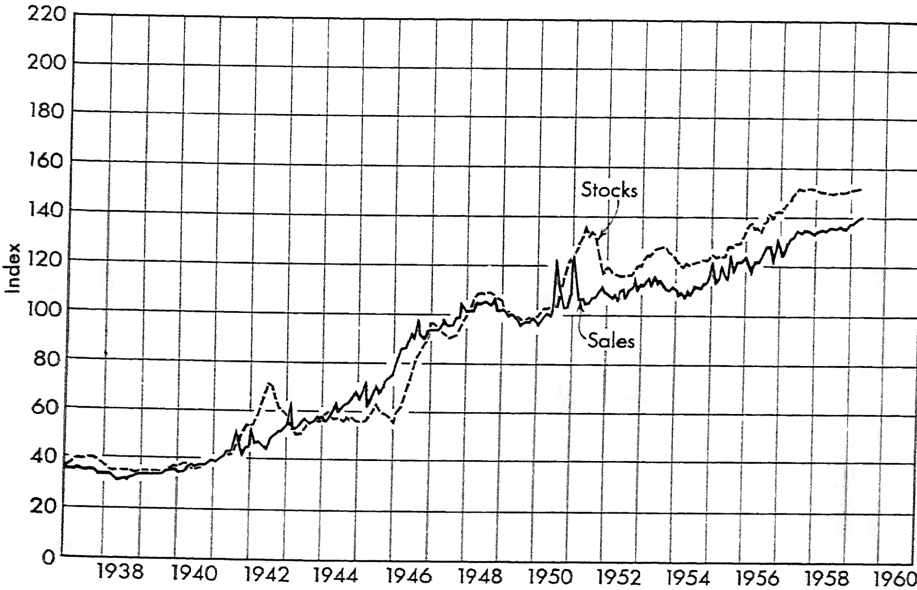

Figure 6–4. ***Department stores, sales, and stocks; 1947–1949 = 100.*** (Courtesy U.S. Department of Commerce, Bureau of the Census)

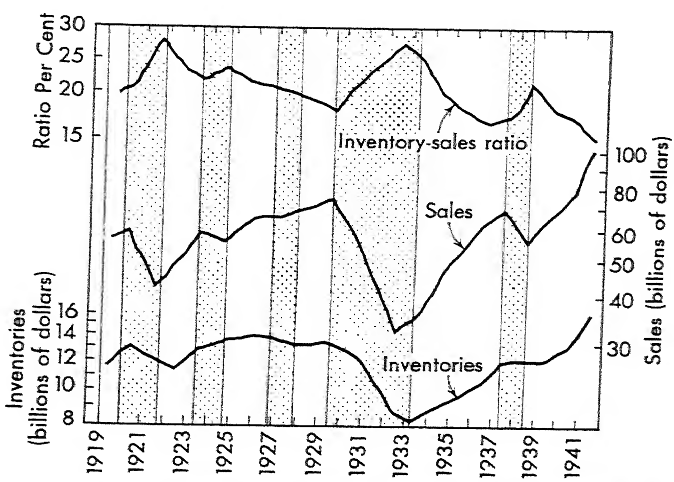

Figure 6–5. ***Inventories and sales and their relation to business cycles.*** (From A. Abramowitz. *Inventories and Business Cycles*, Chart 12, National Bureau of Economic Research Inc., Gallery Press, New York, 1960)

for 1919–1941. The shaded areas correspond to contraction periods in business and are characterized by the fact that the curve of inventory-to-sales ratio has an upward trend, while in periods of expansion this ratio tends to decrease.

Making the Forecast

We have listed above the various factors that the forecaster has to bear in mind when attempting to make a sales forecast. Naturally, some of these factors may be more predominant than others, depending on the product, the industry, and the circumstances. Experience tells which factors have to be meticulously analyzed and which need only a superficial study. The first question that has to be settled, therefore, is: What data should be used for the sales forecast? Such data can be obtained in two ways: published sources and research activities of the firm's sales department or outside agencies engaged for the purpose.

1. Statistics from available publications of public institutions, professional bodies, and others. Valuable statistics are gathered by various reliable sources, and innumerable facts, usually of a general nature, are then readily available to the firm with little expenditure of time or money. Data of national character are usually published by several government departments (e.g., Census of Manufactures (U.S. Bureau of the Census); Survey of Current Business (U.S. Department of Commerce); publications of the Central Statistical Office of the U.K., etc.). Data about general trends and the state of business are also found in economic and financial papers and journals, while specific information about the industry and the fluctuations in its sales are sometimes provided by trade or manufacturers' associations. A useful booklet entitled "Market Research Sources," gives a list of current publications in which abundant statistical information for market research in the United States is available.

2. Facts collected by the firm's own resources, mainly from the firm's past sales and other records, collected by those in direct contact with customers (salesmen, retailers, agents, etc.), by sampling polls (questionnaires by mail or interview, telephone, etc.), and by experts who specialize in assessing new markets, techniques, or products. This research can be carried out by the firm's own people or by agencies, or both, depending on the scope, scale, and degree of specialization required for tackling the job.

After the facts have been assembled, the task of evaluating the data commences, and this is carried out by using statistical methods and inference. Indeed, the knowledge of statistical methods is required right at the start, especially when sampling techniques are employed for the purpose of gathering data, in order to ensure the validity and relevance of the information obtained and a reasonable degree of accuracy within the limitations of time and money that are available. Thus the essence of this analysis is a critical evaluation of the methods of gathering the facts, and of attempts at predicting trends and future behavior. A few current methods of prediction are listed below.

The market share

It has already been mentioned that market share may sometimes serve as a guide to sales forecasting. An example of a study in the market share is given in Table 6–3, where the percentage share of a company is quoted for ten years. In spite of the fluctuations in this share, no trend is apparent, and on the average the company manages to secure 12.5 per cent of the market. Suppose that in the tenth year the total volume of sales of this industry was $40 million and that from general economic trends and the position of the industry as a whole, an expansion of sales of 20 per cent was expected for the eleventh year. In view of this projected expansion, it was decided to launch an entirely new model, which had been developed by the "Ron Company" for a number of years. The company's sales soared to $6.4 million in the eleventh year. Is it reasonable to conclude that this increase in business was due to the new model?

Table 6–3

The Market Share of "Ron Company"

Year	*Share in % of Total Sales of the Industry*
1	12.6
2	13.0
3	11.9
4	12.4
5	13.0
6	13.2
7	12.1
8	11.6
9	12.7
10	12.5
Average in past 10 years	12.5

Now, if the sales volume of the industry came to $48 million (20 per cent above $40 million), it means that the company secured (6.4/48) 100 = 13.3 per cent of the market. The percentage figures cited in Table 6–3 and $\pm 2\sigma$ control limits are shown in Fig. 6–6. These control limits mean that for 95 per cent of the cases in a stable and normal situation, the share percentage would be between 11.5 and 13.5. It appears that the figure for the eleventh year would be still within the control limits and could easily be attributed to chance alone. That does not mean to say that the new model did not contribute to the company's welfare by its position in the market, perhaps in the face of competitors' new models and latest innovations. Had it not been launched, perhaps the company's share would have receded, but to talk in the present circumstances about the company gaining position in the market is being unduly optimistic, in spite of the fact that sales went up from $5 million to an all-time high figure of $6.4 million.

It is clear that even when the percentage share remains fairly constant and when the sales volume of the industry can be accurately determined, it is too much to expect a forecast within close limits on this score alone. In the example cited above we know that the company secures one-eighth of the total market, but this figure of the share must be accompanied by the tolerance of ± 1.0 per cent of the total market, and in 95 per cent of the cases we may expect the figure to fluctuate within these limits. In our case, 1.0 per cent of the total market

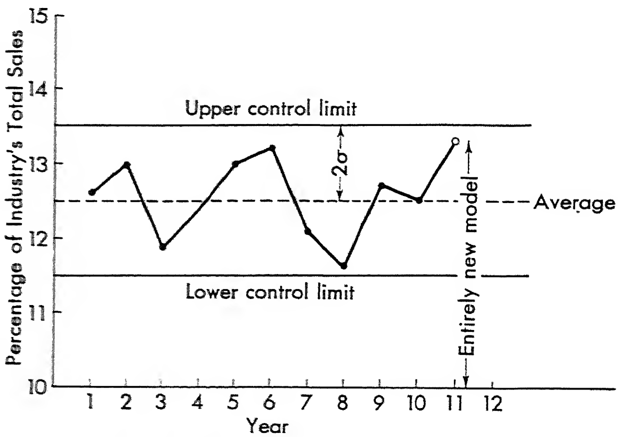

Figure 6–6. ***Market share analysis of "Ron Company".***

corresponds to 8.0 per cent of the sales volume, and coarse as this figure may seem, it is unreasonable to expect a higher degree of accuracy from this method under the described circumstances. When the picture of market share is more erratic and subject to wide variations, the use of this method becomes rather limited, as in the case of some consumer nondurable goods (such as certain foods, chemicals, and cosmetics).

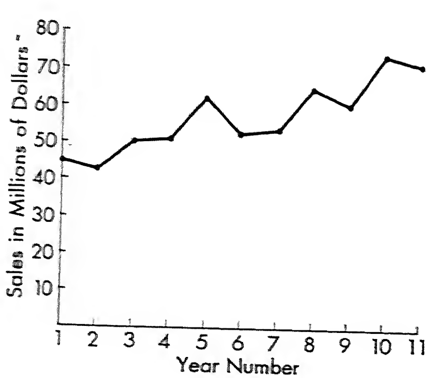

Figure 6–7. ***Sales record of A & Z Corporation*** (See Table 6–4)

Sales trend analysis

Within the framework of general economic trends a study of the company's sales records is indispensable for forecasting.

What are the main principles by which prediction should be guided? Perhaps the safest thing to say is that prediction primarily depends on the appraisal of the future situation in relation to the past. If it is established that a pattern of sales repeats itself fairly accurately in a given period of time, then the study of past behavior is the answer to the future. Not only can the forecast be made fairly accurately in that case, but it can be made a long time in advance, which is a great advantage to management planning. A somewhat similar situation occurs when the trend of events is quite obvious, though the pattern does not repeat itself. Suppose we had sales records of a company as given in Table 6–4 and Fig. 6–7. In spite of the fluctuations it appears that sales are increasing as a linear function, which may serve as a basis for forecasting. The first problem, therefore, is to describe this trend quantitatively.

Table 6–4

Annual Sales of A & Z Corporation

Year	*Sales in Millions of $*
1	45.0
2	42.5
3	50.1
4	50.6
5	62.0
6	52.0
7	53.5
8	64.3
9	60.1
10	73.6
11	71.0

Many methods can be used to fit a straight line to a given scatter, which suggests a linear trend, but the most widely acceptable method is that of the least squares. By this method a straight line is defined in such a way that the sum of the squares of the differences between the ordinates of the suggested line and those of the given points is at a minimum. The data in Table 6–4 may be represented by coordinates, so that the first year is denoted by $x_1 = 0$, the second by $x_2 = 1$, etc., and the appropriate sales figures by $y_1 = 42.5$, $y_2 = 50.1$, etc. The straight line that we want to fit in (called the *regression line*) is expressed by the equation

$$Y = a + bx \tag{6–1}$$

where a is the intercept on the Y axis (at $x = 0$) and b the slope of the line

(see Fig. 6-8). If Y_1, Y_2, etc., are the ordinates of this line at x_1, x_2, etc., th condition of the least squares demands that

$$(Y_1 - y_1)^2 + (Y_2 - y_2)^2 + \cdots + (Y_{10} - y_{10})^2 = \min$$

or

$$\Sigma(Y - y)^2 = \min \tag{6–2}$$

and with this condition the position of the regression line (i.e., the values o a and b) can be determined.

First substitute the value of Y from Eqs. 6–1; thus the condition is

$$\Sigma(a + bx - y)^2 = \min$$

or

$$\Sigma(a^2 + b^2x^2 + y^2 + 2abx - 2ay - 2bxy) = \min$$

The variables are the two unknowns a and b. Of the family of lines having a given slope b, one satisfies this condition when

$$\frac{\partial}{\partial a}\,\Sigma(a^2 + b^2x^2 + y^2 + 2abx - 2ay - 2bxy) = 0$$

hence

$$\Sigma\,(a + bx - y) = 0$$

or

$$\Sigma(Y - y) = 0 \tag{6–3}$$

which merely means that the sum of the differences in ordinates is zero. If there are n points, the last condition can be expressed as

$$na + b\Sigma x = \Sigma y \tag{6–4}$$

Similarly, of the family of curves that have a known intercept a, the one that satisfies the condition of least squares has a slope b, so that

$$\frac{\partial}{\partial b}\,\Sigma(a^2 + b^2x^2 + y^2 + 2abx - 2ay - 2bxy) = 0$$

hence

$$\Sigma(ax + bx^2 - xy) = 0$$

or

$$a\,\Sigma x + b\,\Sigma\,x^2 = \Sigma\,xy \tag{6–5}$$

a and b can now be determined from the Eqs. (6–4) and (6–5), yielding

$$a = \frac{(\Sigma y\,\Sigma x^2) - (\Sigma x\,\Sigma xy)}{(n\Sigma x^2) - (\Sigma x)^2} \tag{6–6}$$

$$b = \frac{n\Sigma xy - (\Sigma x\,\Sigma y)}{n\Sigma x^2 - (\Sigma x)^2} \tag{6–7}$$

The computation of a and b for the above example is illustrated in Table 6–5.

Table 6–5

Computations for the Determination of a Regression Line
A & Z Corporation (See Table 6–4)

Year	x	y Sales ($ in millions)	x^2	xy
1	0	45.0	0	0
2	1	42.5	1	42.5
3	2	50.1	4	100.2
4	3	50.6	9	151.8
5	4	62.0	16	248.0
6	5	52.0	25	260.0
7	6	53.5	36	321.0
8	7	64.3	49	450.1
9	8	60.1	64	480.8
10	9	73.6	81	662.4
11	10	71.0	100	710.0
Totals	$\Sigma x = 55$	$\Sigma y = 624.7$	$\Sigma x^2 = 385$	$\Sigma xy = 3{,}426.8$

Results from this table can be substituted into Eqs. 6–6 and 6–7:

$$a = \frac{(624.7 \times 385) - (55 \times 3{,}426.8)}{(11 \times 385) - (55)^2} = 43.0$$

$$b = \frac{(11 \times 3{,}426.8) - (55 \times 624.7)}{(11 \times 385) - (55)^2} = 2.8$$

Thus, the equation of the line is

$$Y = 43.0 + 2.8x \tag{6–8}$$

and it is plotted in Fig. 6–8.

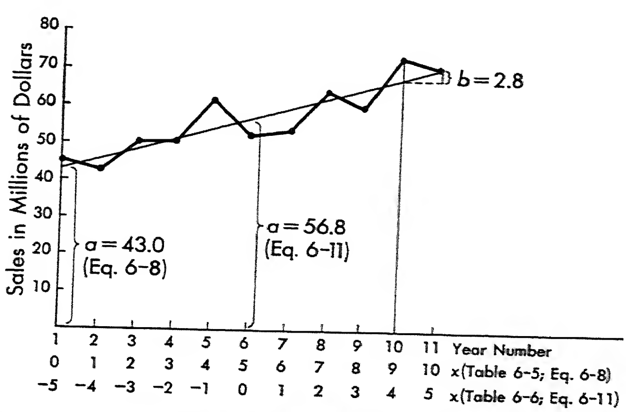

Figure 6–8. ***Sales regression line for A & Z Corporation***
(See Tables 6–5 and 6–6)

The computation may be somewhat simplified when x is selected in such a manner that $\Sigma x = 0$, so that the formulae 6–6 and 6–7 are reduced to

$$a = \frac{\Sigma y}{n} \tag{6–9}$$

$$b = \frac{\Sigma xy}{\Sigma x^2} \tag{6–10}$$

It is still necessary to find Σx^2, Σy, Σxy, and this is shown in Table 6–6. The location $x = 0$ is simply taken at the middle point of the given range for the x's, when these are known at uniform intervals. In our case $x = 0$ is therefore taken at the sixth year.

Table 6–6

Modified Computation for a Regression Line
A & Z Corporation (See Table 6–4)

Year	x	y	x^2	xy
1	−5	45.0	25	−225.0
2	−4	42.5	16	−170.0
3	−3	50.1	9	−150.3
4	−2	50.6	4	−101.2
5	−1	62.0	1	− 62.0
6	0	52.0	0	0
7	+1	53.5	1	+ 53.5
8	+2	64.3	4	+128.6
9	+3	60.1	9	+180.3
10	+4	73.6	16	+294.4
11	+5	71.0	25	+355.0
Totals	$\Sigma x = 0$	$\Sigma y = 624.7$	$\Sigma x^2 = 110$	$\Sigma xy = 303.3$

It is apparent from Table 6–6 that now $\Sigma x = 0$, so that Eqs. 6–9 and 6–10 can be used:

$$a = \frac{624.7}{11} = 56.8$$

$$b = \frac{303.3}{110} = 2.8$$

and the equation of the line is

$$Y = 56.8 + 2.8x \tag{6–11}$$

Comparison of the two results, Eqs. 6–8 and 6–11 reveals that the slope is the same and only the intercepts are different. This is only to be expected, as the origin is placed differently in the two cases. These intercepts (43.0 and 56.8, respectively) are signified by the ordinates at $x = 0$, but in the first case the ordinate is related to year 1; in the second, to year 6 (see Fig. 6–8). To see

whether condition 6–3 (regarding the sum of the differences in coordinates of the line and the given data) is satisfied, examine the calculations carried out in Table 6–7.

Table 6–7

Examination of the Regression Line

Year	Y (*calculated by Eq.* 6–8)	$Y - y$	$(Y - y)^2$
1	43.0	−2.0	4.0
2	45.8	3.3	10.9
3	48.6	−1.5	2.3
4	51.4	0.8	0.6
5	54.2	−7.8	60.8
6	57.0	5.0	25.0
7	59.8	6.3	39.7
8	62.6	−1.7	2.9
9	65.4	5.3	28.1
10	68.2	−5.4	29.2
11	71.0	0	0
		$\Sigma(Y - y) = 2.3$	$\Sigma(Y - y)^2 = 203.5$

The fact that $\Sigma(Y - y)$ is not exactly zero is due to the inaccuracies in calculations, since arithmetically $b = 2.76$ and not 2.8, and this slight discrepancy accumulates as we proceed along the regression line. This also accounts for $Y_6 = 57.0$, whereas through the calculations of Table 6–6, the intercept $a = Y_6 = 56.8$. The inaccuracy, however, is very small and it can be justifiably ignored. The reader can try to repeat the above calculation for $\Sigma(Y - y)$ by using Eq. 6–11 instead of Eq. 6–8 and show that the discrepancy is thereby greatly diminished. This is because the accumulated error due to the computational inaccuracy in determining b is smaller when an ordinate of a point on the regression line is calculated at the middle of the range rather than at one end of it.

The standard error of estimate can now be found by the formula

$$s = \sqrt{\frac{\Sigma(Y - y)^2}{n}} \tag{6–12}$$

This standard error of estimate simply implies that 95 per cent of the data are expected to fall within limits of $\pm 2s$ of the regression line. In our case

$$s = \sqrt{\frac{203.5}{11}} = 4.3$$

Here, again, it should be noted that the computational inaccuracy of b causes a slight error in s, but within the limits of accuracy of our data, this result is adequate. Control lines of $\pm 2s$ are shown in Fig. 6–9. Now, if the sales forecast for the twelfth year is $74 million, being the ordinate of the regression line for

year 12, and the actual sales turn out to be $80 million, the error in forecasting can be attributed to chance fluctuations and not to the inadequacy of the forecasting method. What in fact is predicted in this case is the trend of events, not every individual event by itself. When the actual sales figures are ascertained, their deviations from the mean trend can be checked by the control chart. If the points are in control, the fluctuations can be attributed to chance variations and the method for forecasting may remain unaltered. If a point falls outside the control lines, the reason for this departure from the general trend must be sought, and the situation can then be re-evaluated (such as in the case of the thirteenth year in Fig. 6–9).

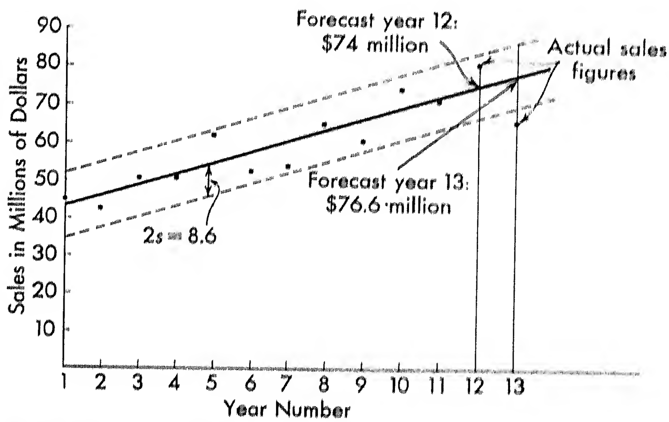

Figure 6–9. ***A control sales chart with a regression line.***

The case is more complicated when trends are not linear but are related to functions of a higher degree or are periodic in nature. Basically, however, the approach is the same: The past trend is expressed in a mathematical form, which provides a tool for predicting the future trend. Fluctuations are watched with the aid of control limits, deviations are studied for possible causes, and the mathematical formulation of the general trend is critically evaluated and modified as more actual sales figures become known.

Forecasting in seasonal demand

The study of seasonal demand is very important for production and inventory control. The nature of sales fluctuations from one season to the other in terms of amplitude and regularity and the general trend (if any) on which these fluctuations are superimposed present an intriguing problem to forecasters. Here, again, the past is analyzed with the view to finding some pattern that may give a clue to the future behavior of the system.

First, to illustrate a seasonal demand without the effect of general trends, a

three-year sales record of a company is given in Table 6–8 and Fig. 6–10. A preliminary examination of this record shows that the three years do not substantially differ from each other. Sales soar in December and dip in July, the

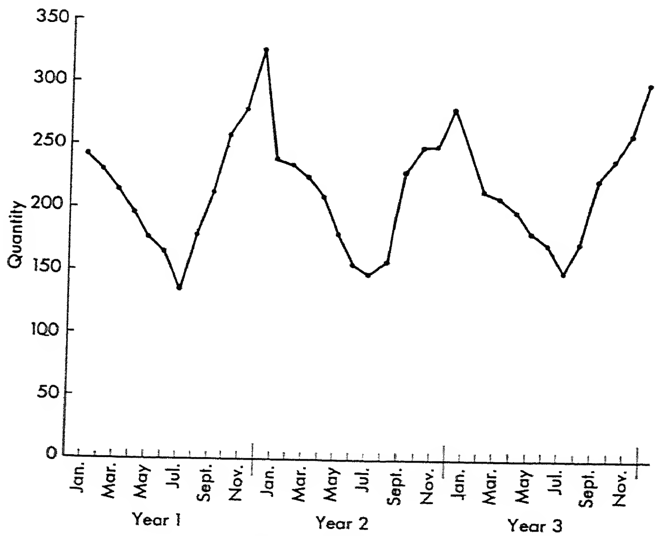

Figure 6–10. ***A three-year sales record of B & Y Incorporated*** (See Table 6–8)

ratio between the two months sometimes exceeding 2:1. The annual sales do not seem to fluctuate much, the difference between the first and second year being about 2½ per cent.

Table 6–8

Three Years' Sales Record with a Seasonal Demand; B & Y Incorporated

	Year 1	*Year 2*	*Year 3*
Jan.	242	238	250
Feb.	232	235	215
Mar.	215	226	210
Apr.	196	210	200
May	176	180	182
June	165	156	172
July	135	148	150
Aug.	181	158	175
Sept.	214	231	225
Oct.	261	250	240
Nov.	278	250	260
Dec.	328	280	300
Total	2,623	2,562	2,579

The seasonal cycle becomes more evident when a 3-month moving total is plotted (Table 6–9, Fig. 6–11). A 3-month total for any month is defined here as the sum total of sales in that particular month and in the two preceding it. Thus the 3-month total for March of year 1 is the sum of the sales during January, February, and March of year 1 (i.e., $242 + 232 + 215 = 689$). The 3-month total for April is obtained by taking the total for March, subtracting from it the sales of January, and adding the sales of April ($689 - 242 + 196 = 643$), etc.

Figure 6–11. ***A three-month moving total*** (See Table 6–9)

In this way the 3-month totals are established for each month (Table 6–9) and the moving total can be plotted (Fig. 6–11). Similarly, a 12-month total is defined as the sum total of sales in the last 12 months. The 12-month total of December is thus equal to the annual sales. The 12-month total for January is obtained when sales for the current January are added and the sales of the preceding January are subtracted from the 12-month total for December, etc. The 12-month totals for B & Y Inc. are indicated in Table 6–9, and the moving total is shown in Fig. 6–12.

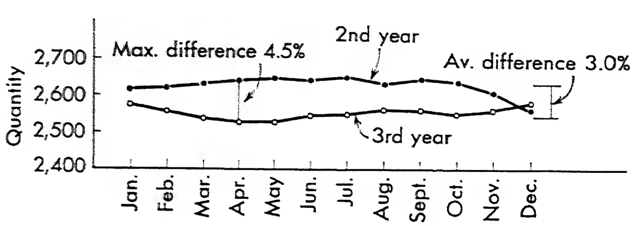

Figure 6–12. ***A twelve-month moving total, B & Y Incorporated*** (See Table 6–9)

Table 6–9

Averages and Moving Totals; B & Y Incorporated

(See Table 6–8)

	3-Month Moving Total				*12-Month Moving Total*	
	Year 1	*Year 2*	*Year 3*	*Average*	*Year 2*	*Year 3*
Jan.	. . .	844	780	812	2,619	2,574
Feb.	. . .	801	745	773	2,622	2,554
Mar.	689	699	675	688	2,633	2,538
Apr.	643	671	625	646	2,647	2,528
May	587	616	592	598	2,651	2,530
June	537	546	554	546	2,642	2,546
July	476	484	504	488	2,655	2,548
Aug.	481	462	497	480	2,632	2,565
Sept.	530	537	550	539	2,649	2,559
Oct.	656	639	640	645	2,638	2,549
Nov.	753	731	725	736	2,610	2,559
Dec.	867	780	800	816	2,562	2,579

The curve of the moving total has the advantage that it removes the crests and valleys of the monthly fluctuations and shows the seasonal regularity that repeats itself in the three years. The number of months that should be taken for a moving total depends on the length of the cycle. As a rule, the more months (or other time units) are included in the moving total, the smoother the final curve, the more it damps crests and valleys, and the less details it can offer about the cycle characteristics. It is like studying geographical topography: When the observer stands on the surface of the earth, he can see all the undulations, but his vision is limited. As he rises above the surface, his vision has a wider scope, the details that obstructed his sense of judgment become slightly obliterated and can be more objectively placed and related to the general topography of the area. But when the observer rises too high, details become too blurred, until it becomes impossible to compare heights of areas. In order not to lose sight of too many details of the demand cycle, a 3-month moving total is generally adopted when the seasonal cycle is of 12-month duration. It is readily seen from Table 6–9 and Fig. 6–12 that when a moving total covering the whole cycle is taken (in this case 12 months), the seasonal fluctuations disappear completely. This moving total, however, can be used to bring out long-term cycles and general annual trends in the sales volume.

The 12-month moving total in Fig. 6–12 appears to be fairly steady. The average of this total in the third year is lower by 3.0 per cent than that of the second year, and the maximum discrepancy between the two years (in April) is 4.5 per cent. As no apparent trend of this curve can be ascertained (the yearly average of the totals seems to be greatly affected by December sales), the fourth year may be assumed to resemble the past, provided there is no reason to expect changes in the state of business in general or in the type of commodities this firm is marketing.

The forecast in this case can be based either on the average for each month of the past three years (Method A) or on the average 3-month moving total (Fig 6–13) (Method B), depending on what we believe is the basic behavior of

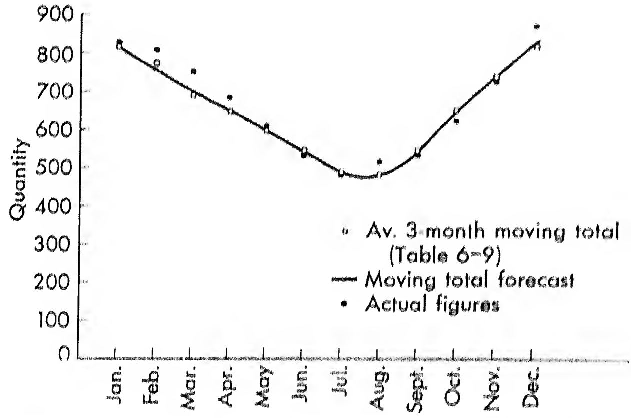

Figure 6–13. ***A three-month moving forecast for B & Y Incorporated*** (See Table 6–10)

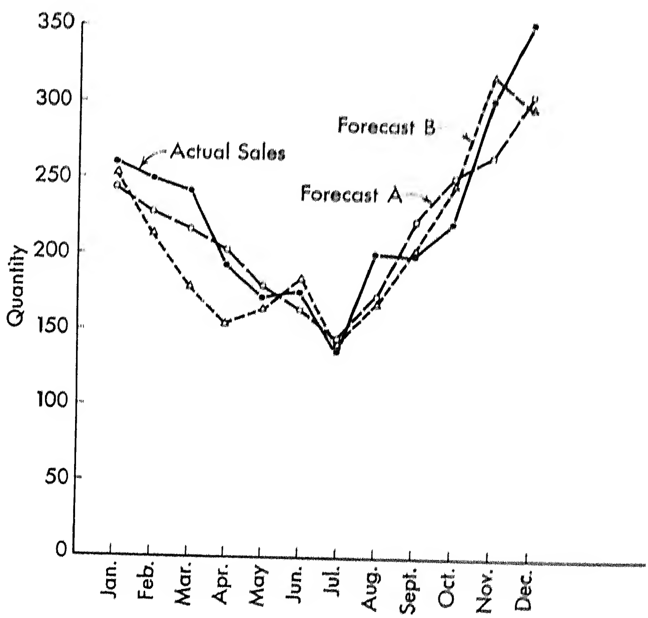

Figure 6–14. ***Sales forecasts for B & Y Incorporated*** (See Table 6–10)

the system. If we have reason to believe that monthly sales are independent of previous months, the first method should be adopted, while preferring the second method implies that we perceive a compensating mechanism in the system, whereby undersales last month will tend to increase sales this month, etc. The two methods are shown in Table 6–10, where the forecasts are compared with actual sales figures. The absolute error (arithmetic addition of errors, irrespective

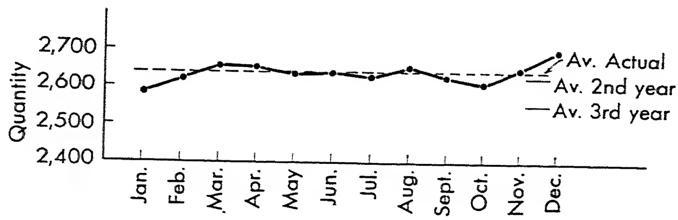

Figure 6–15. ***Twelve-month moving total fourth year, B & Y Incorporated*** (See Table 6–10)

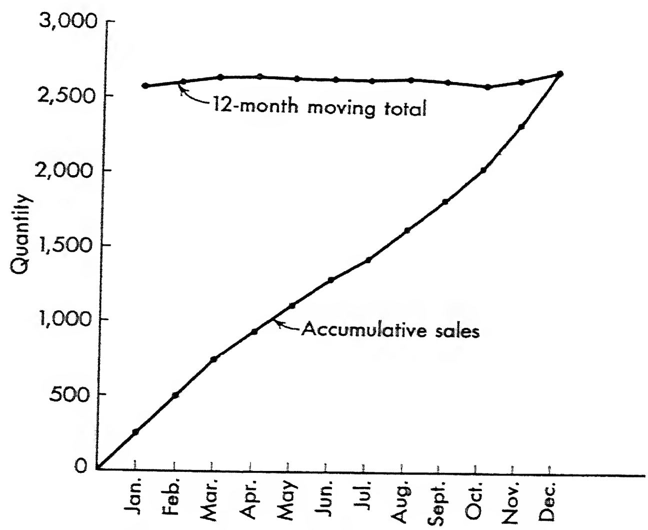

Figure 6–16. ***Accumulative sales (four-year), B & Y Incorporated***

of their sign) is about 10 per cent for both forecasts, but it appears that Method A is preferable here because the total error is smaller than that obtained by Method B. The comparison of forecasts with actual sales figures is given in Figs. 6–14 and 6–15, and the accumulative sales curve is shown in Fig. 6–16.

Table 6–10

Sales Forecast for B & Y Incorporated

(See also Table 6–9)

	Method A		*Method B*					
	Monthly Average of Last Three Years	*Monthly Forecast*	*3-month Moving Forecast*	*Previous 2-month Sales*	*Forecast = difference between last two columns*	*Actual Sales (see Fig. 6–14)*	*Error* Forecast (A)	*Error* Forecast (B)
Jan.	243	240	810	560	250	260	− 20	− 10
Feb.	227	230	760	560	200	250	− 20	− 50
Mar.	217	220	700	510	190	242	− 22	− 52
Apl.	202	200	650	492	160	193	+ 7	− 33
May	179	180	600	435	170	171	+ 9	− 1
June	164	160	540	364	180	175	− 15	+ 5
July	144	140	490	346	140	138	+ 2	+ 2
Aug.	171	170	480	318	170	200	− 30	− 30
Sept.	223	220	540	338	200	200	+ 20	0
Oct.	250	250	640	400	240	220	+ 30	+ 20
Nov.	263	260	740	420	320	300	− 40	+ 20
Dec.	303	300	820	520	300	350	− 50	− 50
				Total		2,699	−129	−179
				Absolute error			265	273
				Total error, %			− 4.8	− 6.6
				Absolute error, %			9.8	10.1

A trend analysis in seasonal demand

The case of a seasonal demand with an upward trend can be tackled in a similar manner. The 12-month moving total in Fig. 6–17 evidently shows an upward trend in sales and this trend is linear, as far as one can judge, the yearly increment being Δ units. This means that, on the average, the sales volume of each

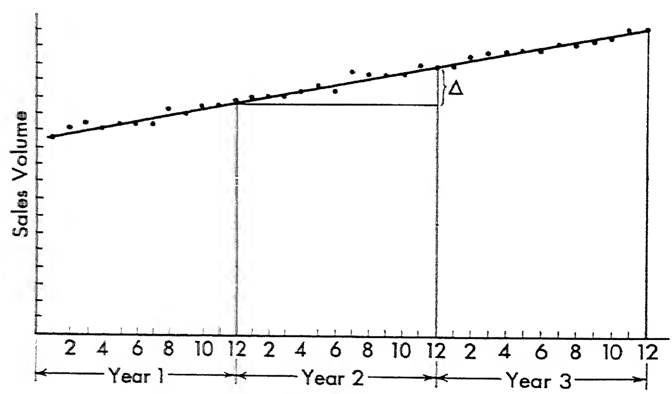

Figure 6–17. ***A twelve-month moving total with an upwards sales trend***

month will exceed the corresponding month in the preceding year by Δ. The sales record for the past three years are given by the matrix:

Month	*Year 1*	*Year 2*	*Year 3*
Jan.	$v_{1,1}$	$v_{2,1}$	$v_{3,1}$
Feb.	$v_{1,2}$	$v_{2,2}$	$v_{3,2}$
Mar.	$v_{1,3}$	$v_{2,3}$	$v_{3,3}$
.			
.	$v_{1,j}$	$v_{2,j}$	$v_{3,j}$
.			
Dec.	$v_{1,12}$	$v_{2,12}$	$v_{3,12}$

where $v_{i,j}$ is the sales volume in the jth month of the ith year, etc. Had the general trend remained stationary, this record would read in terms of the next year's sales as follows:

Month	*Year 1*	*Year 2*	*Year 3*	*Average*
Jan.	$v_{1,1} + 3\Delta$	$v_{2,1} + 2\Delta$	$v_{3,1} + \Delta$	$V_1 = \frac{1}{3}(v_{1,1} + v_{2,1} + v_{3,1}) + 2\Delta$
.				
.				
.				
Dec.	$v_{1,12} + 3\Delta$	$v_{2,12} + 2\Delta$	$v_{3,12} + \Delta$	$V_{12} = \frac{1}{3}(v_{1,12} + v_{2,12} + v_{3,12}) + 2\Delta$

In other words, all the figures have been "translated" to next year's level; hence the forecast based on the monthly average of previous years is simply F_1 (forecast for January next year) $= V_1$; $F_2 = V_2$; etc. Similarly, when the existing record consists of n previous years, the forecast becomes

$$F_j = \frac{1}{n}\sum_{i=1}^{n} v_{i,j} + \frac{n+1}{2}\Delta \tag{6–13}$$

and other types of trends (nonlinear, cyclic, or combination of both) can be dealt with in very much the same way.

An interesting example of seasonal demand with an upward trend is illustrated in Fig. 6–18, where monthly figures for domestic electricity consumption for

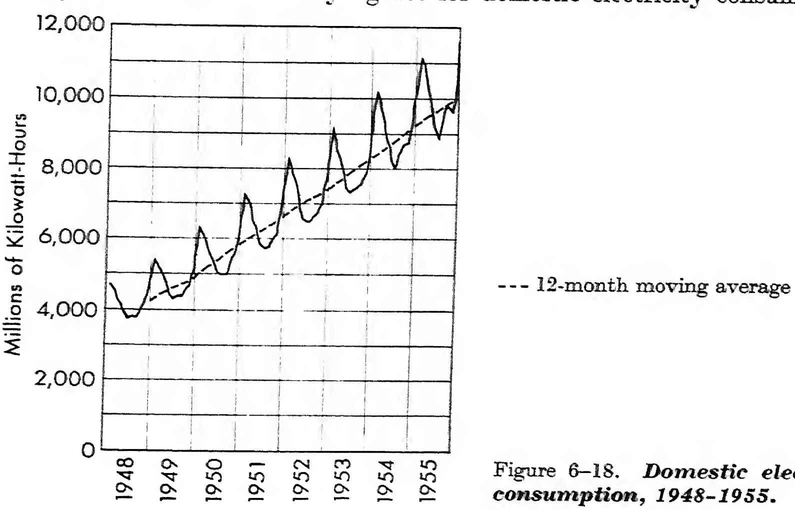

Figure 6–18. ***Domestic electricity consumption, 1948–1955.***

1948–1955 are plotted. Peak consumption is registered in the winter months, mainly in January, and a trough is recorded in the summer season. In the 12-month moving average curve the seasonal fluctuations disappear and the trend seems to be so clear that a graphical extrapolation is fairly easy, although mathematically its function may be more difficult to express because the average curve is not linear.

The use of indicators and correlation analysis

Analysis of trends as a function of time is a fascinating problem, and the interested reader will find a lot of literature published on the subject in recent years. This analysis may be simple enough when definite trends as a function of time can be ascertained. However, in most cases the system is sensitive to other factors, and the purpose of a correlation analysis to show the relation between cause and effect. Suppose we had product sales that follow the Federal Reserve Board Index after a time lag of three months, and that the relation between sales and the FRB Index is suggested by Fig. 6–19. This correlation implies that once the FRB Index is published, the sales volume can be predicted well in advance. Naturally, the correlation has to be constantly checked, and the curve must be modified as more data become available, but as long as the correlation is valid and suggests a cause-and-effect pattern of events, the FRB Index in this case may be considered what is termed the *predictor* or *indicator*.

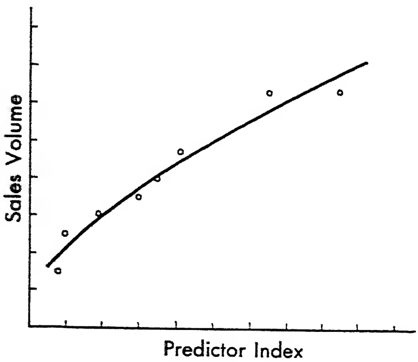

Figure 6–19. ***A simple correlation analysis.***

In the absence of clear-cut cause-and-effect systems, forecasters often resort to the use of any predictor that appears to indicate in advance trends in the behavior of the system. For instance, when a correlation analysis reveals that the sales of a commodity x moves synchronously with those of a given company's product, the curve for x leading by a known time interval, then x may be a very useful predictor. Sometimes one commodity has to be chosen to predict an upsurge in sales and another to indicate an impending decline. The selected predictors, however, are sometimes so remotely related to the system under consideration that one can only shake one's head with skepticism at the loudly proclaimed reliability and accuracy that are attributed to any forecasting mechanism in which these predictors play a predominant role.

When the expression of sales in terms of one variable is inadequate—and this is usually the case—a number of factors have to be taken into account in what is called *a multiple correlation analysis*. An example of a graphical presentation of such a relationship in terms of two variables is shown in Fig. 6–20. The sales function $V = f(x,y)$ is determined after a correlation with factors x,y is carried out. Thus, for a situation where $x = 160$, $y = 14$, we may expect a forecast as shown by the arrow in Fig. 6–20. The mapping of such curves can often be made possible only when a considerable amount of reliable data have been accumulated.

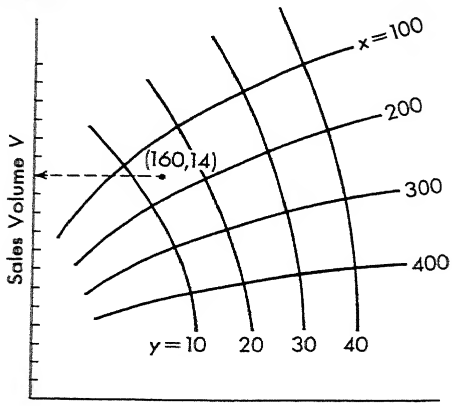

Figure 6–20. ***Sales volume as a function of two variables x and y.***

In a multiple correlation analysis, when more than two variables are used, the forecast is normally expressed as a formula in terms of several variables. Often this is a linear expression, such as

$$F = a_0 + a_1x_1 + a_2x_2 + \cdots + a_nx_n \tag{6–14}$$

where the a's are coefficients and the x's are the predictors. This is an attractive method, but its main difficulty lies in the selection of the relevant variables. Mathematicians have shown time and again that they are capable of producing excellent expressions that accurately fit past events. To conclude that these will also fit the future, or rather that the future will fit into their framework, is often too presumptuous. Apart from the fact that different expressions can be devised to represent the same past, it has been stressed by many authors that the crucial step in multiple correlation analysis is the choice of variables. This preappraisal—with its assumed objectivity, natural prejudices, or mere ignorance of the facts—determines the success or failure of the analysis far more than the accuracy of the mathematical methods used to arrive at the final expressions.

Combination of methods

When great benefits are to be derived from accurate forecasts (as, for instance, in consumer goods), companies very often prefer to rely on more than one method. Results obtained from different sources and by different techniques can thus be compared, and from its past experience the company can judge the relative

values and reliability of the methods used. A company may, for instance, have the following methods:

1. A forecast extrapolated graphically from trend of sales
2. A simple or multiple correlation analysis
3. Total of sales managers' (or agents' or retailers') estimates
4. Total of product by product (and/or model by model) forecasts

Results from the four methods may then be compared and analyzed by an authoritative body within the company and a procedure set up to finalize the forecast figures.

Effects of Forecast on the Production Order

It would be erroneous to assume that the monthly forecasts can be issued to the production department in the form of a production order. The discrepancies between the forecasts and reality are reflected in the stock level, and it is often desirable to take this fact into account when the production orders are worked out.

A case of such an effect of the forecast on the production order is shown in Table 6–11, which is related to the example of seasonal demand discussed above. Suppose it is decided to have a safety stock of an average monthly sales (in this case 200 units, column 1). The available stock at the beginning of the month (column 2) is the same as the balance at the end of the preceding month (column 5), which is obtained simply by taking into account the difference between production and sales figures (or: column 5 = column 2 + column 3 − column 4). The target stock desirable at end of next month is the safety stock plus the forecast figure of the next month but one (the forecasts are taken from "Method A" column, Table 6–10). The quantity required from the production department is, therefore, obtained in the following way:

Stock at end of month (column 5) *minus* quantity believed will be sold next month (column 6) *plus* quantity that should be produced next month (column 8) *equals* target stock at end of next month (column 7) or

$$\text{column } 8 = \text{column } 7 - \text{column } 5 + \text{column } 6.$$

Example

The quantity ordered from the production department at end of August for September is

$$450 - 386 + 220 = 284 \sim 280 \text{ units}$$

This example is based on several assumptions that perhaps should be pointed out here:

1. There is no time lag in obtaining information about sales volumes in the current month and the figures are available when the production order is to be specified—which in reality is not always so.

2. No lead time is required to put the order into effect, and the quantities produced are indeed equal to those specified in the previous respective months (see Table 6–11).

Table 6–11

Effect of Forecasts on Production Orders

	1	2	3	4	5	6	7	8
Month	*Safety Stock*	*Available Stock at Beginning of Month*	*Produced This Month*	*Sales This Month*	*Balance at End of Month*	*Forecast Next Month (Table 6–10, Method A)*	*Target Stock at End of Next Month = Safety Stock + Forecast of Month Next but One*	*Order to Produce Next Month*
Dec.		450	300	300	450	240	430	220
Jan.	200	450	220	260	410	230	420	240
Feb.	200	410	240	250	400	220	400	220
Mar.	200	400	220	242	378	200	380	200
April	200	378	200	193	385	180	360	160
May	200	385	160	171	374	160	340	130
June	200	374	130	175	329	140	370	180
July	200	329	180	138	371	170	420	220
Aug.	200	371	220	200	391	220	450	280
Sept.	200	391	280	200	471	250	460	240
Oct.	200	471	240	220	491	260	500	270
Nov.	200	491	270	300	461	300	450	290
Dec.	200	461	290	350	401	250		

The method, however, can be easily modified to overcome these difficulties. When final sales figures are not available, estimates can be used, and if production schedules have to be planned more in advance and the desirable quantities in column 8 cannot always be complied with, the computation should be adjusted accordingly.

The effect of the forecast on stock variations is shown graphically in Fig. 6–21. For the sake of simplicity it is assumed here that the quantity produced each month is transferred to stock only at the end of the month; hence the saw-shaped curve with the vertical upsurges, which indicate the monthly quantities produced. Inaccuracies in forecasting lead to fluctuations in the minimum stock, the level of which is quite often below the specified desirable safety stock. Figure 6–22 shows that for a hypothetical case, when forecasts are accurate and coincide with the actual sales figures, the specified safety stock is invariably reached at the end of the month. This difference between the two cases has an effect on the average quantity held in stock and is of prime importance when decisions about the size of the safety stock have to be taken. Naturally, in a system that has a pattern as shown in Fig. 6–22, the risk of running out of stock is smaller than that in Fig. 6–21, and a smaller safety stock may be allowed (assuming all the other conditions and considerations are the same).

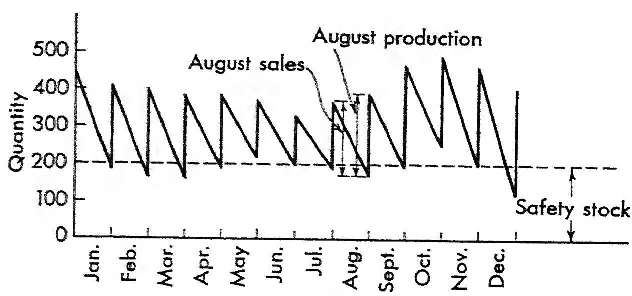

Figure 6–21. ***Effect of forecast on stock variations.***

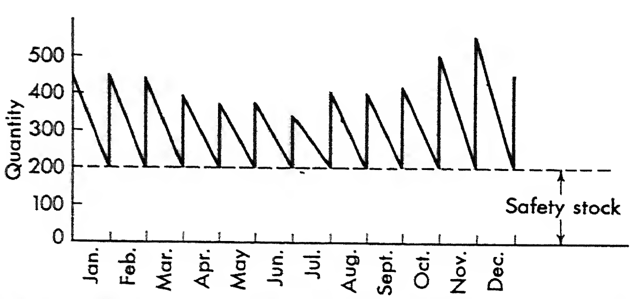

Figure 6–22. ***Effect of forecast on stock variations (an hypothetical case of accurate forecasting).***

Accuracy of Forecasts

What accuracy is achieved in sales forecasting in practice? An interesting survey of sales-forecasting practices, which was carried out by the American Management Association in 1956, covered 297 companies, about two-thirds of which had an annual sales volume less than $50 million (the median of annual sales being $27 million). Some results of this survey are given in Table 6–12. Of the 248 companies that responded to the question of forecasting accuracy, it appears that all the companies belonging to the class of consumer nondurable goods (group G) could boast of having forecasts within 10 per cent. In all the other groups, however, the scatter of errors was rather wide, and while some companies seemed to enjoy an accuracy of 0 to 2 per cent, others seemed to be off the mark by 30 and even 50 per cent. The average error was 8 per cent, while the median was down to 5 per cent.

Table 6–12

Accuracy of Sales Forecasting in Practice

(AMA Survey in U.S., 1956)

Group	*Type of Product*	*Number Surveyed*	*Number Responded*	*Error of Forecast (%)*		
				Average	*Median*	*Range*
A	Capital and industrial goods; e.g., machine tools, telephone, broadcasting, railway and materials handling equipment	38	32	9.2	6	2–29
B	Industrial and office accessories; e.g., pumps, instruments, light office machinery, industrial furniture	29	26	5.9	4	0.5–30
C	Components, machine parts, and assemblies	75	58	11.0	9	0–50
D	Materials for manufacturing; e.g., chemicals, metals, plastics, rubber	47	42	7.5	6.5	1–28
E	Industrial and office supplies; e.g., lubricants, paper and packaging materials	12	11	7.1	5	0.5–20
F	Consumer durable goods; e.g., automobiles, electrical appliances	34	30	8.7	8	0–22
G	Consumer nondurable goods; e.g., foods, drugs, cosmetics	38	29	4.2	4	0.3–10
H	Miscellaneous; e.g., agricultural equipment, military aircraft	9	7	7.7	4	2–15
I	Services and trade; e.g., transportation, insurance	15	13	6.2	3	1–30
	Total	297	248	8.0	5	0–50

However, it must be borne in mind that the errors cited are deviations of the actual *total* sales of each company from the *total* sales forecast. Some of these companies deal with thousands of products each, and it is therefore obvious that the forecasting experience of the company varies greatly with the products. In a report to the American Management Association conference on sales forecasting in 1956,[3] the Corning Glass Works, Corning, New York, stated the following errors in total sales forecasts for the company (comprising 22 sales departments organized on trade-channel lines and covering a variety of consumer, technical, and electrical products):

Forecast for 1952, − 8% (i.e., 8% below actual sales)
Forecast for 1953, − 9%
Forecast for 1954, +13%
Forecast for 1955, + 4%

It was stated, however, that "in a breakdown of the individual components, the record shows that, for some sales departments, our forecasts have been very good; for others, they have been quite wide off the mark."

Case Studies in Sales Forecasting

Numerous case studies in forecasting have been published; some are detailed, some lack relevant information, some relate methods that have been specially developed for one particular firm or industry and which are of little use to other firms. But essentially their importance lies in their evaluation of their own methods, in the accuracy achieved, and in the price the company has to pay for inaccurate forecasts. Two examples are related below.

Capitol Records, Inc., Hollywood, California[4]

This company's products include "single" records (disks usually with one selection on each side) which are released as 7-inch 45 rpm, and "album" records (with several features), which are made in long-playing 12-inch $33\frac{1}{3}$ rpm and in 7-inch 45 rpm. The company reported having 26 branches and 10 distributors.

The main problem is inaccurate forecasting because the market may vary considerably in size and the product very often has a short life. Most of the records have a sales volume of 25,000 units, but once in a while a record may sell more than a million units in 90 days. Overproduction due to high sales expectation leads to obsolescence; underproduction results in lost sales opportunities, which may sometimes be quite considerable if "hit" records fail to be recognized as such when forecasts are made. In order to rectify the shortcomings of underestimates in the forecast, the production schedule has to be very flexible indeed to respond to market demands, but owing to the extremely short-lived popularity

[3] American Management Association: Sales forecasting, uses, techniques and trends (Special report No. 16, 1956).

[4] From reports published in the AMA case studies in Production Forecasting, Planning and Control (Manufacturing Series No. 223, 1957).

of some records, this is not always possible. Hence the major task of forecasting is to try to minimize obsolescence on the one hand and risks of underproduction on the other, and this in the face of a highly variable and unpredictable market.

First the company had to establish trends in public preference of the various types of products offered to the market. It was found that the 78-rpm disk market was contracting and that the 12-inch $33\frac{1}{3}$ rpm disk was becoming more and more accepted, though the preference of size and speed of disk was still a function of the repertoire. Although definite trends in this category still seem to be discernible, the question of forecasting obviously becomes more complicated when, in addition to trying to tell how much will be sold of each song, the forecast must also provide the breakdown as to sizes and speeds of records.

The sales forecast committee at Capitol Records Incorporated prepares semiannual forecasts: a final forecast for the coming six months and a tentative one for the succeeding half-year. The methods used are:

1. Field forecast, based on forecasts prepared by district sales managers
2. Sales trends, based on past record sales by product lines
3. Over-all phonograph sales forecast, prepared by the market research department and based on predicted share of the market percentage
4. Album product forecast, prepared by the market research department and based on analysis of promotion methods in the future as related to successes in the past

The committee correlates results of all these methods and produces the final forecast.

In short-term forecasting (one month in advance), on which the record release schedule is based, the progress and success of each artist are closely studied and actual past sales show whether an artist is a consistent performer in terms of sales, in which case future sales of his records may be more accurately predicted. The distribution of an advance batch is another method for testing public reactions before the general distribution of the record. These reactions may help in reassessing the short-term sales forecasting.

It is very interesting to mention the accuracy that has been achieved by these forecasting methods in an industry so vulnerable to the public whims and often unaccountable preferences. No data are given in report to the AMA, but later the company released the following figures of errors in its semiannual forecasts for its potential business:

Date of Forecast	*For the period*	*Error of Forecast in Relation to Actual Sales*
November, 1956	Jan.–June, 1957	−11%
April, 1957	July–Dec., 1957	+ 2%
October, 1957	Jan.–June, 1958	+15%

The error of +15 per cent in the forecast for January to June, 1958, was attributed to the effects of the Spring recession of 1958, which were greater than anticipated, and to the influence on the market of stereophonic disks, which were

introduced earlier than anticipated. The forecasters believe that by currently reviewing their methods, they have been constantly improving their accuracy.

Eastman Kodak, Rochester, New York[5]

The company produces a variety of products, but the report to the AMA is confined to photographic products (which account for two-thirds of the total sales). These include durable goods, nondurables, and services. The techniques used for sales forecasting vary from product to product.

First a general economic survey is prepared twice yearly, and it includes an economic forecast for the coming year and for a five-year total in terms of gross national product, disposable income, and several selected indicators. After approval by top management, the review is passed on and explained to division and department managers in written form.

Sales forecasting is generally done by two methods.

The trend cycle method: i.e., by extrapolation of past trends. The results are reviewed in the light of the economic forecast and modifications are also made after examining selected predictors that have proved reliable in the past.

Multiple correlation method: i.e., by use of mathematical formulae, which include what seem to be the main variables (such as demographic factors, price ratios, and economic indicators).

After the first analysis, the company uses the following procedure:

1. A statistical projection is prepared, based on this analysis and on the economic forecast, and then modified by such factors as the competitive situation, size of dealer inventories, pricing policies, and sales promotion plans.

2. Conferences are held with executives responsible for the company's policy, who can contribute and adjust the forecasts.

3. The Finished Products Committee (made up of a vice-president and top managers responsible for sales, production, advertising, finance, etc.) gives its final approval.

These methods have been constantly evaluated and modified, and the company claims that in the period 1932 to 1956, the sales forecast deviated from actual sales by less than 10 per cent in all but 4 years (the exceptions: 1932, 1937, 1941, 1950, the last two being connected with United States entering a war), while in 15 years the deviations were less than 5 per cent.

Summary

Sales forecasting is an indispensable preplanning tool, that guides the firm's activities and determines the efficient utilization of its resources. It provides data on which management decisions have to be taken: decisions relating to production volumes, inventories, budgeting, pricing, and long-term planning of the firm's future development. Market research is basically a study of the factors

[5] From report published in the AMA case studies in Production, Planning and Control (General Management Series No. 188, 1957).

that restrict consumption, the main ones being: the product and its limitations, the consumer, competition and saturation, distribution methods, and the state of business as a whole. In making the forecast, several methods are used, such as general economic trends, share of the market, sales trend analysis, simple and multiple correlation analyses, sales managers' estimates, or combinations of these methods for correlation purposes. Predicting the future, however, is a difficult task, and the validity and accuracy of the collected data do not always come up to the production engineer's expectations. A survey in the United States of some 250 firms indicates that the average forecasting error amounts to 8 per cent (the median being 5 per cent), with wide variations reported within industry groups.

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Problems

1. Discuss the importance of sales forecasting for production scheduling.
2. Devise a questionnaire for the purpose of finding the limitations of a specific domestic appliance. The questionnaire should be simple, easy to comprehend and to answer, short enough to maintain the interviewee's interest and co-operation, while at the same time long enough to ensure an adequate picture of customers' attitudes.
3. Select a commodity which in your view emphasizes the problem of quality as a function of replacement. Discuss the various aspects that should be weighed by the management of the company concerned when the question of materials quality is to be specified for a new model of your selected product.
4. Why should terms of sales affect sales? Suppose a new hire-purchase system is being studied. Outline the procedure that should be followed in order to evaluate its quantitative effect on the sales forecast.

5. Use the method of the least squares to decide whether in the example cited in the text for market share analysis (Table 6–3, Fig. 6–6) any trend of the market share can be established.
6. In Table 6–3 add the data for the eleventh year (13.3%) and do the following:
   (i) Find the new average and verify by the least squares whether a trend in the share of the market is discernible.
   (ii) Modify Fig. 6–6 in the light of the additional data.
   (iii) What do you expect the market share to be in the twelfth year?
7. Figure 6–9 gives the regression line with control limits, calculated on the given data of 11 years, as specified in Table 6–4. It is now known that the sales for the twelfth year is \$80 million. Correct Fig. 6–9 accordingly, and forecast the sales for the thirteenth year.
8. Add to Table 6–4 the figures for year 12 (\$80 million) and year 13 (81 million).
   (i) Find the regression line.
   (ii) Forecast the sales for year 14.
9. The sales of C & X Corporation in eight years were (in millions of \$); 5.2, 4.5, 9.8, 9.2, 11.0, 18.5, 17.5, 22.0.
   (i) Find the regression line, following the method outlined in Table 6–5.
   (ii) Plot the regression line and the control limits.
   (iii) Check your results, following Table 6–7.
   (iv) Use the modified computational method (as per Table 6–6), using $x_1 = -3.5$ (for the first year), $x_2 = -2.5, \ldots, x_8 = +3.5$.
10. Develop a method for plotting a regression line of the second degree, expressed as $Y = a + bx + cx^2$, and determine the coefficients a, b, c by the criterion of the least squares.
11. From the data in Fig. 6–18,
    (i) Find an expression for the regression line of the 12-month moving average (note that it is not linear).
    (ii) Forecast the average consumption of electricity in 1956–1960.
    (iii) Check your forecast against figures for these years, published in the Survey of Current Business (or any other source).
    (iv) Modify your expression accordingly.
12. From the data given in Fig. 6–18 make a monthly forecast for 1956–1960. Note that the seasonal pattern is magnified with time. Assume a constant rate of magnification per annum and base your forecast on:
    (i) The regression line you obtained in the previous problem.
    (ii) A linear trend based on 1952–1955.
13. It was suggested that a simple correlation analysis should be carried out to establish the rate of increase in domestic electricity consumption as a function of the rate of increase in population. What do you think?
14. Would you support a forecasting method based only on a multiple correlation analysis that proved quite reliable for six years? Why?
15. Read several case studies in sales forecasting and discuss the merits and limitations of the use of indicators.
16. Discuss the implications of forecasting on the manufacture of durable and non-durable goods and point out the difference in approach, if any, to forecasting methods in the two cases.

17. In the hotel business it is useful to study the statistics of movement of passengers into the country. Passenger movement into and out of the United Kingdom in recent years is shown in the following table (figures in thousands):

	1948	*1949*	*1950*	*1951*	*1952*	*1953*	*1954*	*1955*	*1956*	*1957*	*1958*	*1959*
Total passengers arriving:												
By sea	2,084	2,365	2,449	2,631	2,676	2,678	2,910	3,226	3,288	3,398	3,592	3,772
By air	502	583	710	865	922	1,120	1,259	1,462	1,850	2,192	2,342	2,655
Total visitors arriving	504	563	618	712	733	819	902	1,037	1,107	1,180	1,258	1,395

All figures in thousands (Annual Abstract of Statistics, U.K., 1960). Analyze these figures and determine the regression lines for forecasting purposes.

18. Figure 6–21 is based on the calculations shown in Fig. 6–8. Construct a similar table of calculations for the hypothetical case when forecast figures coincide with actual sales figures and check your results by means of Fig. 6–22.
19. In Fig. 6–21, which describes the fluctuation of stock due to forecasting, the average stock for the year is 405 units, as compared with an average stock of 424 units in the hypothetical case of accurate forecasting (Fig. 6–22). As each unit in stock involves certain storage costs, the system described in Fig. 6–21 is less costly than the one in Fig. 6–22. Should it therefore be concluded that, from the point of view of storage costs, inaccuracy in forecasting is a blessing in disguise? Explain.
20. In Table 6–11 it was assumed that the quantities produced each month are the same as those specified in the production order (column 8). Suppose that, owing to scheduling considerations, it is not always possible to comply with these orders and some deviations are inevitable. Should the actual production figures (column 3) read Dec. 300, Jan. 200, Feb. 250, Mar. 200, Apr. 200, May 150, June 150, July 200, Aug. 200, Sept. 300, Oct. 250, Nov. 250, Dec. 300 (the total production quantity for the year equals the sum of the monthly orders), how would the stock level be affected? Construct the table on the lines of Table 6–11, plot the stock variations, and compare with Fig. 6–21.
21. In its publication, "National income and expenditure, 1958," the U.K. Central Statistical Office quotes the following index numbers of prices and costs (1948 taken as 100):

	1948	*1949*	*1950*	*1951*	*1952*	*1953*	*1954*	*1955*	*1956*	*1957*
Home total costs per unit of output	100	102	103	110	121	124	127	130	139	145
Price of final output	100	103	106	119	126	127	128	133	140	144

Is it reasonable to conclude that prices kept pace with rising costs?

7

PLANT LAYOUT

A plant layout is an arrangement of facilities and services in the plant. It outlines relationships between production centers and departments. A layout may either evolve gradually, or it may be planned for future operations. The main purposes of a planned layout are to:

Integrate the production centers into a logical, balanced and effective production unit.

Facilitate satisfactory movement of materials and personnel and an efficient control mechanism of such transportations.

Provide a logical distribution of functional facilities in the plant.

Be adaptable to possible changes in the plant's production program; either changes in product design or changes in the required output may require rearrangement of equipment or expansion of the plant's facilities.

Ensure proper allocation and utilization of space to the production centers and to services departments.

Allow convenience of operation both to operators and supervisors.

All these objectives cannot always be simultaneously attained; for instance, a demand for minimum transportation is often at variance with an ideal functional grouping. Likewise, a flexible layout adaptable to changes may be incompatible with an integrated one. Thus, before arriving at a well-balanced solution, alternative layouts have to be carefully analyzed and their advantages and disadvantages studied in the light of these objectives.

Plant layout provides the broad framework within which production and many administrative activities have to take place, and as such has an important bearing on utilization of facilities, on manufacturing methods, on control mechanisms, and on production costs. Layout has also a marked effect on capital expenditure, and the costs of special installations have to be carefully weighed against potential savings in the cost of labor, machine time, handling, and services. The major effects of plant layout on various aspects of production management may be summarized as follows:

1. The layout determines the location of departments and production centers, their proximity to each other and to various services, and hence the efficient utilization of available space.

2. It outlines the nature of flow in the plant and affects the distances traveled by materials and personnel; hence it is concerned with the time, effort, and costs spent on transportation.

3. It affects the types of handling systems, their integration in the over-all production program, and the costs of their installation.

4. It specifies the location, accessibility, and size of stores, and also the space and location of temporary storage for work in process.

5. It greatly affects the amount of work in process and work awaiting further processing; hence it involves the total production time and the capital tied up in work in process.

6. Machine utilization is partly determined by layout. This is reflected in output per machine hour, in the total machine capacity that is required, and in capital tied up in equipment.

7. Operator's span of activities, responsibilities, auxiliary tasks, walking time, fatigue, and efficiency may be dictated by layout considerations.

8. Maintenance procedure, schedules, and costs as well as policy of repairs, reserves, and replacement may be affected.

9. The amount of supervision required and the degree of specialization necessary in supervision is sometimes dependent on layout.

10. Production planning and control systems may be greatly affected, particularly the complexity of routing, machine loading, expediting, and the paper work involved in control mechanisms.

11. Effect on time lag between inspection and corrective actions has to be studied, as this may be reflected in amounts of rejected work.

Flow Systems

The flow of materials through the plant is one of the major factors that determines the type of layout. The flow of materials governs the cost of the materials handling, the amount of work in process, the capital and space tied up by work in process, and the length of the total production time. All these are weighty factors indeed, and each one is enough to justify a careful study of flow systems. Furthermore, the rate of performance and coordination of operators may have to be related to the rate of flow, and line production in particular may impose considerable physical and mental strain on the operators. Supervision and control mechanisms may also be affected. Simple flow lines are ideal for control purposes, and visual supervision is often possible. A complicated flow system usually implies a complicated control system. Quite often a plant layout design starts with the flow system, around which services and other facilities are added and buildings designed or modified accordingly, but sometimes the flow must be adapted to existing buildings. Flow systems can be classified into horizontal and vertical flows.

Horizontal flow lines

The five basic types shown in Fig. 7–1 are:

1. *I-flow, or line flow,* is the simplest form of flow. Materials are fed at one end and the components leave the line at the other. The I-flow is economical in space and convenient in I-shaped buildings (see example in Fig. 7–2).

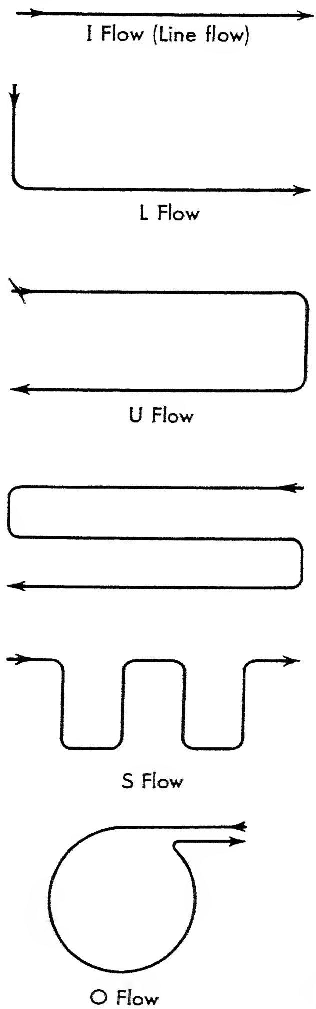


Figure 7-1.
Basic horizontal flow systems.

2. *L-flow* is similar to the I-flow and is mainly adopted when an I-line cannot be accommodated in the available space.

3. *U-flow* has both the feeding to and ejection from the line at the same end. When the line occupies the whole flow, a U-line is convenient, since it allows

Figure 7–2. ***Examples of I- and U-flow. (Top) I-flow in a body trim conveyor line for automobiles.*** (Courtesy Vauxhall Ltd. England) ***(Bottom) U-flow in an assembly line for winding small motors.*** (Courtesy Brown, Boveri & Co. Ltd, Baden, Switzerland)

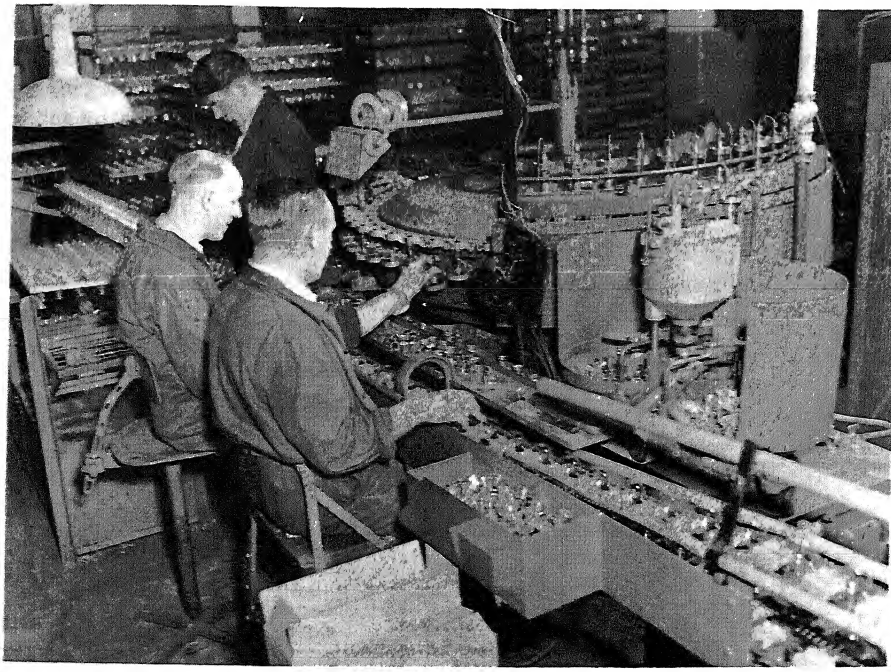
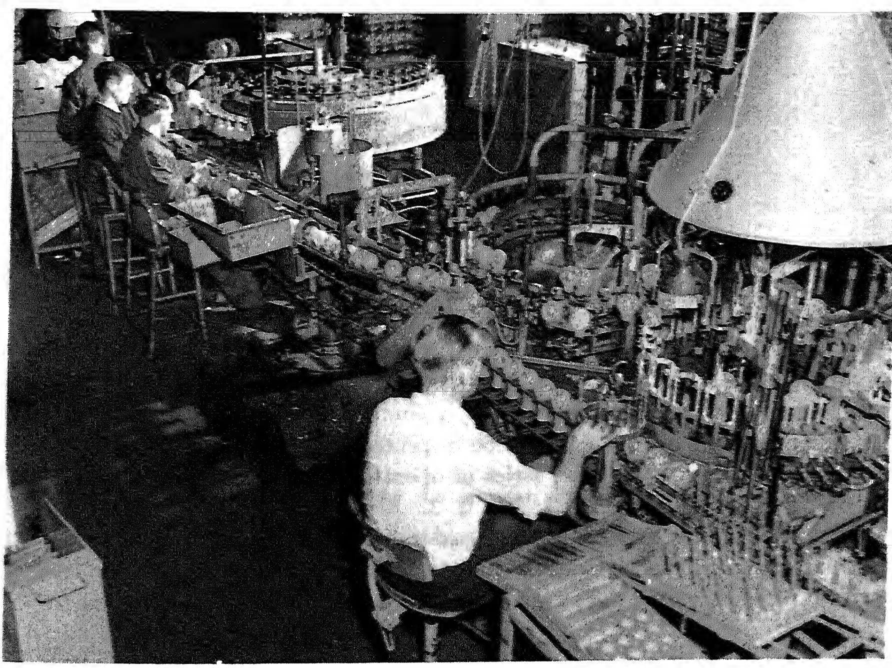


Figure 7–3. ***Examples of O-flow. (Top) Filling apparatus with soldering control for sockets. (Bottom) Combination of circular and line flow in automatic production of electric bulbs.*** (Courtesy of Philips, Eindhoven, Holland)



both receiving and dispatching of goods to be made at one side. It is also easier for supervision than an I- or L-line. An example of a U-line is shown in Fig. 7–2.

4. *S-flow* is adopted when the production line is so long that zigzagging on the plant floor is necessary. The S-flow provides efficient utilization of space and is compact enough to allow effective supervision.

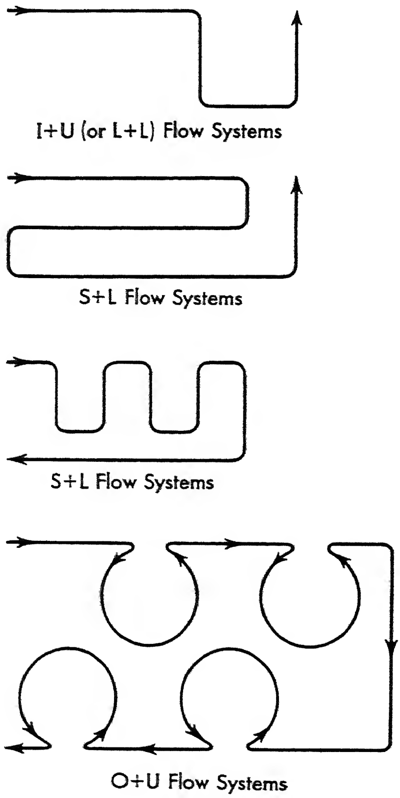


Figure 7–4. ***Examples of combinations of basic horizontal flow systems***

5. *O-flow* is used when operations are carried out on a rotary table, or a rotary handling system. The components are passed from one working station to the other, and when they leave the O-line, a complete set of operations has been performed, and the components can be inspected before they are passed on to a second O-line for an additional series of operations or to an assembly line. An example of an O-flow is shown in Fig. 7–3.

The basic flow lines are frequently used in various combinations in industry; some examples are given in Figs. 7–4 and 7–5. Two additional aspects of horizontal flow lines should be mentioned:

Unidirectional or retractional flow

In a unidirectional flow, the material is transferred from one machine to another without having to move again along the same path (Fig. 7-6). In retractional flow, the flow is repeated, i.e., two or more nonconsecutive operations are performed on the same machine. Evidently this aspect of flow is determined by considerations of machine utilization. In retractional flow, the available machine time is more fully employed, but schedules have to allow for repeated machine setting and for the fact that intermittent localized halts occur in the production line each time a machine is switched over from one operation to another.

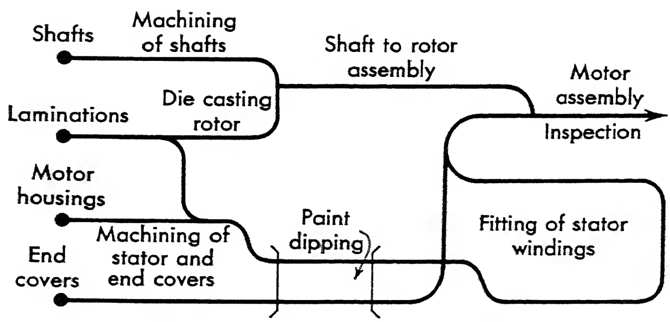


Figure 7-5. ***Flow line in the manufacture of motors, showing the combination of basic flow lines.*** (Courtesy Brown, Boveri & Co. Ltd., Baden, Switzerland)

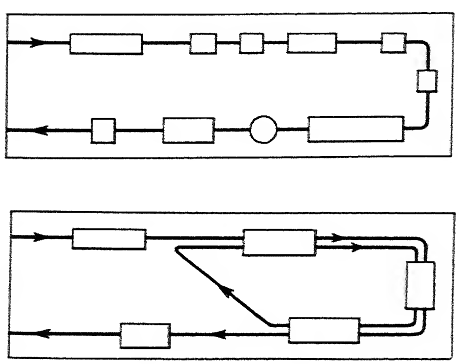


Figure 7-6. ***Unidirectional and repeated flow.***

Integration of flow lines into an assembly line

For a continuous flow system to operate smoothly, where several flow lines feed assembly lines, the logical pattern of integration can be likened to a river, where several estuaries combine into the main stream (Fig. 7-7). The rate of

flow of the main assembly line is the dominant factor in the integration of all the lines into one unit. If the rate of flow of one of the feeding lines is higher than the main line, production in the feeding line has to be intermittent, and the feeding line can be used to produce several components or subassemblies in batches. For example, in Fig. 7–7 the main assembly line is supplied with six subassemblies: three are continuously fed into the main line by lines I, III, and IV, and the other three (A,B,C) are manufactured in batches by line II and supplied to the main line from temporary storage locations.

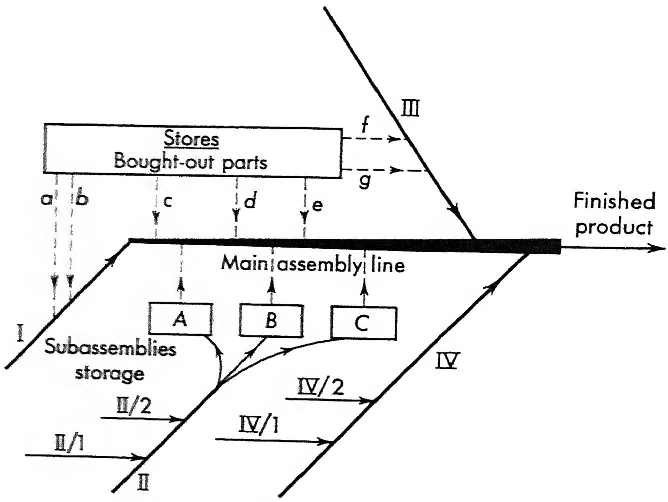


Figure 7-7. *A "river" convergent flow.*

I, II, III, IV	Production lines feeding the main assembly line.
II/1, II/2	Sublines feeding II.
IV/1, IV/2	Sublines feeding IV.
a, b, c, d, e, f, g	Bought-out components or assemblies fed to assembly lines.
A, B, C	Subassemblies produced consecutively in batches on line II.

Vertical flow lines

In multistory buildings a carefully planned flow is of particular importance for materials handling systems and control mechanisms to operate effectively. Six basic aspects of vertical flows are shown in Fig. 7–8.

Processing downwards or upwards

The flow patterns seem similar for these two cases, but in downward processing the raw materials have to be taken to the top floor, whereas in upward processing the finished product ends up at the top floor. This aspect has an important bearing on the handling system. When processing downward, many gravity

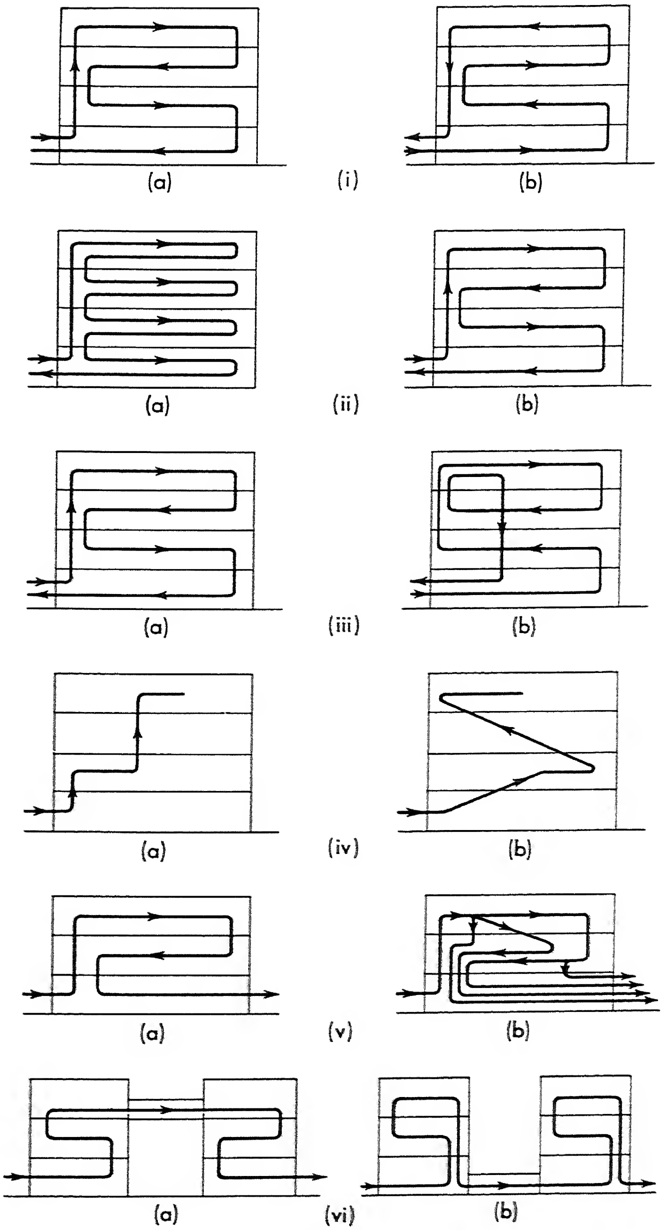


Figure 7-8. ***Six basic aspects of vertical flow systems.***

(i) Processing downward or upward.
(ii) Centralized or decentralized elevation.
(iii) Unidirectional or retractional flow.
(iv) Vertical or inclined flow.
(v) Single or multiflow.
(vi) Flow between buildings: elevated or ground flow.

handling systems (roller lines, chutes, pipes, buckets, hand-operated lifts, etc.) can be used, and these are economical in installation, operation, and maintenance. In some cases, however, processing downward may not be practical. In metal-working industries, if the raw material consists of large raw castings, processing downward would mean installing the heavy machine tools on the top floor and carrying swarf and scrap back to ground level—a very costly setup. The normal practice is to have heavy machine shops on the lower floors and assign only light machines and light assembly lines to the higher floors, thereby achieving comparatively low costs in installation and foundations and restricting vibration effects in the light machine shops.

In some cases, working upward is dictated by the nature of the process; for example, in one type of manufacture of plane glass by a continuous process, the glass is pushed upward through successive sets of rollers, and the finished glass sheet is cut into large panes as it emerges on the top floor.

Centralized or decentralized elevation

In a centralized elevation all the handling systems, either upward or downward, are concentrated at one end of the building. This method facilitates economic supervision and maintenance of the handling systems, sometimes even reducing the installation costs considerably, and is very useful when the flow on each floor is a U-flow. However, when the flow is, say, a line flow (I-flow), a centralized elevation system results in extra materials handling on each floor because the work has to be returned to the elevators. In a decentralized elevation method, handling on each floor can be greatly reduced and more flexibility in design of the flow lines is possible, but the method is more costly in installation, maintenance, and space.

Unidirectional or retractional flow

In the retractional flow shown in Fig. 7–8 (iii), after processing on the third floor the work has to return to the fourth floor for a second time. As in a horizontal flow, a vertical retractional flow yields better machine and space utilization, but handling is more costly than in unidirectional flow.

Vertical or inclined flow

Vertical flow is carried out by elevators, chutes, and buckets, and is often economical in space. Inclined flow, which is also done by conveyor belts and chain systems, is sometimes more adaptable to flow systems.

Single or multiflow

A single flow system consists of one flow line. In a multiflow system, several production lines feed one assembly line and converge into one flow line. Alternatively, as shown in Fig. 7–8 (v), the material is divided into several streams after the first operations, and each stream is directed to different processes, so that several products finally emerge.

Flow between buildings

When several buildings form links in one production line, the flow from one building to the next may be either an elevated flow or a ground flow. Ground flow is cheaper to install and is particularly suitable when processing upward in the second building. If in the second building, processing is of the downward type, a ground flow system implies that the material has to be elevated to the top floor in the second building. Excessive handling may thereby result, and in such cases an elevated flow transfer between the buildings may be preferable. Another advantage of an elevated flow is that it frees the ground space for traffic and storage purposes.

Types of Layouts

Layouts can be classified into three types, depending mainly on the type of production used in the plant.

1. Product layout or line production, in which machines and auxiliary services are located along the product flow line. The layout is suitable for continuous types of production and can employ one of the basic horizontal flow lines or their combinations.

2. Process layout, in which machines and services are grouped according to their characteristic functional purpose; for instance, all turning, welding, painting, etc., are performed in separate turning, welding, and painting departments. A process layout is mainly employed in job and batch production.

Table 7–1

Advantages and Limitations of Types of Layouts

Advantages	*Limitations*
Product Layout	
1. Layout corresponds to sequence of operations, resulting in smooth and logical flow lines.	1. Layout is determined by the product and leaves little room for flexibility. A change in product design may need major alterations in layout.
2. Reduced materials handling, since the machines are so located as to minimize distances between consecutive operations.	2. The "pace" is determined by the slowest machine; hence speed of machines is deliberately reduced or machines have excessive idle time.
3. Small amounts of work in process, as the work from one process is directly fed into the next.	3. A breakdown of one machine may lead to a complete stoppage of the line that follows that machine.
4. Less space is occupied by work in transit and for temporary storage.	4. Comparatively high investment is required, as identical machines (a few not fully utilized) are sometimes distributed along the line; also, machines may be required to stand by in case of breakdowns.
5. Total production time per unit is short.	5. Supervision is general but not specialized.
6. Simple production planning and control systems and simplified supervision.	
7. Little skill is usually required by operators at the production line; hence training is simple, short, and inexpensive.	

Table 7–1 (Continued)

Advantages and Limitations of Types of Layouts

Advantages	*Limitations*
Process Layout	
1. Flexibility in equipment or manpower allotment for specific tasks. Thus load distribution can be controlled, and this is particularly of importance in case of breakdowns, for maintenance schedules and for multiproduct manufacture.	1. Long flow lines; hence more expensive handling.
2. Better utilization of machines available time; consequently less machines are required.	2. Comparatively large amounts of work in process, waiting for the next operation.
3. Comparatively low investment in machines is required.	3. Space and capital are tied up by work in process.
4. Each section can benefit from specialized supervision.	4. Production, planning, and control systems are more involved.
5. The diversity of tasks offers a more interesting and satisfying occupation for the operator.	5. Total production time is longer, owing to time consumed in materials handling and to waiting times.
	6. Because of the diversity of the jobs in specialized departments, higher grades of skill are required.
Static Product Layout	
1. Material movement is reduced to a minimum.	1. Movement of machines and equipment to the production center may be costly and time consuming.
2. The task is usually carried out by a gang of operators; hence continuity of operations and responsibility is ensured.	2. Positioning of the material or object or machines may be cumbersome and costly.
3. Production centers often operate independently of each other, and effective scheduling can be planned to ensure minimum total production time.	3. Machines and equipment utilization is usually low, owing to handling and positioning time, even if schedules provide for successive employment of the equipment at several production centers.
	4. High grades of skill are required.

3. Static product layout, or layout by fixed position, in which the product is too big or too heavy to be moved from one process to the other and is consequently fixed in one place. The machines and men are brought to the product to perform the required operations. This layout is typical of job production in constructional work, in shipbuilding, in the fabrication of large tanks or pressure vessels, etc.

Each of these types of layout has its advantages and limitations, as shown in Table 7–1, but combinations of these types are in use very often in industry. Even, for example, when a product layout is basically desirable, it is sometimes necessary to allow for a repeated flow, as shown in Fig. 7–6, in order to avoid excessive machine idle time and extra capital expenditure.

The difference between a product and a process layout is illustrated in Figs. 7–10 and 7–11. The product in question consists of 3 components, and involves 17 operations and 5 inspections in a machine shop, as shown in Fig. 7–9.

Component C — Bar stock

Distances (Ft)

Product Layout	Process Layout	Compromise Layout	Operation
8	8	8	(11) Saw
10	10	10	(12) Turn
14.5	18.5	14.5	(13) Drill
8.5	15	8.5	(14) Tap
8.5	26	8.5	(15) Turn
20	13.5	20	(16) Grind
9.5	18	9.5	4 Inspect
9.5	9.5	9.5	
88.5	118.5	88.5	

Component B — Sheet metal

Distances (Ft)

Product Layout	Process Layout	Compromise Layout	Operation
8	8	8	(4) Cut
31	47	31	(5) Trim
3	3	3	(6) Bend
15	20	15	
57	78	57	

Component A — Bar stock

Distances (Ft)

	Product Layout	Process Layout	Compromise Layout	Operation
	8	8	8	(1) Saw
	10	16	10	(2) Turn
	7.5	23	7.5	1 Inspect
	7	40	7	(3) Drill
	57	78	57	
	24.5	14	24.5	(7) Weld
	8	30	10	2 Inspect
	6	21	9	(8) Turn
	8.5	25	18.5	(9) Mill
	8.5	16	8.5	(10) Drill
	18.5	34	18.5	3 Inspect
	88.5	118.5	88.5	
	9.5	12	9.5	(17) Assemble
	11.5	11.5	11.5	5 Final Inspection
	10	10	10	To store
TOTAL	283	477	298	

Figure 7–9. ***Comparison of distances traveled in a product, a process, and a combined layout.*** (See Figures 7–10, 7–11, 7–12)

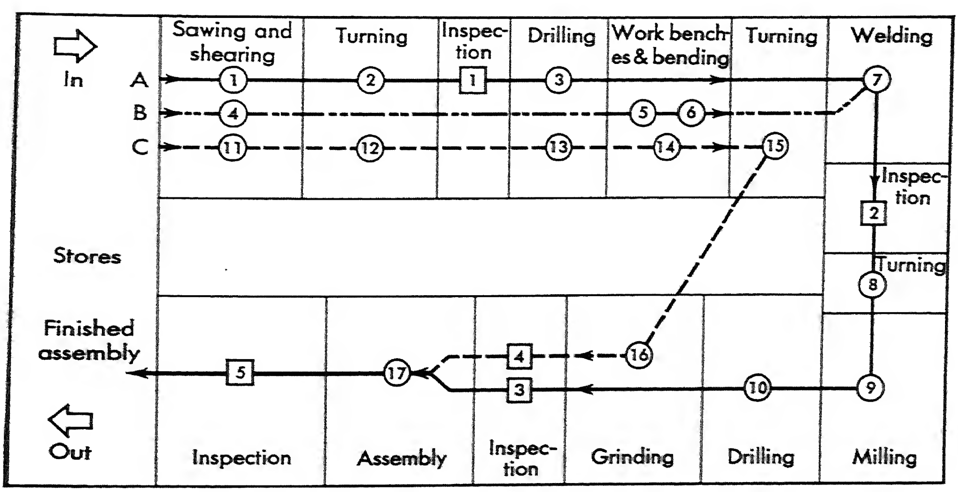


Figure 7–10. ***A product layout.***

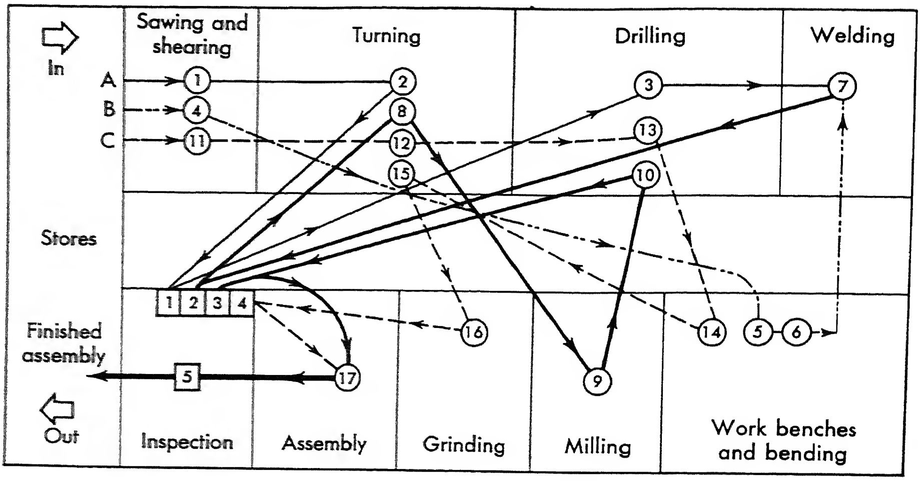


Figure 7–11. *A process layout.*

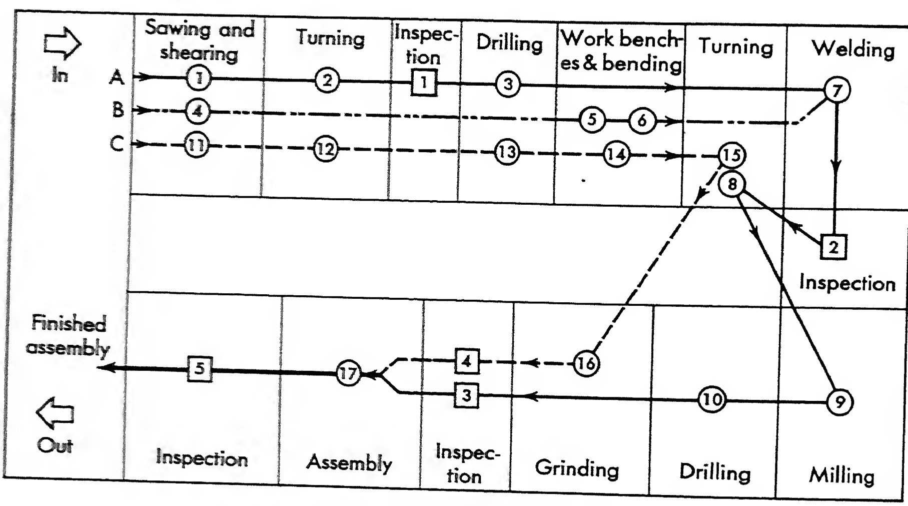


Figure 7–12. *A compromise layout.*

The flow is basically a U-flow. The raw materials are supplied from the stores and the metal bars and sheets are first cut to size. In the functional layout (Fig. 7–11) there are nine production centers to which all the components are brought according to the process chart, the inspection in this case being also centralized. In the product layout (Fig. 7–10) the U-flow line is strictly adhered to and the total distance traveled by materials is considerably shorter, but turning

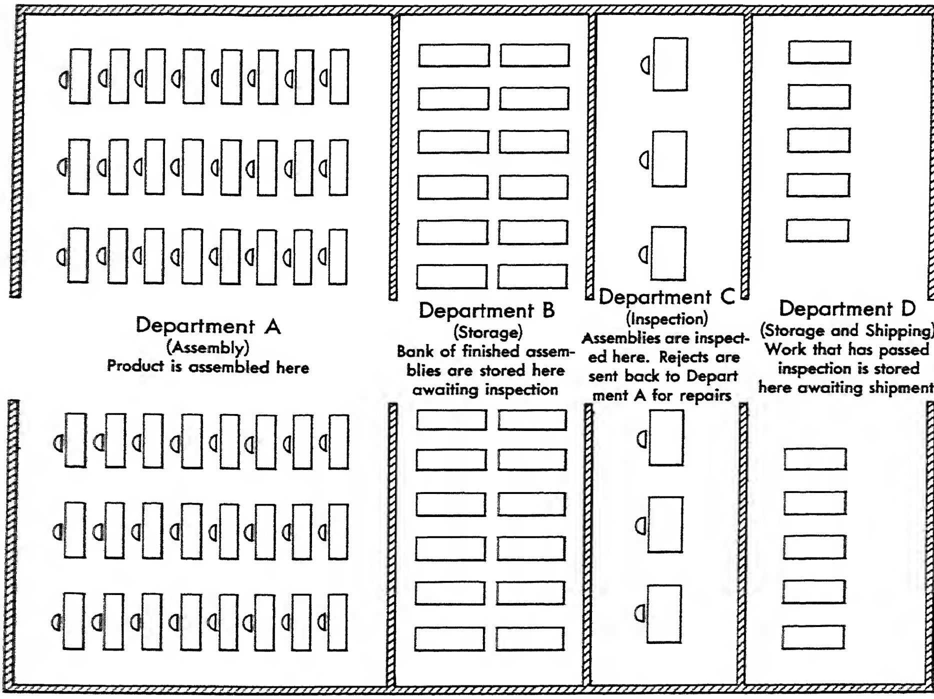
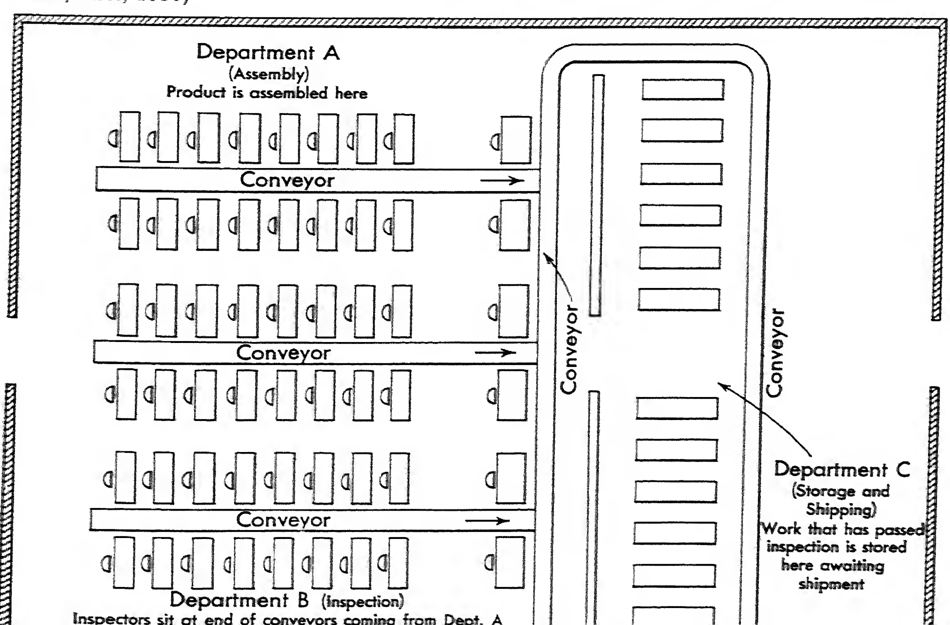


Figure 7–13. ***(Top) Old layout with a separate inspection department C. Work awaiting inspection is accumulated in department B. (Bottom) New layout, providing continuous flow from assembly to final stage. Inspectors sit beside the conveyors, which eliminate accumulation of work in progress awaiting inspection.*** (Reprinted with permission from Ralph M. Barnes, *Motion and Time Study*, John Wiley & Sons, Inc., 1956)



operations, for instance, have to be performed at three different locations along the production line. The comparison between the two layouts can be summarized in the following table:

	Product Layout	*Process Layout*
Number of operation stations	11	8
Number of inspection stations	4	1
Total number of stations	15	9
Total distance covered by materials (based on the assumption that floor area is the same for the two cases)	283 ft.	477 ft.

In the product layout great saving in materials handling can obviously be effected. At the same time the utilization of the lathes and the drilling machines have to be analyzed to ascertain that the distribution of the operations along the line does not lead to excessive machine idle time. In the suggested process layout, for instance, it is also clear that inspections are responsible for a great amount of handling (208 feet), and therefore a decentralized inspection system, i.e., inspection along the line, may be more suitable. In this way a reasonable solution for a layout can be arrived at, which is quite often a compromise between the two methods. An example for a solution that could perhaps satisfy the case discussed above is shown in Fig. 7–12, where two turning and two drilling stations are specified, leading to the following summary:

Number of operations stations	10
Number of inspection stations	4
Total number of stations	14
Total distance covered by materials	298 ft.

The location of inspection and its integration into the production line may have a considerable effect, not only on the cost of handling in the plant but on the amount of work in process and on the time lag between inspection and corrective actions. In Fig. 7–13a we have an example where assemblies are completed in department *A*, stored at *B* waiting for inspection, inspected at *C*, and stored again at *D*. The amount of work accumulated at *B* causes a serious delay between production and inspection, with the result that if too many rejects are found, which would require machine adjustment at *A*, this adjustment takes place after a considerable amount of rejects or scrap has accumulated at *B*. When a layout as in Fig. 7–13b is adopted, the inspection is carried out immediately after production, and information about corrective actions can quickly be fed into the production department. The flow system ensures that work in process is greatly reduced and economies in space can consequently be effected.

Another example of a layout for inspection immediately after the operation is carried out is shown in Fig. 7–14, where the inspection stations are located in front of the appropriate machines. The finished components are fed through a chute to the inspection station, where they are sorted out and the good components are collected in a container under the inspection bench. Figure 7–15

illustrates a final inspection layout, which allows for several inspection stations in series along the conveyor belt. Each station is responsible for checking one particular aspect of the product, such as the inside, or the outside. If the product is acceptable, it is passed on to the next station; if it is rejected, it is transferred to a rejects line for repairs, after which it is fed again to the first inspection station on the conveyor belt.

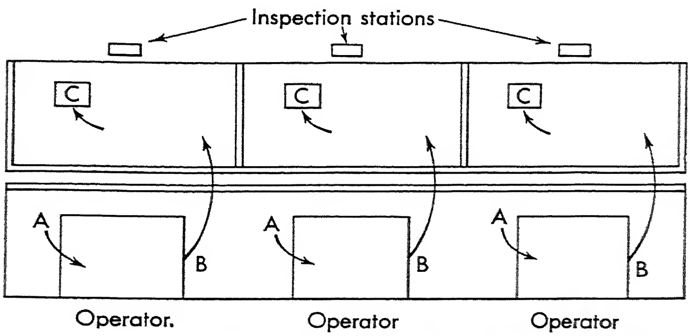


Figure 7–14. ***Layout for inspection: a bench at rear of machine eliminates some handling.*** (Reproduced with permission from "Inspection in Industry," a productivity report of the Anglo-American Council on Productivity, 1953)

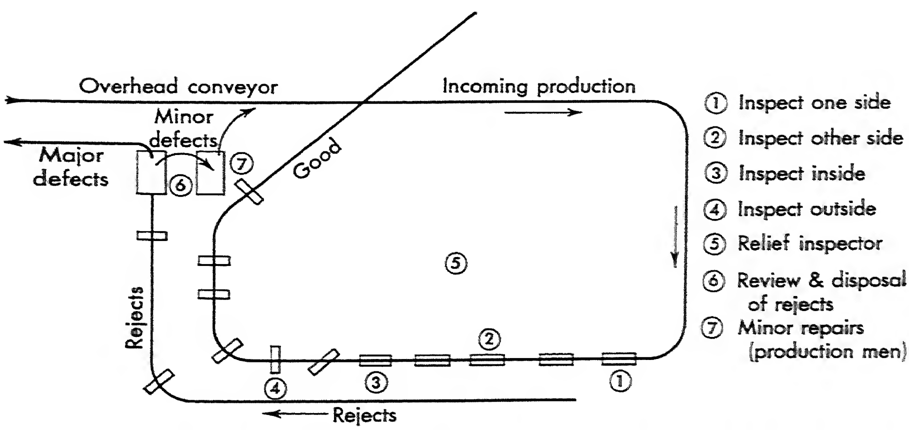


Figure 7-15. ***Layout for final inspection on conveyor belt.*** (Reproduced with permission from "Inspection in Industry", a productivity report of the Anglo-American Council on Productivity, 1953)

Machine layout

The general layout of the plant can be broken down to a detailed planning of the position of each machine, its relation to other machines, and to materials handling systems. The layout of the machine not only affects the efficiency of

the operation performed on it, but also may have a bearing on subsequent operations and on the rate of production of the whole line. One major consideration in machine layout is that of space, and the problems that have to be studied in this respect were summarized by Alford and Bangs[1] as follows:

Room for the worker operating the machine or machines.

Allowance for projection, overhang, or overtravel of machine parts, such as the table of a planer.

Allowance for projection of work, such as bars fed to a screw machine.

Room for industrial trucks to deliver and remove parts that are large or are handled on skids, pallets, or in tote boxes.

Space for floor conveyors or chutes in a fixed product layout.

Room to get large work on and off machines. Often this handling is done by a hoist, jib crane, or overhead traveling crane for the use of which there must be the proper space allowance.

Area for the storage of the maximum-sized lots of work to be done, and for work completed and awaiting removal. Most frequently these areas are necessary in the actual doing of work to provide the place from which to get parts for processing and to put them as they are finished.

Place for workbench, work table, tool rack, or other equipment containing the worker's tools, supplies, drawings, etc.

Room to get at any part of the machine which may require adjustment or changing in the course of operations.

Quick access to safety stops in case of accident to worker or breaking or jamming of the machine.

Access to the machine for inspection, maintenance, oiling and repairs, and for the removal of any part, such as a shaft, without moving the machine from its position.

Allowances made necessary because of proximity to columns, walls, partitions, stairways, elevator approaches, etc., which may require the providing of extra area, or for the waste of area, because of the size or shape of the machine.

Apart from space, additional aspects have to be considered in machine layout:

Problems of installation, including foundations, power, water, exhaust systems.

Convenience of loading and unloading materials, the integration of these operations in the handling systems and the effect of the layout on the flow lines.

Convenience in operating the equipment and supervising it, to ensure maximum efficiency and minimum operator fatigue.

The proper relation of the machine to the other machines in the production center, including allowance for multimachine supervision by one operator or for additional tasks the operator has to perform in the production center, in order to ensure a smooth integration of the machine in the center and reduction to a minimum of operator walking time.

Lighting, ventilating, and safety requirements that have to be met by the layout.

[1] Alford, L. P., and Bangs, J. R.: *Production Handbook* (The Ronald Press Co., 1952).

Materials handling

Materials handling systems are the means by which the flow in the plant is sustained and hence form a vital part of plant layout design. Materials handling is used for horizontal, vertical, or combined horizontal and vertical movement and may be classified into the following main groups:

Cranes for handling material or components above the ground, either for the purpose of freeing the floor from handling devices, in order to save space, or because the material is heavy, bulky, or awkward to move by other methods.

Conveyors for handling components or bulk material, including belt conveyors, roller conveyors (gravity or power operated), chutes (very convenient for handling between floors), screw conveyor (often for bulk material, both horizontally and vertically), chain conveyor (from which components are suspended or on which buckets are attached), or a pipe system for moving powder, grain, or liquid material under pressure.

Trucks, hand or power operated, including fork lift trucks with their multitude of attachments, platform tracks, tractors, and trailers.

Gravity handling systems are naturally much cheaper than power-operated ones, and usually are more adaptable to changes in layout. All handling systems impose space requirements which must be taken into account in layout planning, especially problems of clearances of cranes and maneuverability of trucks.

Figure 7–16 is an example showing the effect of flow on a plant layout. Another case study[2] illustrating the introduction of a mechanized materials handling system is given in Fig. 7–17. in which 25,000 lb. of yarn per week were supplied to two knitting departments, in cases weighing 350 lb. gross each. The main yarn store was located outside the main building and handling was carried out by hand trucks. Each knitting department used a temporary yarn store (occupying space of 800 sq. ft.), from which material was transported to the machines. When a mechanized overhead conveyor system was installed with carriers capable of holding 25 lb. each and a procedure worked out for loading the conveyor, receiving at the knitting departments and returning trays to the main store, the output of the plant increased by 10 per cent. The layout remained essentially unchanged, but the temporary yarn stores were eliminated and the cost of handling greatly reduced.

Effect of automation on layout

The fully automatic factory is still a dream of the future, but automated individual sections or production lines are becoming more and more frequent in industry and are having considerable impact on the layout of plants. The automated line provides for:

1. A constant flow of materials through the system
2. Automatic loading, positioning, and unloading

[2] The Institution of Production Engineers Journal, December 1956, p. 764.

3. Automatic handling between operations
4. Inspection after certain predetermined operations
5. Automatic sorting of good components from bad ones
6. Adjustment of machines and processes as dictated by inspection results.

The significant implication of automated production lines is the reduction of work in process and of temporary storages to a minimum, resulting in substantial saving in space (up to about 50 per cent) and costs of materials handling.

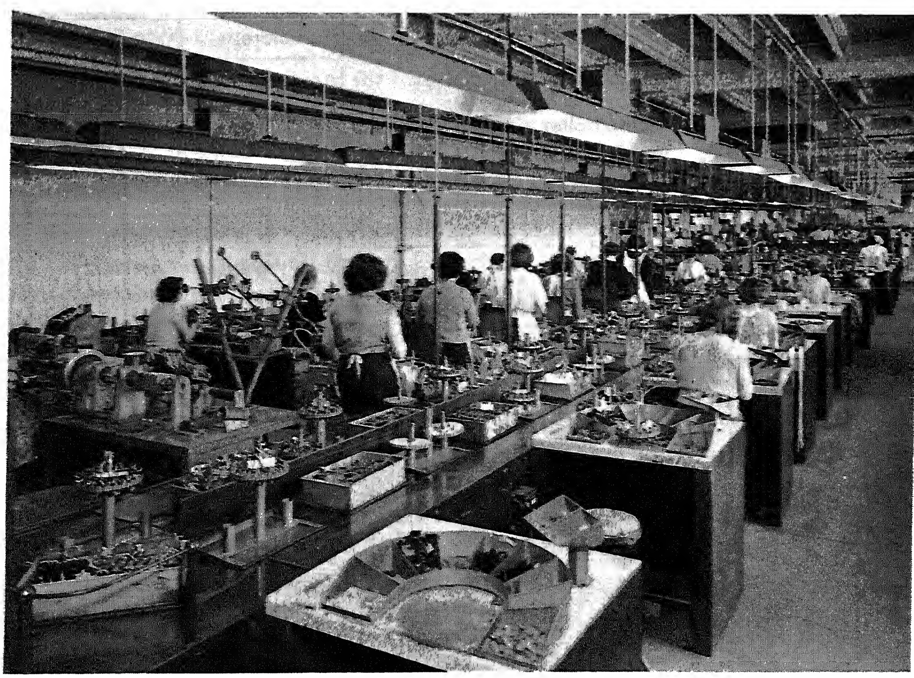
Automatic installations are usually associated with major capital expenditure, and naturally the question of flexibility immediately arises, i.e., the adaptability of the automated line to variations in design or to multiproducts. Clearly, if the line is a rigid system, which requires major changes in layout structure for even minor changes in product design, automation can be justified only for very high volumes of production, where the line can turn out the same components for an appreciably long time. If, however, flexibility in automated lines can be achieved it can be adopted for batch production and have a marked effect on small and medium size establishments. The integration of these lines into one layout, the output capacity ratio of automated feeding lines to nonautomated assembly lines, the maintenance procedure, and general equipment policy, all these remain major problems affecting both layout considerations and production planning and control systems in industry.

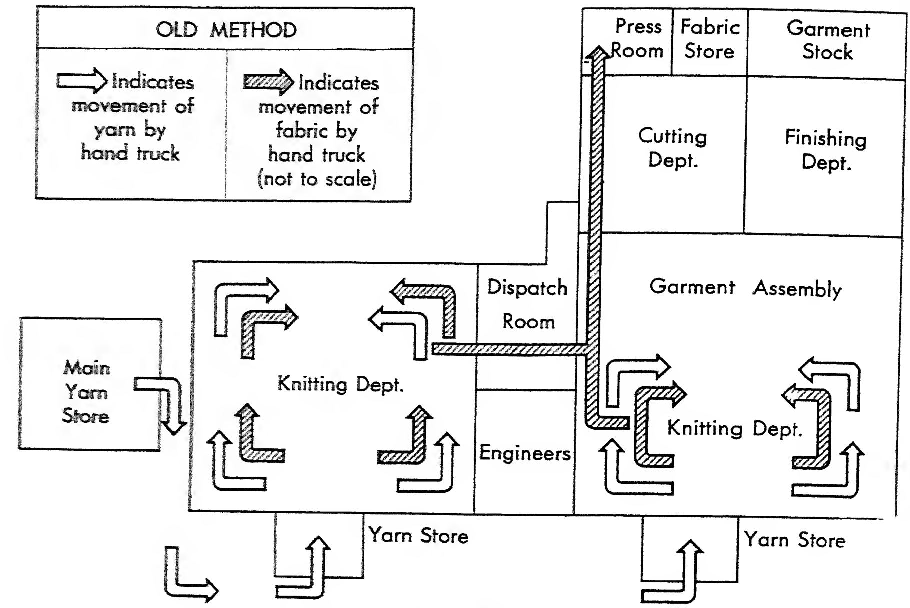
Symptoms of a bad layout

A plant layout usually grows and develops. Not every minor change in the production program or additions of machines or sections justify redesign of the plant layout, but these changes accumulate and gradually alter the basic pattern of flow lines in the plant, until replanning of the layout becomes inevitable. The symptoms of a layout in need of redesign are:

1. Congestion of materials, components, and assemblies
2. Excessive amounts of work in process
3. Poor utilization of space
4. Long transportation lines
5. Production bottlenecks at certain machines while similar or identical machines have idle times
6. Excessive handling by skilled operators
7. Long production cycles and delays in delivery
8. Mental or physical strain on operators
9. Difficulties in maintaining effective supervision and control.

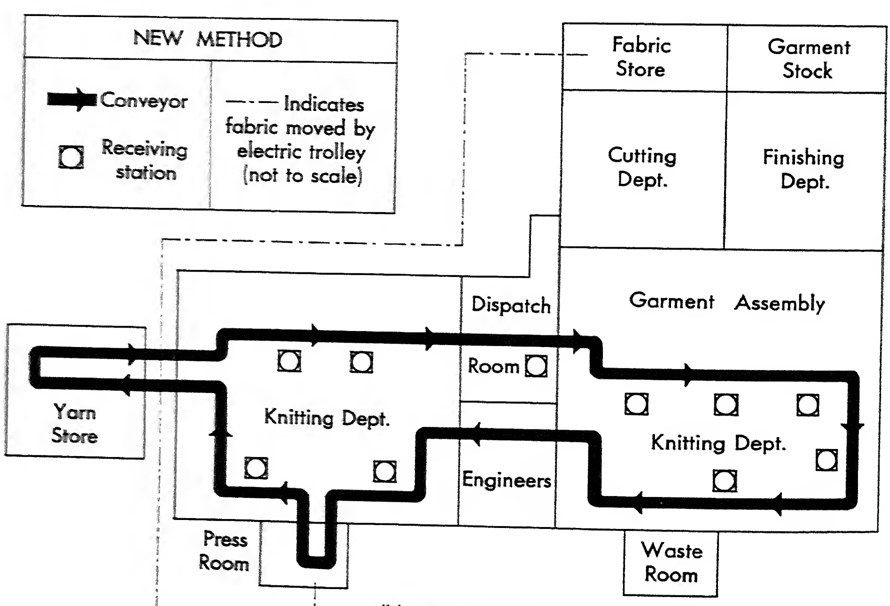
Figure 7–16. ***The effect of flow on layout. These are flows of a winding shop in the manufacture of electric ceiling fans. Spools used to be delivered to the girls at the benches by small trucks, whereas in the new layout the handling is performed by the conveyors. The improved version of the work place layout is also clearly seen in the bottom photograph.*** (Courtesy General Electric Co., Ltd, Birmingham, England)





(a) Old Layout

Figure 7–17. ***The effect of mechanized handling on layout.*** (Courtesy Cooper & Roe, Ltd, Eagle Works, Nottingham, England)



(b) New Layout

Factors to be considered in layout planning

When a new or modified layout has to be designed, a great number of factors must be analyzed before any decision can be made on the pattern of the flow lines and type of layout and communication systems. Some of the major factors to be considered are:

Hazards: nature of risks due to moving parts, projecting machine elements, suspended weights, air pollution at the production centers; other physical or chemical risks, and the type of precautionary measures required to ensure the safety of personnel and plant.

Type of production: job, batch or continuous; also whether the production is by a continuous process or is an assembly line, and whether it involves single or multiflows.

Type of operation: wet or dry, using heavy or light machinery, involving swarf, scrap, reprocessing, etc.; characteristics of the production and service centers.

Sequence of operations: dependency of one operation on another, rigidity of the sequence, reprocessing and its effect on the flow being unidirectional or retractional.

Integration of production: single flow or multiflow, relation of parts of the flow system to each other, coordination of sublines that feed major assembly lines.

Type of product: its weight, volume, physical state (solid, liquid, or gas), its durability, susceptibility to transportation, and difficulty connected with its storage.

Type of inspection: centralized or decentralized, its effect on amounts of work in process, and machines readjustment.

Management policy: plans for future output and expansion, changes in product design and variety.

Templates and models

In planning a new layout or modifying an existing one, templates and models (normal scale $\frac{1}{4}$ or $\frac{3}{8}$ inch to 1 ft.) are very useful for the analysis of several contemplated alternatives. They save the time and effort required for preparing a separate drawing for each alternative and enable the engineer to visualize the characteristics, merits, and weaknesses of the layout under consideration.

Templates can be black and white or colored and either of the black type (which gives only over-all dimensions of the equipment, including allowances but no details) or of the two-dimensional (plane) type (which gives the outline of the equipment in heavy line, other details in light line, and clearances in broken dash line). Two-dimensional templates are obviously much superior to block templates, which often lack the fundamental information required in planning a layout.

Figure 7–18. ***Cast aluminium models for layout planning.*** (Courtesy of Visual Planning Systems, Ltd, Alperton, England)

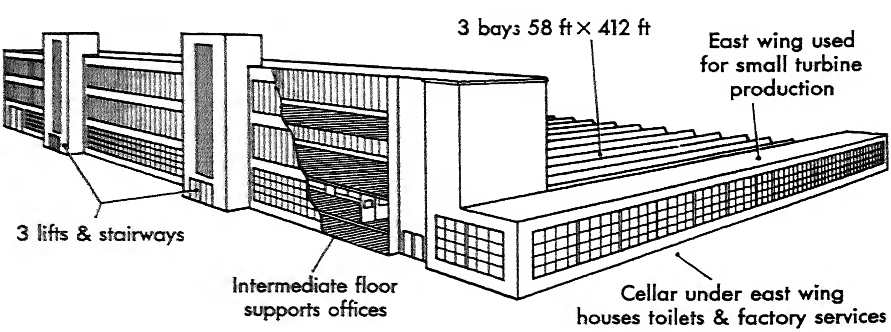
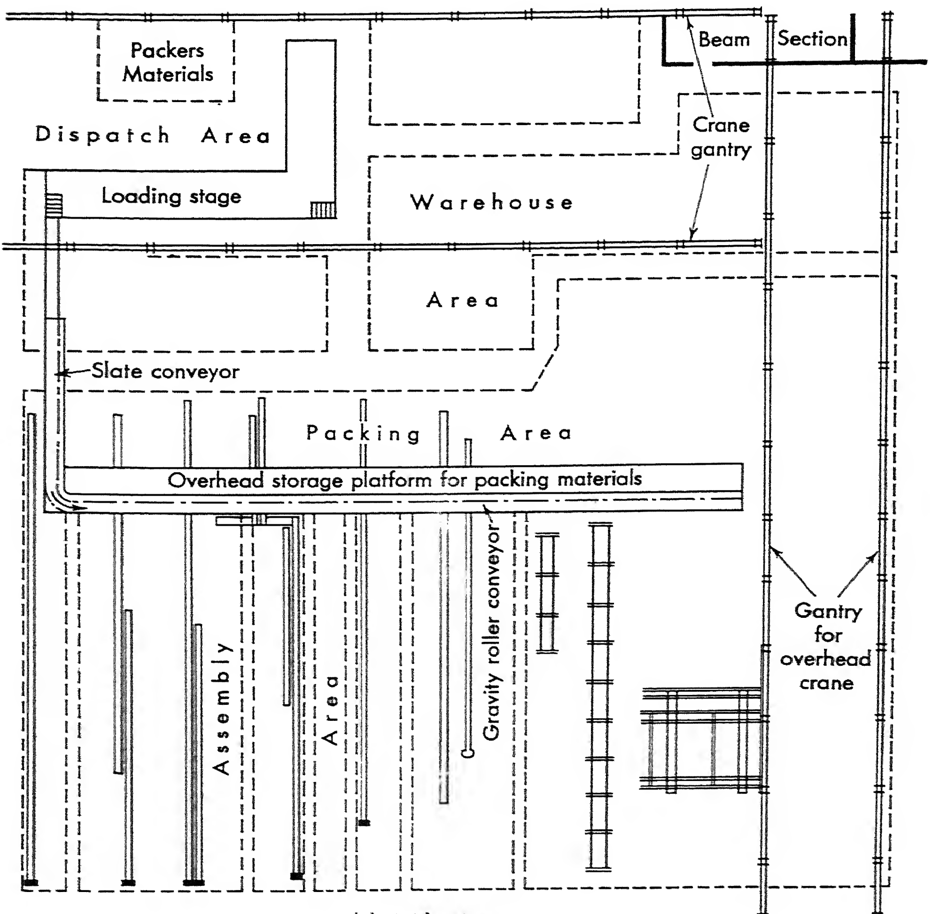


Figure 7–19. ***Arrangement of a new factory.*** (Courtesy Brown, Boveri & Co., Ltd, Baden, Switzerland)

Three-dimensional models (see Fig. 7–18) make the suggested scheme easier to visualize, particularly to nonengineers (see Figs. 7–19, 7–20, 7–21, 7–22). Machine allowances and clearances, however, cannot be marked on models, and when these are important, the models are stuck on plane templates on which the clearances are marked. The combination of models and plane templates is by far the best, but are naturally more costly than the other methods.



(a) A Plan View

Figure 7–20. ***Layout of an assembly department. The components are transferred from the components stores to the assembly lines and then to the packing area. The packed goods are brought to the warehouse by fork lift trucks. There are two overhead traveling cranes and several conveyors, the location of which is easily visualized in the three-dimensional model.*** (Courtesy W. & T. Avery Ltd, Leeds, England)

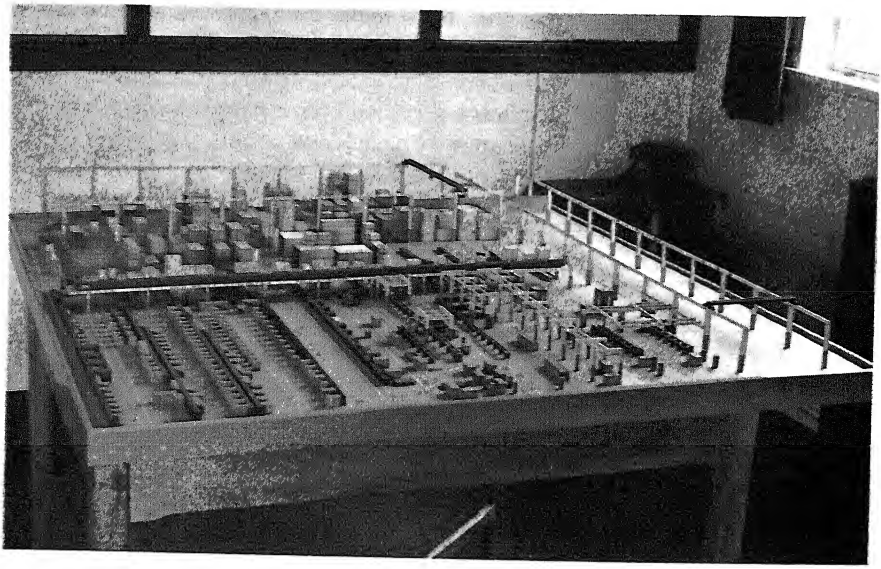


Figure 7–20 (*continued*) (*b*) A three-dimensional model: (1) assembly lines; (2) packing line and overhead platform for packing materials; (3) the warehouse.

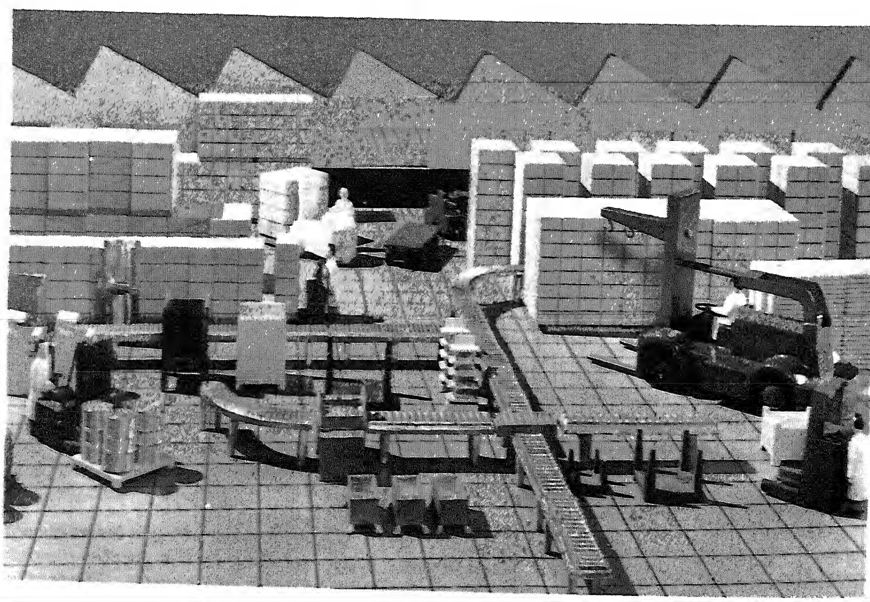


Figure 7–21. ***Store layout planning with the aid of three-dimensional models.*** (Courtesy of Visual Planning Systems, Ltd, Alperton, England)

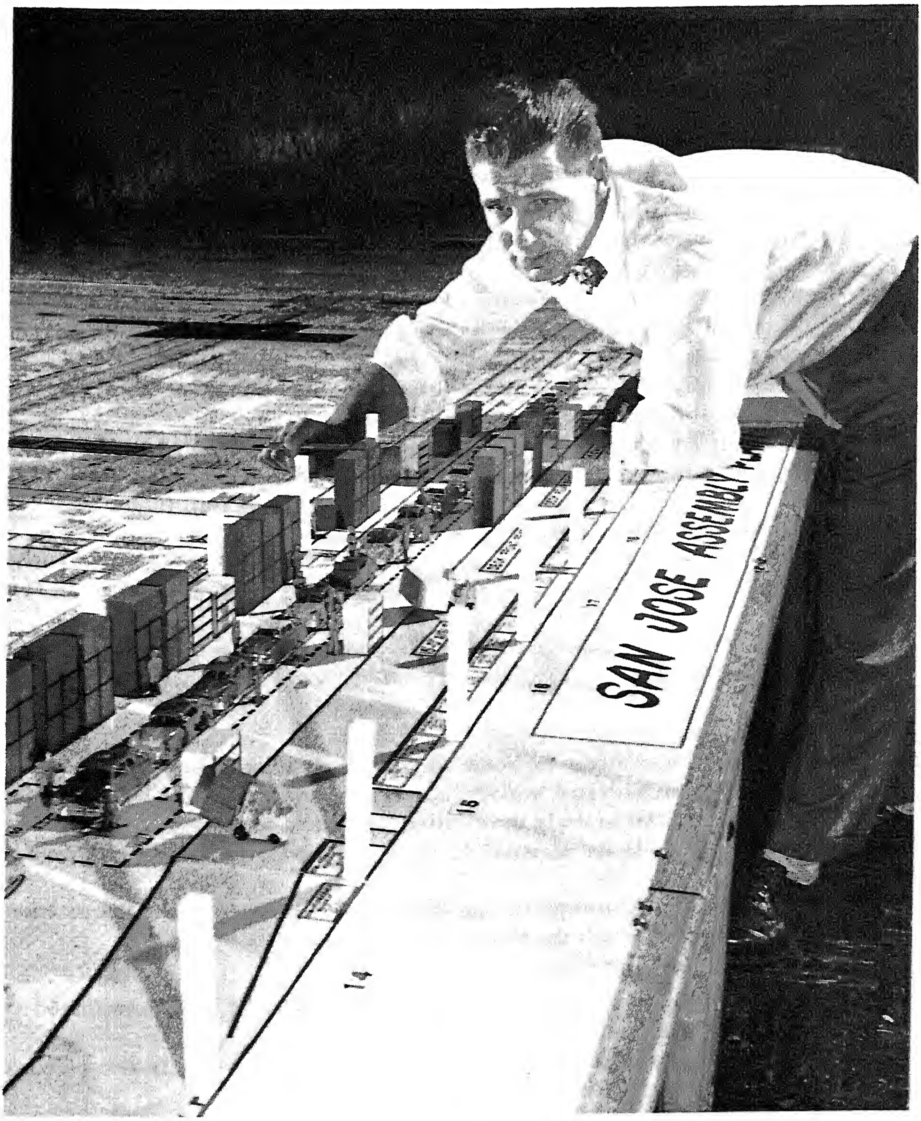


Figure 7–22. ***Planning an assembly plant with models.*** (Courtesy Ford Motors Co., Dearborn, Michigan)

Summary

A plant layout has a marked effect on utilization of space and equipment, on transportation problems, on the amount of work in process, and on production control. A major factor in layout is the flow system and its relation to materials handling methods. There are several basic characteristics of horizontal and

vertical flows which have to be considered when designing a flow system. Layouts are classified as being of the product type (most suitable for continuous production or manufacture of large batches), process or functional type (mainly suitable for batch or job production), and static product type (for the manufacture of products that are awkward or costly to transport in the shop). The effect of automation on layout is mainly associated with saving in space and handling, and reduction of work in process and problems of adaptability to product variety and changes in design.

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Problems

1. A plant has been in existence for some 25 years and has expanded considerably in the past 8 years, with new workshops in several annexes. It has been decided to set up a committee to study the existing layout and to make the appropriate recommendations. As an assistant to the chief production engineer you are being asked:
   (i) To specify which departments should be represented in the committee.
   (ii) The method which the study should follow.
   (iii) The terms of reference.

2. A plant is presently organized on a process layout basis. It is contemplated to replace the layout as a product layout.
   (i) Prepare a statement for management in which you specify the circumstances that would justify such a changeover.
   (ii) Assuming that the changeover is to take place, what data would you require before planning the stages of such a change?

3. (i) What are the basic horizontal flow systems and what is their significance?
   (ii) Can the basic horizontal systems be used in vertical flow? Why?
   (iii) Discuss the possible effect of automation on flow patterns in existing buildings with S-flows.

4. Discuss the possible implications of materials handling systems on production planning and control procedures.

5. What problems have to be studied for machine layout?
6. In planning a new layout in an existing building, would you start by outlining the flow systems first and fitting each machine layout into it (i.e., by analysis), or would you start with the individual machines layout (synthesis)? Explain the merits of your recommended method.
7. The management has decided that mechanized materials handling systems have to be introduced into a plant where excessive manual handling is used. A well-known manufacturer of conveyors has offered to send one of his engineers to study the problems of materials handling in the plant and to give his recommendations free of charge. Bearing in mind that his findings may be biased toward the introduction of a multitude of conveyors, what steps should the management take to ensure that it is presented with an objective picture before final decisions are taken?
8. In a plant, two production lines for major assemblies feeding the main assembly line will be automated. What effect can this have on the production planning and control department?

8

EVALUATION OF MATERIALS AND PROCESSES

The selection of materials and processes naturally has an important bearing on the production efficiency of the plant and on the final cost analysis of the product. All the issues associated with such a selection must therefore be carefully considered at the design stage. Though the desirable product characteristics are obviously of prime consideration, the significance of the maxim, "design for production," should never be overlooked.

What are the responsibilities of the production planning and control department regarding the selection of materials and processes? The production people should be consulted at the design stage, since their experience with regard to the behavior of materials and capabilities of available production processes often greatly contributes to the designer's realistic analysis, before the design is finalized. Once the materials and major processes have been specified, no changes may be introduced by the production departments without formal approval of the engineering department. The role of production planning at this stage is confined to machine allocation, which involves only minor decisions in process selection. However, a constant evaluation of materials and processes is required so that the primary decisions can be modified as more suitable materials or methods are evolved, or as more data become available about the factors that are considered "uncertainties" in the initial analysis.

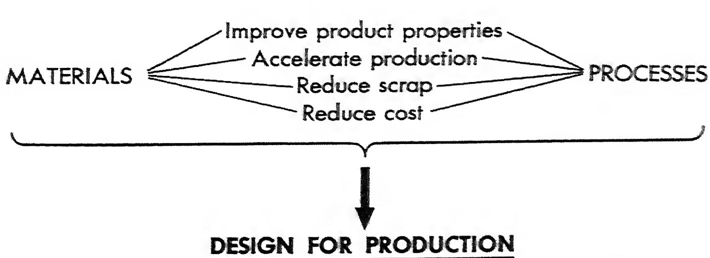


Figure 8–1. *Value analysis, or how to get more for less.*

In the evaluation of processes and materials, the four main factors that have to be considered are the product properties, the rate of production, the reduction of scrap, and last but not least, the economic analysis (Fig. 8–1). These factors are naturally not independent of each other. Any of these considerations may lead to possible changes in design, and these have to be taken up by the engineering and research and development departments.

Value Analysis

The purpose of value analysis is to improve the qualities of a product while maintaining or reducing its costs—in short, to get more for less. The first step in value analysis is to find out precisely the qualities that the product is supposed to possess, and why. These characteristics should then be compared in detail with the materials and production operations, in order to find out what material and which operation are responsible for which characteristic. This comparison may reveal material qualities that are not exploited or operations that seem duplicative. By critically analyzing the value of materials and processes, new avenues may be explored, with the aim of developing more efficient methods (i.e., more efficient in the sense that all the ingredients of input, in the form of properties of materials, machine operations and labor, really contribute to the attainment of the final goal).

Consideration of new techniques and materials

Conservatism in production engineering is a familiar phenomenon. Designers are reluctant to change techniques that have been used repeatedly with success in the past. In part this reluctance is due to ignorance of all the available possibilities, ignorance that stems from lack of experience in diversified methods of manufacture. But in many cases this conservatism is associated with anxieties and uncertainties as to what result may really be brought about by a change in method. The installation of a new method or a new machine is always accompanied by unforeseeable difficulties, and people have to adjust themselves to new ideas or new conditions and may require some time before they regain their confidence and feel masters of the situation. Concern regarding the advisability of abandoning a working method in favor of one that has still to prove its merit under the special circumstances is always genuine, often justified, and may well account for the rule of tradition in processes and methods that is often encountered in industry.

This is why value analysis has repeatedly produced spectacular results. The General Electric Company has established a value-analysis service section, which has resulted in saving several million dollars a year for several years. The U.S. Navy reported saving several millions through its value analysis department. The Caterpillar Tractor Company claims saving over a million dollars annually, and the Joseph Lucas (Electrical) Company in Britain reported saving up to \$300,000 per annum. Many more examples could be cited, and it appears that

manufacturers are becoming more conscious of the obvious and effective service that may be derived from value analysis.

Value analysis tests

Figure 8–2 illustrates ten tests in value analysis, as suggested by the Anglo-American Council on Productivity.[1] Each product or component is subjected to the following tests:

1. Does its use contribute value?
2. Is its cost proportionate to its usefulness?
3. Does it need all its features?
4. Is there anything better for the intended use?
5. Can a usable part be made by a lower cost method?
6. Can a standard product be found which will be usable?
7. Is it made on proper tooling, considering quantities used?
8. Do materials, reasonable labor, overhead, and profit total its cost?
9. Will another dependable supplier provide it for less?
10. Is anyone buying it for less?

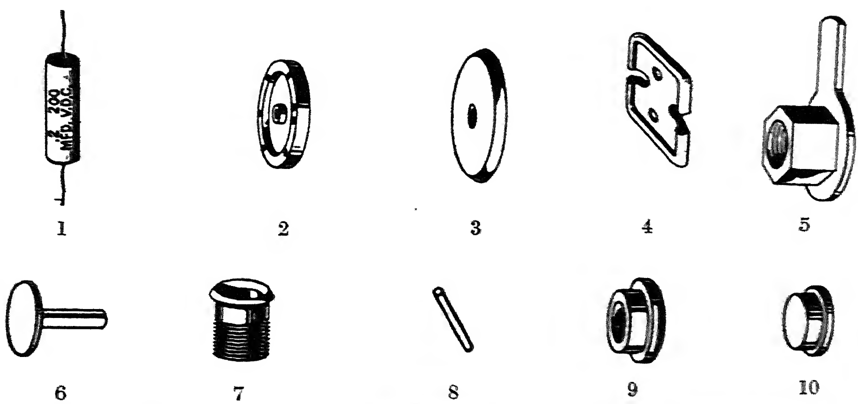


Figure 8-2. ***Ten tests for value analysis.*** (From "Design for Production," Anglo-American Council on Productivity, 1953)

These tests cover the main facets that have to be considered. The question of materials and processes evaluation will now be discussed in more detail.

Efficient Utilization and Selection of Materials

In the manufacture of some products, materials constitute quite a sizable portion of the total production costs, and a careful study of material utilization may result in substantial saving. Reduction of scrap may not only affect direct cost of materials, but it may also reduce the costs of handling, storing, and disposal of scrap, saving in space, labor costs, and machine time. An interesting

[1] Design for production (Anglo-American Council on Productivity, productivity report, October, 1953).

survey carried out by the Institution of Production Engineers,[2] London, revealed that in the manufacture of metal components, the material utilization in recent years in Britain has been as follows:

Material	*Range of Material Utilization* (%)[3]
Bar parts	12–80
Sheet metal	66–98
Brass sections	65–90
Iron castings	53–93
Aluminium alloy castings	67–93
Steel stampings	46–86
Average for complete set of parts in light precision assemblies	73

Although this survey was confined to the metal working industry, the wide range of utilization suggests that, while some firms are materials conscious and keep a close check on scrap and swarf, there is plenty of room for improvement in many other establishments.

Criteria for selection of materials

What criteria should guide the selection of suitable materials? Materials are primarily selected for their physical and chemical properties in order to ensure that the part will function satisfactorily. When several materials are suitable, and when no other requirements have to be met, the cheapest material is usually selected. There are, however, numerous examples in industry where materials have not been chosen with adequate consideration for optimal processing conditions. In all cases where the specifications of the materials impose a restriction of any kind on the process or on the machine, the critical study of these specifications is worth while, in order to ascertain quantitatively to what extent the restriction can be relaxed. Among the examples commonly found in industry the following should be briefly mentioned.

Machine speeds

Certain materials may impose restriction on machine operating speeds, such as in cutting speeds of machine tools and presses or flow through extruders due to selection of materials having low machineability, or in rate of flow incurred by the plastic properties or high viscosity of the material. Many case studies have shown that a selection of suitable materials allows an appreciable increase in operating speeds, so that even if more expensive materials are involved, the extra cost is sometimes offset by the savings effected through the shorter production cycle.

[2] Material utilization in the metal working industries (Institution of Production Engineers, London, 1955).

[3] Material utilization is defined as the proportion of purchased materials converted to final product.

Hot processes

Another consideration is restriction on hot processes, either when the temperature required is too high and involves lengthy heating operations or when the temperature range of the operation is too narrow and repeated heating has to be undertaken because of rapid cooling. The time required for heating, especially when several heating stages are involved, and the plastic or viscous properties of the material at the process temperature may be vital factors governing the production cycle.

Surface characteristics

Limitations caused by the surface characteristics of the material may require preparatory treatment prior to the main operation, particularly in cases of welding, plating, or painting. The necessary characteristics of the surface may

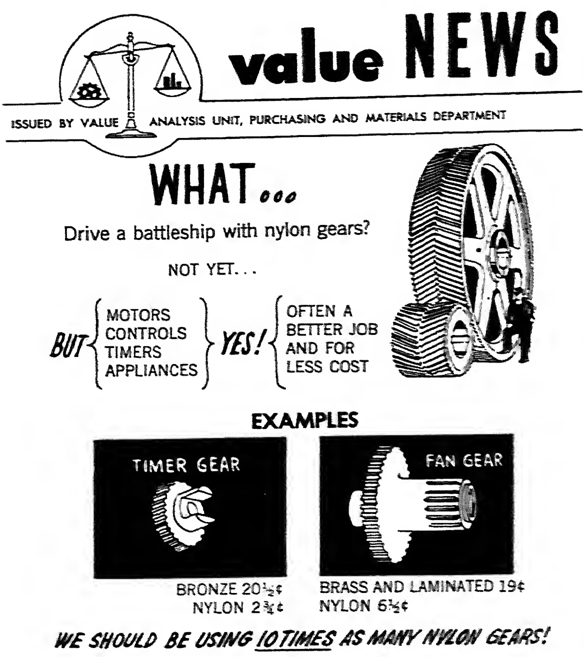


Figure 8–3. ***A leaflet for designers, encouraging the use of nylon gears.*** ("Design for Production," Productivity Report published by the British Productivity Council, formerly the Anglo-American Council on Productivity, 1953)

include the physical state of cleanliness, the surface texture, surface finish, and the chemical composition of the skin, or some specified combinations of these four factors. To obtain a clean surface, the material may have to undergo dusting or brushing operations, sand blasting, immersion into tanks containing detergents and other degreasing agents, or rinsing in stationary or flowing liquids.

To obtain the required surface finish and texture, mechanical working processes have to be used, such as roughing, grinding and polishing. The skin properties sometimes play a crucial role in welding and soldering or when adhesives are used. The skin (usually an oxide) may have to be removed, or alternatively, a new layer of some suitable material may have to be added. All these treatment operations are sometimes very costly and time consuming, but by careful selection of materials and specifications some of the preparatory treatment may be completely eliminated. The following examples are a few of those found in industry.

1. When using strips or sheets with a highly polished surface that have to be joined by adhesives, a certain roughness is required if the adhesion is to be strong enough. The polished surface, which is often attained at a cost, is in this case a disadvantage. A correct specification may reduce the price of the material and save a roughing operation.

2. A small shaft in a measuring instrument was ordered from a supplier. This component had to be nickel-plated before assembly, but it was found that prior to plating, it had to be surface-ground and then polished by barreling. The supplier had to grind the shaft prior to delivery and the surface finish of the component had to conform to the specifications he obtained. By restating these specifications, he could supply the shafts with a higher surface finish at a relatively small increase in costs, while the second grinding operation with the extra setup that was involved, could be eliminated.

3. In the manufacture of electronic instruments a 1-inch mild-steel strip was used to produce pressings. With a hole drilled at each end, one function of the pressings was to serve as a connection tag for several terminals. The soldering of the wires to the tag proved to be somewhat difficult, owing to the skin oxide, and many faulty connections were found during inspection. When a tinned strip was tried, it was found that the change in material specification had no effect on the press operation, while the rejects due to dry joints in soldering virtually disappeared.

New materials

New materials become available for a variety of new uses every day. Synthetic resins, for instance, have in recent years, come into many uses which had been hitherto confined to metals and their alloys. Epophen epoxide resins are now even being used in making tools and jigs, so that in spite of the lower tensile and impact strengths and poorer hardness qualities of these materials (compared with steel), they have to be seriously considered for application in many cases, owing to the relatively short time required for production of the tools and jigs. Other advantages include the appreciable reduction of machining and thereby release of machine tool capacity in the tool shop and to the reduction of total production costs.

Dimensional specifications

Improper dimensional specifications of the materials often increase the amount of scrap appreciably. Some examples are as follows:

1. For the manufacture of seamed tubes of 2-inch internal diameter, a plant used a strip 7-inches wide. First the strip was fed through a saw, trimming the width to 6.3 inches, so that when the strip passed through the rollers that gave it the required cylindrical form, its internal diameter was 2 inches as required. The swarf, which amounted to about 10 per cent of the purchased material, consisted of a very long, narrow strip that soon became untenable, owing to its volume. It was therefore necessary to use a small press, which was placed right beyond the saw, in order to chop the emerging narrow strip into bits of about 1 inch, and in this form the swarf was convenient for disposal. A study of the dimensional specifications of the strip revealed that a special strip 6.3 inches wide could be ordered, but since this dimension was not standardized, it would cost 6 per cent more, even though this strip was narrower than the original 7-inch one. The saving of 10 per cent scrap, the sawing and chopping operations, the release of the saw and the press, and the elimination of swarf handling, all these greatly outweighed the extra cost of the new strip.

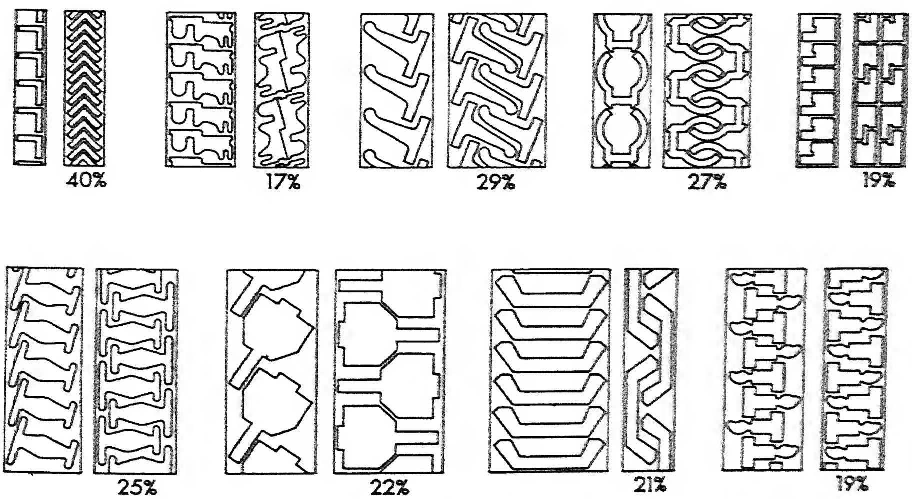


Figure 8–4. ***Nesting for presswork for better utilization of materials. Percentage figures indicate the saving effected by the new arrangements.*** (Reprinted with permission from James F. Young, *Material and Processes*, John Wiley & Sons, Inc., 1954)

2. The specification of metal strips for blanking in press-work is greatly affected by what is called "nesting" of the components on the strip. Figure 8–4 shows several examples of material saving by proper nesting and its effect on the width of the strip that should be purchased.

3. A common example is the use of bar stock of excessive diameter for turning operations. In Fig. 8–5 the component has a maximum diameter for 0.800 inch, and the use of $1\frac{1}{8}$-inch bar stock is very wasteful, first because of the large amount of swarf that has to be manufactured and secondly because of the machine time that is required to machine down the $1\frac{1}{8}$-inch diameter to 0.800-inch diameter. A more appropriate bar stock in this case would be $\frac{7}{8}$ inch or even $\frac{13}{16}$ inch.

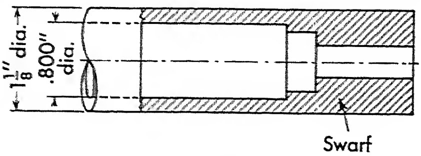


Figure 8–5. ***Excessive swarf caused by selection of bar stock of too large a diameter.***

Material Utilization of a Product or Assembly

The material utilization of a component has been defined as the ratio of the amount of material comprising the component to the amount of material that goes into the production process. This figure is easily found by the *ratio of weights*.

Even in the case of a product or an assembly comprising several such components, this examination by weights ratio is applicable. However, though this figure gives a very general idea as to the level of utilization, it does not provide enough useful information for the purpose of value analysis because it does not show which components involve waste. For the same reason, a simple arithmetic average of the figures for material utilization of the components is generally meaningless; 20 per cent utilization for one component may be far less serious a matter than even 80 per cent utilization for another. A different approach therefore seems advisable: All components are reduced to a common denominator by weighing the utilization figures against the relative production cost of the components, as shown in the accompanying table.

Component No.	*Material Utilization*	*Total Prod. Costs*	*Relative Prod. Costs*	*Partial Utilization Figure*
1	w_1	C_1	$c_1 = C_1/C$	w_1c_1
2	w_2	C_2	c_2	w_2c_2
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
i	w_i	C_i	$c_i = C_i/C$	w_ic_i
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
n	w_n	C_n	c_n	w_nc_n
Total (for the whole assembly)	W	$C = \sum_{i=1}^{n} C_i$	$\Sigma c_i = 1$	$\sum_{i=1}^{n} w_ic_i$

The equivalent utilization for the assembly is obtained by the product of the second and fourth columns for the assembly, namely, $W\,\Sigma c_i$ or $W \cdot 1$, but this

figure is also given by the summation of the partial utilization figures in column 5; therefore

$$W = \sum_{i=1}^{n} w_i c_i \tag{8-1}$$

The partial utilization figures represent the contribution of each component toward the total equivalent figure. The fourth column shows the maximum contribution that could be obtained if each component had a material utilization figure of 100 per cent. By comparing the last two columns it becomes a simple matter to determine which components should be analyzed first. Let us examine the application of this method by an example.

Example

An assembly consists of six components, having material utilization figures of 80, 52, 22, 20, 20, and 95 per cent. The production costs are \$2.00, \$1.10, \$3.12, \$0.52, \$0.10, \$0.04. The accompanying table gives the corresponding data.

Component No.	*Material Utilization* (°)	*Prod. Costs*	*Relative Prod. Costs*	*Partial Utilization Figure*
1	0.80	\$2.00	0.291	0.233
2	0.52	1.10	0.160	0.083
3	0.22	3.12	0.453	0.100
4	0.20	0.52	0.076	0.015
5	0.20	0.10	0.015	0.003
6	0.95	0.04	0.006	0.006
Total		\$6.88	1.001	0.440

The equivalent material utilization figure for the assembly is 44 per cent. It is evident from the table that improving the material utilization of the last three components will not contribute very much, whereas the first three contribute now about 42 per cent out of a potential figure of 90 per cent; the third component particularly should be studied for possible improvement (as it contributes now 10 per cent out of potential 45 per cent).

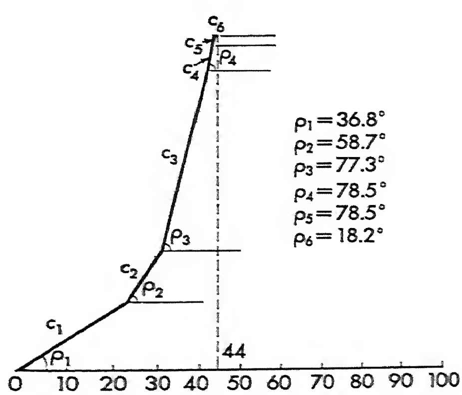


Figure 8–6. ***Material utilization polygon for a product, an assembly or a shop.***

This method is shown graphically in Fig. 8–6. The relative production cost c_1 is drawn at an angle $\rho_1 = \text{arc cos } w_1 = 36.8°$ to the horizontal axis, then c_2 at an angle $\rho_2 = \text{arc cos } w_2 = 58.7°$, etc. The partial utilization figures accumulate on the horizontal axis until 44 per cent is obtained for the whole assembly. In the polygon constructed in this way, the longer the side and the larger the angle, the more worth while it is to study the material utilization of that component.

Materials utilization of a shop or a process

The same method can be employed to find the materials utilization of a shop or a process. Again, the over-all utilization established by the ratio of total material output to total material input, is often useful for obtaining an idea about the amount of scrap and swarf involved. The second method, however, gives an indication as to where action would be most effective. In this case, however, the number of components produced should also be taken into account, since the larger the batch, the greater is the contribution toward the total materials utilization. Hence the preceding computation table has to be modified as follows:

Component No.	*Prod. Volume (units per unit time)*	*Material Utilization*	*Total Prod. Costs per unit*	*Total Prod. Costs per batch*	*Relative Prod. Costs*	*Partial Utilization Figure*
1	Q_1	w_1	C_1	Q_1C_1	$c_1 = Q_1C_1/C$	w_1c_1
2	Q_2	w_2	C_2	Q_2C_2	c_2	w_2c_2
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
i	Q_i	w_i	C_i	Q_iC_i	$c_i = Q_iC_i/C$	w_ic_i
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
n	Q_n	w_n	C_n	Q_nC_n	c_n	w_nc_n
Total (for the whole shop)		W		$C = \sum_{i=1}^{n} Q_iC_i$		$\sum_{i=1}^{n} w_ic_i$

Thus

$$W = \sum_{i=1}^{n} w_ic_i = \frac{1}{C}\sum_{i=1}^{n} w_iQ_iC_i \tag{8–2}$$

C represents the total turnover of the shop in the period of time taken for the study (a week, a month, etc.). From the partial utilization figures (or from the material utilization polygon) the components that require further study may be selected.

It has been assumed in this method that the proportion of materials costs to the total production costs is about the same for all components in the shop, and for many studies this assumption is adequate. When, however, the material

costs content varies widely from one component to another, this factor has to be taken into account. The materials content factor is defined as

$$\beta_i = \frac{\text{cost of material}}{\text{total prod. costs}} \text{ (of the } i\text{th component)}$$

and the partial utilization figure is $w_i c_i \beta_i$, compared with $c_i \beta_i$, which is the maximum that can be achieved with 100 per cent materials utilization. The over-all figure is

$$W = \frac{1}{C} \sum_{i=1}^{n} w_i Q_i C_i \beta_i \tag{8–3}$$

Selection of Processes

The purpose of production processes is to attain one of the following objectives:

To shape the material as nearly as possible to the final desired form and dimensions, in order to save materials, machine time, and labor

To join components into assemblies that possess the required functional qualities

To improve the properties of the material, for instance by heat treatment or by addition of other materials to form alloys, coatings, etc.

Analysis of processes

As in the case of materials, the selection of production processes should be primarily determined by technological considerations, in order to ensure the achievement of these objectives. It is not intended to describe here the various production processes in use and to relate the advantages of each; this would require too much space and therefore the reader is advised to study some of the titles mentioned in the references. Assuming that the reader is familiar with the subject, the following exposition will mainly dwell on the problem of selection between different processes, all of which may be equally acceptable from the technological analysis aspect. Obviously, if only one process is capable of producing the required characteristics, management decision is confined to the question: Should the component be produced in the plant or bought from suppliers? The final decision will mainly rest on the comparative costs of self-production versus procurement. Usually, however, the component can be made by methods A, B, C, etc., some of which may be at the plant's disposal, while others may either involve subcontracts or new installations at the plant. It is therefore necessary to decide:

1. What additional functional qualities result from the processes in question, apart from the minimum requirements set in the specifications, and how much do they cost?

2. How do the processes compare in costs at various production ranges

(following, for example, the method illustrated in Fig. 8–7), and what is their rating? This is useful when processes have to be re-evaluated because of changing conditions, when the best process cannot be exploited, owing to overloading in the plant or reluctance of management to subcontract certain components.

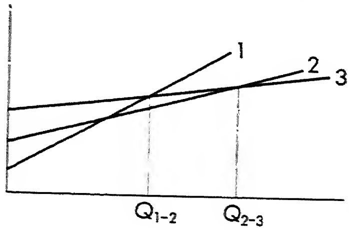


Figure 8–7. ***Initial evaluation of three processes.***

3. Under what conditions is it advisable to increase the capacity of a given production center by acquisitions of additional equipment, while machines operating on an "inferior" process (as determined by the above rating) remain idle or have to be scrapped?

Evidently the value analysis of production processes is very helpful, but the process defined as "best" is essentially "best" under idealized conditions. In the final analysis all the operational factors pertaining to the circumstances prevailing in the plant have to be taken into account. Availability of machine time, shop loading, and fluctuations in the production schedule, expansion possibilities, subcontractors' offers and competence, subcontracting policy—these are some of the factors that have to be carefully weighed, and it may well be that what is best for one plant is not best for another.

Apart from new processes and new materials, which may revolutionize the production or initiate an entirely new approach to production problems, one factor that greatly affects the selection of methods (and sometimes materials) is the quantity to be produced. The effect of quantity on the economic analysis of production was already discussed in Chapter 5. Suppose that in the first analysis three processes (1, 2, 3) are considered suitable from the product characteristic point of view, and the break-even chart is represented by Fig. 8–7. The fixed costs are given by c_1, c_2, c_3, respectively, but the variable production costs decrease as the fixed costs increase, say, owing to increased rate of production. Assuming that all three processes are available and questions of capacity do not affect the analysis at this stage, the following simple rule is adopted:

When $Q < Q_{1-2}$	use process 1	(Q being the quantity to be produced)
For $Q_{1-2} < Q < Q_{2-3}$	use process 2	
For $Q_{2-3} < Q$	use process 3	

For the special cases $Q = Q_{1-2}$ or $Q = Q_{2-3}$, the decision will rest on what is predicted for future development. If now $Q = Q_{1-2}$, but there are indications that quantities will probably increase in the future, process 2 is adopted, etc.

Suppose the quantity to be produced is between Q_{1-2} and Q_{2-3} and that process 2 has been selected. The market has now been expanded slightly beyond Q_{2-3}, and according to the initial analysis of Fig. 8–7, process 3 becomes more economical. The question of changeover must also take into account any capital expenditure involved in such a step, and this is done on the lines discussed in Chapter 5 for changes in models or designs.

An additional factor that has to be considered is the effect of experience and learning. During the period of exploiting process 2, better methods may have been developed and operators may have become more skilled in their job, leading to a situation as shown in Fig. 8–8, where 2 is the initial evaluation of process 2 and $\bar{2}$ is its present evaluation. The break-even point Q_{2-3} is transferred to $Q_{\bar{2}-3}$, so that even when a larger production volume is required, it may well be that process 2 is still good enough. Similarly, situations may arise where re-evaluation reveals that the existing process has ceased to be the best one and should give way to another. The effect of the learning curve and a step-wise fixed costs function on the total costs is further discussed in Chapter 20.

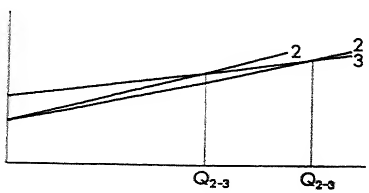


Figure 8–8. ***Re-evaluation of process 2 in relation to process 3.***

Numerous case studies have been carried out, in which the main theme is the evaluation and comparison of processes on the lines described above. Perhaps a few examples will be useful.

Examples

1. The use of cold heading has often been proved to be more economical than machining when the quantities to be produced are big enough. Two factors contribute to the superiority of this process: saving in material and high rates of production. Figure 8–9 describes three case studies in cold heading versus machining.

2. Accurate casting methods are often used to cut down machining, or even to eliminate it altogether. Complex components can be made to close dimensional tolerances with a high-grade surface finish, so that in many cases accurate casting can economically replace sand casting or other production processes. In Fig. 8–10 three instances are cited to illustrate the use of gravity die casting and shell molding instead of sand casting, and investment casting instead of complex milling operations. Some typical parts produced by investment casting are

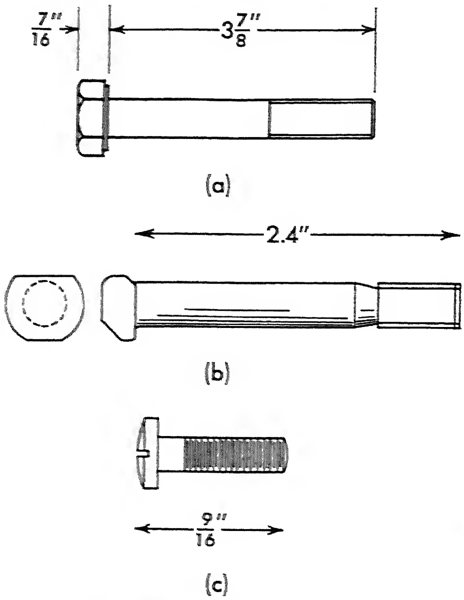


Figure 8–9. ***Case studies in cold heading versus machining.***
("Material Utilization in the Metalworking Industries", Institution of Production Engineers, London, 1955)

(*a*) The screw was originally turned from hexagon steel bar, the material utilization being 33 per cent. Cold heading increased material utilization to 84 per cent and reduced costs by 53 per cent.

(b) When producing the illustrated motor engine connecting rod by cold heading and thread rolling instead of using conventional methods, a saving of 63½ per cent was effected.

(c) The screw was originally machined on a single spindle auto. When changed to cold heading, a saving of about 3,500 pounds of steel resulted for half a million screws. Material utilization in the first method: 23·4 per cent; in the new method: virtually 100 per cent.

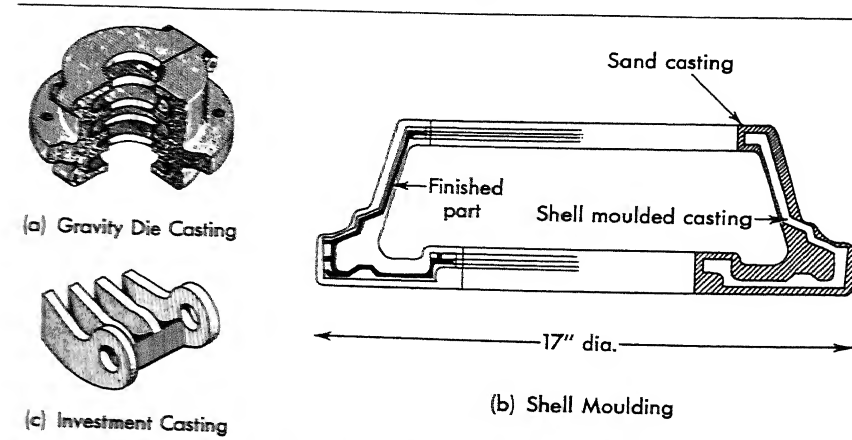


Figure 8–10. ***Examples for the use of accurate casting methods.***
("Material Utilization in the Metalworking Industries", Institution of Production Engineers, London, 1955)

(a) A carbon packing casing for a bearing for a turbine, originally made as a sand casting in gunmetal, was later produced by gravity die-casting in manganese bronze, resulting in 60 per cent saving in production costs.

(b) To produce components of this kind in sand casting, a comparatively thick wall has to be allowed for, and the thin section can later be achieved by machining. In shell molding the cast component requires 65 per cent less material, which is in itself a substantial saving, let alone the saving in machining costs.

(c) This pawl for an adding machine had been produced by machining (including milling, drilling, and reaming operations). When investment casting was selected instead, the total production costs came to one-third of the original method.

shown in Fig. 8–11. When large quantities are involved, pressure die casting with permanent molds becomes an economical proposition. With a very small rate of scrap, high dimensional accuracy, and high rate of production, pressure die casting can be used to make very complicated shapes (Fig. 8–11).

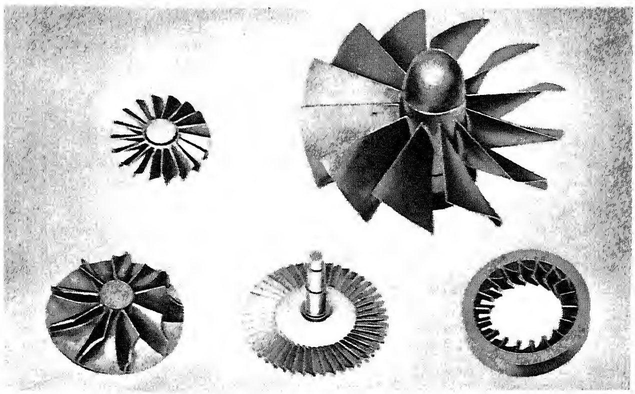


Figure 8–11. ***Parts produced by accurate casting methods.*** (Reprinted with permission of United Carbide International Co.)

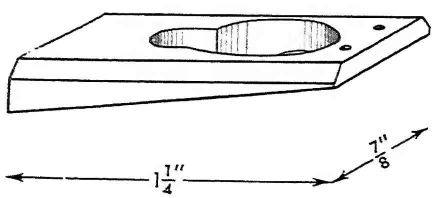


Figure 8–12. ***An example of the use of powder metallurgy. The illustrated part was first produced by milling a solid piece of steel. Milling is a lengthy operation, and by changing the process to powder metallurgy, a saving of 74 per cent in production costs was obtained.*** ("Material Utilization in the Metalworking Industries", Institute of Production Engineers, London, 1955)

3. The powder metallurgy process may effectively replace casting and machining processes. Figure 8–12 describes a case where the use of powder metallurgy reduced costs by about 74 per cent. Some typical parts produced by powder metallurgy are shown in Fig. 8–13. Apart from its economic advantages, components produced by powder metallurgy may be self-lubricating (owing to the porous structure of the material), a feature that lends the process superior functional advantages in the case of bush bearings, etc.

4. A thick metal washer, $\frac{1}{2}$-inch OD, as illustrated in Fig. 8–14, may be produced in two ways:

(i) Machining in a lathe, using $\frac{1}{2}$-inch diameter bar stock, in two operations: hole drilling, parting off. This method is time consuming, even when a multiple

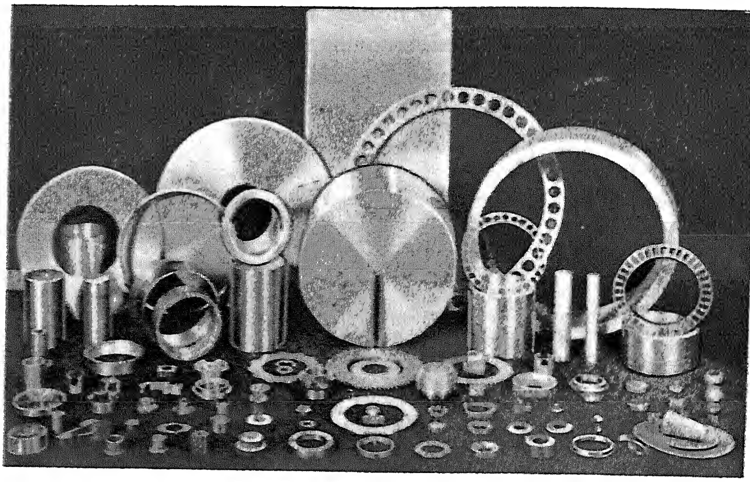


Figure 8–13. ***A group of typical parts produced by the powder metallurgy process.*** (Courtesy Amplex Division, Chrysler Corporation)

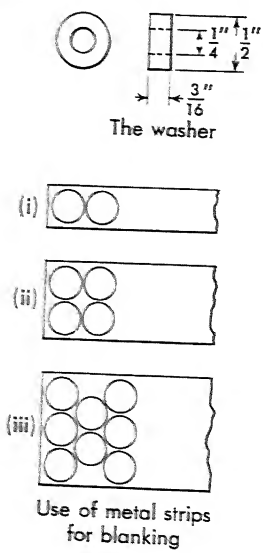


Figure 8-14.
Blanking a thick washer.

parting-off tool is used, and the amount of swarf (assuming a tool width of $\frac{3}{32}$ inch, not counting the hole) is

$$\frac{\frac{3}{32}}{\frac{3}{32}+\frac{3}{16}}\,100 = 33\%$$

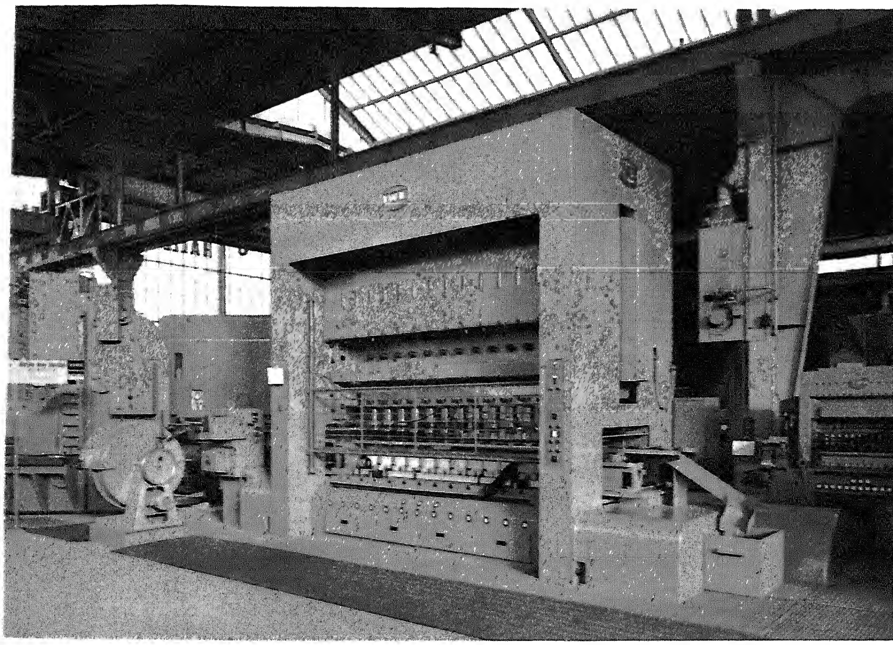
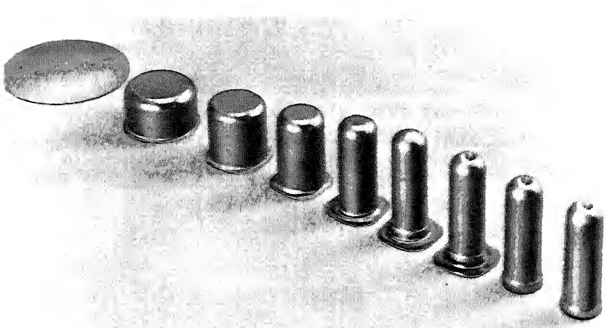


Figure 8–15. ***A transfer press and sample products.*** (Courtesy IWK, Karlsruhe, Germany)



(ii) Blanking by press, where the material wastage is roughly (assuming a strip $\frac{1}{2}$ inch wide is used, and not counting the hole)

$$\frac{(0.5)^2 - \frac{1}{4}(0.5^2\pi)}{0.5^2}\,100 = 21.5\%$$

This figure can be reduced, since more washers can be blanked from a wider strip.

5. Press work is a very efficient production process when large quantities of a component are required. Very often, several press operations are needed, and a transfer press for multistage operations may economically replace a battery of presses in the shop (Fig. 8–15). Some case studies on transfer presses suggest that they become worth while even when they are not fully utilized, owing to saving effected in floor space and materials handling.

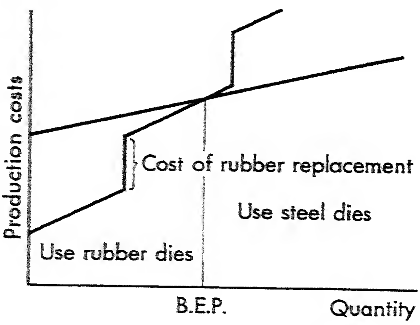


Figure 8–16. ***Use of rubber dies versus steel dies in press work.***

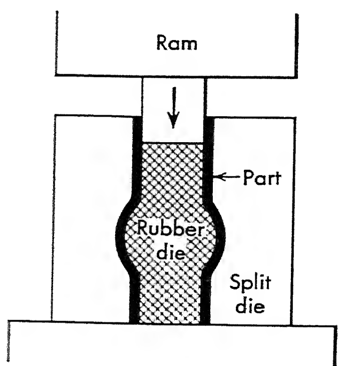


Figure 8–17. ***Bulging with a rubber die.***

6. Press work in small quantities may be very effective with the use of rubber dies. The rubber die has a life of 10,000 to 20,000 operations, so that when larger batches are involved, the dies have to be replaced. The rate of production by rubber dies is relatively low, and therefore this method cannot hope to compete with conventional steel dies (Fig. 8–16) in producing large quantities. There are, however, some applications for which rubber dies are ideally suitable; for instance, for bulging, as shown in Fig. 8–17.

7. Punching in sheet metal working has been found to be more economical than drilling, even when several drilling operations are performed simultaneously on a multispindle machine. In some cases the drilling has to be carried out after pressing of sheet metal, and this may involve lengthy setting-up operations. The use of piercing units (incorporating a punch and a lower die block), for the pressing shown in Fig. 8–18, allows all the holes to be punched simultaneously by operating all the piercing units in one ram travel. The units can be standardized for various operations and their location can be facilitated by V-slots. In this way costly dies or lengthy drilling operations can be saved.

8. The spinning operation may become an excellent substitute for press work. Though it requires a highly skilled operator (whereas in press work semiskilled

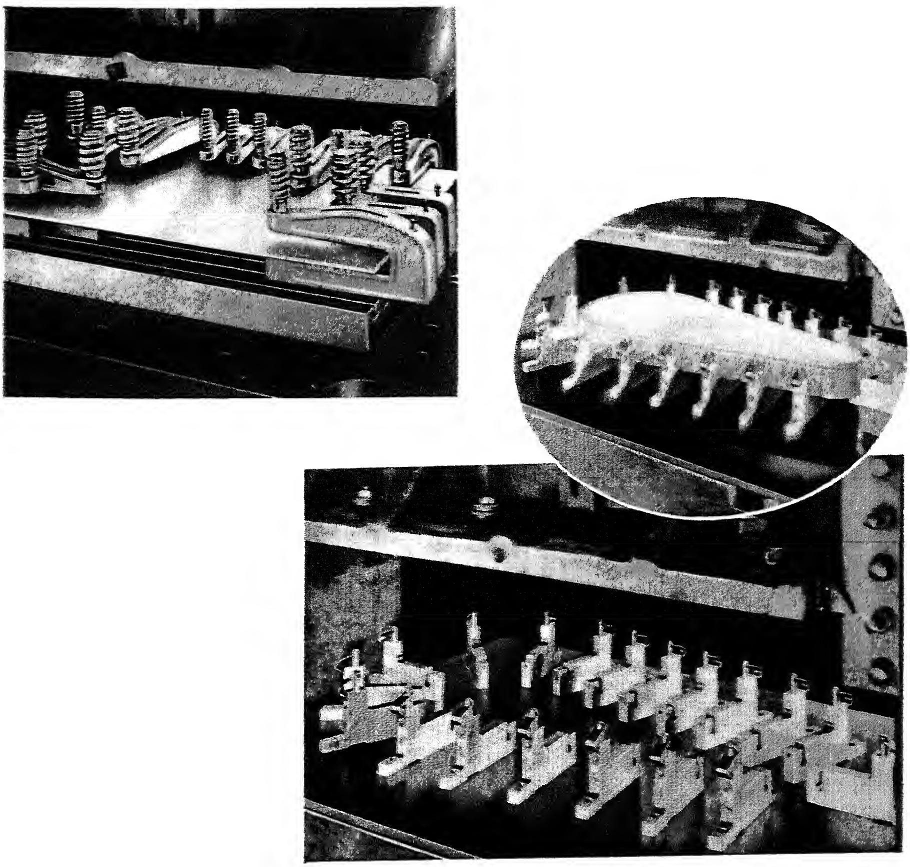


Figure 8–18. ***Use of vertical and horizontal units as a substitute for drilling. Units are located in the appropriate positions and the holes are pierced in one travel of the ram.*** ("The British Pressed Metal Industry", Anglo-American Council on Productivity, 1953)

workers may often be adequate) and the operation is distinctly slower, spinning involves low-cost dies, and it may therefore become more economical when small batches are required. The cost-function comparison is somewhat similar to the case in Fig. 8–17. Spinning becomes particularly advantageous when it replaces multistage deep-drawing operations which require several sets of tools or when frequent changes in design are desirable. To change press steel tools for a new design is a very costly affair, while in wooden dies for spinning the opposite is the case. For example, an aluminium lighting reflector (Fig. 8–19) having a large diameter would require a fairly large press, which would be considerably more costly than a spinning lathe with its comparatively cheap tools. If the quantity to be produced does not exceed several thousand pieces, spinning may be far cheaper.

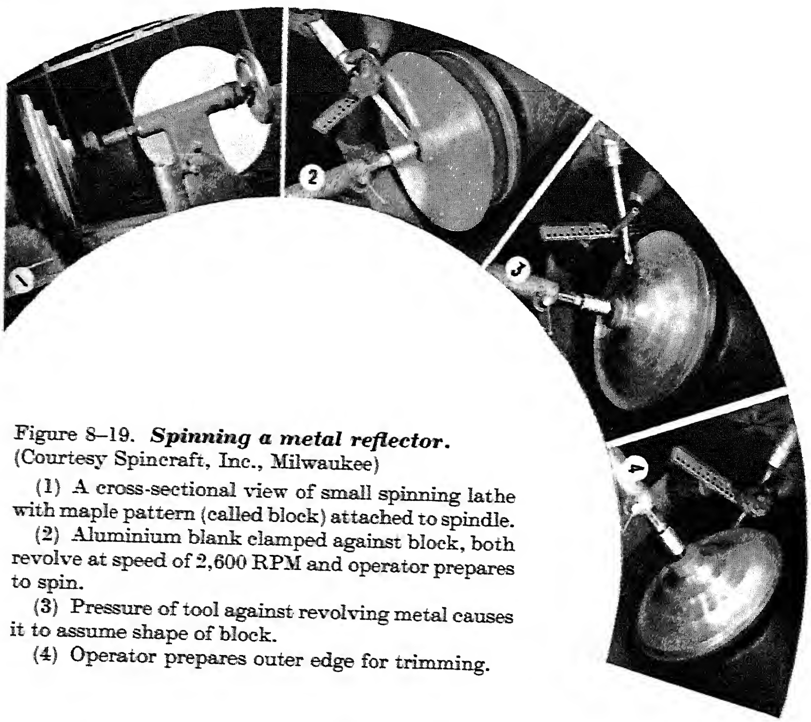


Figure 8–19. ***Spinning a metal reflector.*** (Courtesy Spincraft, Inc., Milwaukee)

(1) A cross-sectional view of small spinning lathe with maple pattern (called block) attached to spindle.
(2) Aluminium blank clamped against block, both revolve at speed of 2,600 RPM and operator prepares to spin.
(3) Pressure of tool against revolving metal causes it to assume shape of block.
(4) Operator prepares outer edge for trimming.

9. The use of welding for construction work is becoming widespread, and welding is rapidly replacing other joining methods (riveting and bolting) and casting, the preference of welding to other joining processes being mainly based on a cost analysis. Compared with casting, the welded construction has some outstanding advantages:

(i) It uses far less material, which in many cases constitute a substantial item in total costs.

(ii) It need not be assembled in the plant. The parts can be transported to the site where the final construction has to be erected, and assembled there. This may appreciably reduce transportation costs (which are high for bulky and awkward constructions) and free space at the plant.

(iii) It is lighter and therefore easier to handle.

Two examples, where welding was substituted for casting, are shown in Fig. 8–20. Apart from a spectacular reduction in the weight of the structures, the costs in each case were reduced by about 30 per cent.

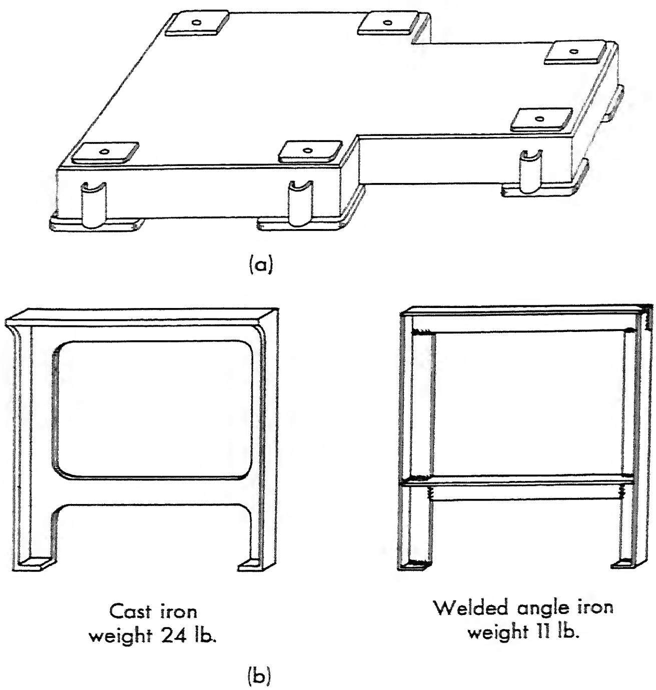


Figure 8–20. ***Welding replacing casting.***
("Material Utilization in the Metalworking Industries", Institution of Production Engineers, London, 1955)

(a) A common machine bedplate. Use of welding saved approximately fifty pounds in weight and 32.5 per cent in production costs, compared with casting.

(b) A bench leg commonly used in engineering plants. Change from casting to welding reduced weight from 24 to 11 pounds and reduced costs by 30 per cent.

10. An interesting example for combining operations is shown in Fig. 8–21, where turning and grinding were substituted by machining on a shaving machine. Changes of this kind may be worth while even when the suggested operation is slower than those being used currently, as in many cases the saving of handling, loading, and unloading between operations is very substantial.

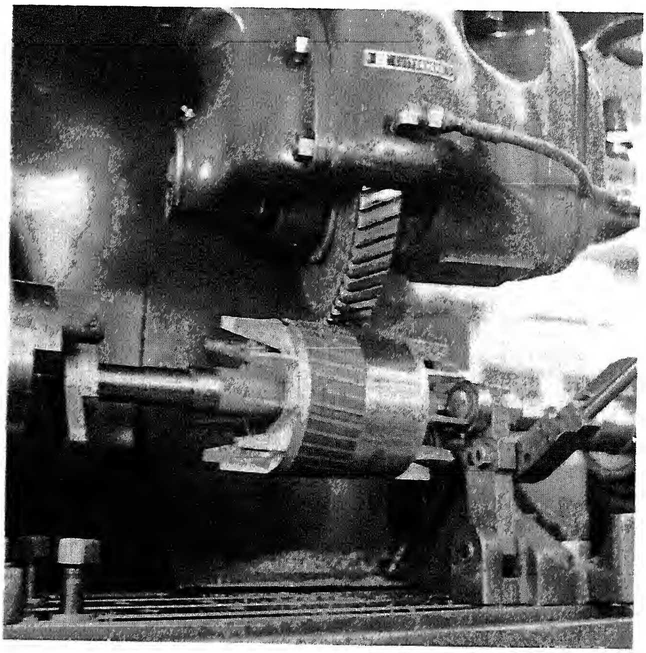


Figure 8–21. ***Production of rotors for F.H.P. motors. Former method: turning and grinding; new method: rotary machining on a shaving machine; saving: ten minutes per rotor.*** (Courtesy Brown, Boveri & Co., Ltd, Switzerland)

Design for Production

The exploration and evaluation of various alternatives in the selection of materials and processes should have an effect on the design of the product, as already illustrated in Fig. 8–1. This is a vital feedback: The experience gained during the manufacturing period of the product should be closely scrutinized with the view of introducing modifications into the design to make it easier and cheaper to produce. The examples enumerated above illustrated how such goals may be achieved through reduction of scrap and by employing a cheaper production method; some of them indicated the need for design modifications for that purpose.

Scope of modifications

Design modifications may be required to effect solution of production problems. Among these are the following:

(i) Modifications in the design of the component, to facilitate the use of higher production rates, better methods or materials; easier handling, loading,

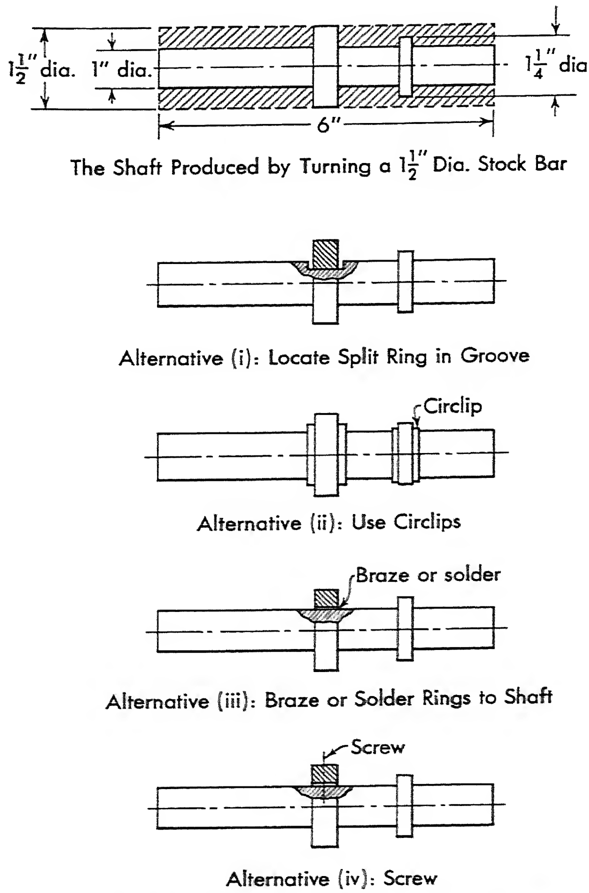


Figure 8–22. ***The production of a shouldered shaft.***

and positioning in the machine; quicker unloading; easier inspection to enable detection of faults at various stages of the production; easier and quicker machine setting, etc. Naturally, such modifications have to be checked from the consumer's point of view, to ascertain that the functional qualities of the product are thereby not impaired.

(ii) Combination of several components into one, in order to reduce or eliminate processing operations of individual components and the need of matching and assembly operations.

(iii) Division of a component into several parts, especially when the component involves excessive machining operations, lengthy positioning, or complicated inspection.

Several examples follow to illustrate modifications in design on these lines.

Examples

1. A shaft 1 inch in diameter is to be produced, having shoulders of $1\frac{1}{2}$-inch and $1\frac{1}{4}$-inch diameter, respectively, as shown in Fig. 8–22. The original method consisted of turning down steel bars of $1\frac{1}{2}$-inch diameter. The scrap amounted to

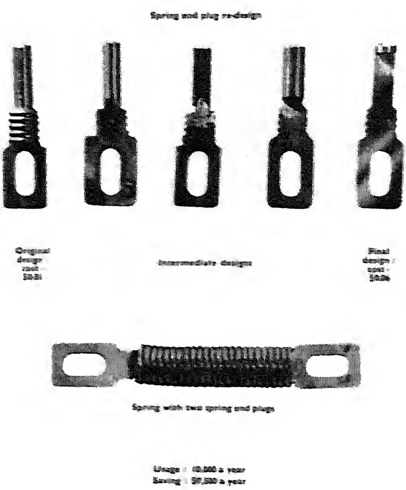


Figure 8–23. ***Modifying a design. Progressive design modifications to reduce production costs of spring end plug in voltage regulator (a transformer component).*** ("Design for Production", Anglo-American Council for Productivity, 1953)

slightly above 50 per cent of the stock bar, and the machining consumed a considerable amount of lathe time. The following alternative methods were considered:

(i) Use 1-inch bar as raw material, cut shallow grooves in position of the shoulders, and insert split rings that can be slid across a 1-inch shaft and sprung back to locate in the grooves. Only 5 per cent of the material has to be machined in this method.

(ii) Use 1-inch bar and position the rings by circlips, with or without the grooves mentioned in the preceding method.

(iii) Eliminate the turning operation and join the rings to the 1-inch bar by brazing or soldering.

(iv) Screw the shoulders to the shaft.

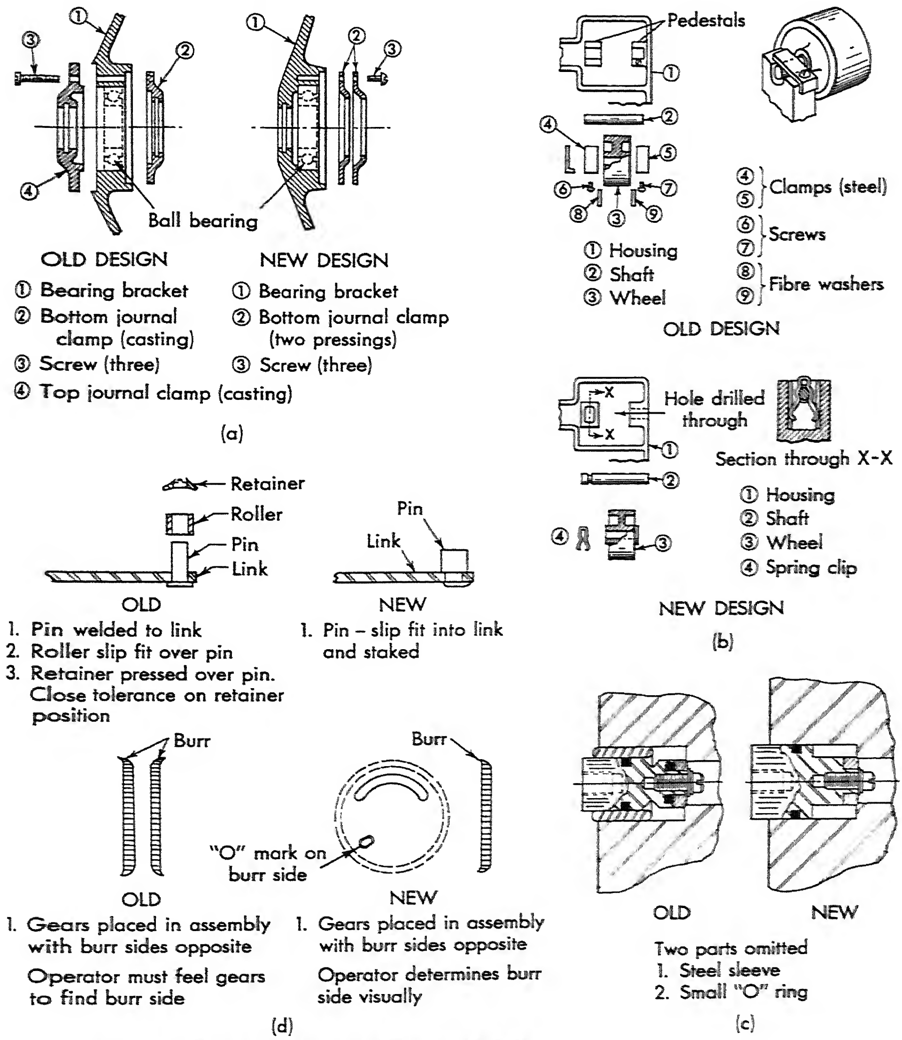


Figure 8–24. ***Examples of design modifications:***

(a) Bearing bracket journal clamp | (J. Rawicz-Szczerbo, "Value Analysis", J. Institution
(b) Wheel mounting of a vacuum | of Production Engineers, 1958)
(c) Valve assembly | (Courtesy Process Department, General Motors Corporation,
(d) Fan assembly | Detroit)

All four methods have one feature in common: They strive to reduce or eliminate scrap and machining time by introducing an operation for locating rings on the shaft. The simplest and cheapest alternative is the use of circlips without grooves on the shafts, and in the case study referred to, this method proved to be satisfactory. Had the shoulders been subjected to side thrust, brazing or soldering would have been preferable. The fourth alternative is generally more costly than the third, since the shafts have to be drilled and tapped. This example shows how the change in production process may require a change in design, which sometimes involves a change in the number of components in the assembly. In this case a one-piece shaft is replaced by a shaft plus two rings, not counting the circlips that are required by the second alternative.

2. The spring end plug of a voltage regulator, shown in Fig. 8–23, was redesigned several times. Finally a cold pressing was substituted for a component turned from a bar, resulting in a substantial reduction of production costs. The intermediate design stages are typical of the process analysis that is often carried out in cases of this kind. In spite of the comparatively low annual consumption of this part, a saving of $7,500 a year was achieved.

3. A ball-bearing bracket for a ceiling fan (Fig. 8–24) was originally designed to take two clamps, between which the bearing was held. In the new design the bracket and one clamp are combined so that the operations of machining this clamp and screwing it on the bracket are eliminated, and the total costs are thereby reduced.

4. A wheel mounting of a vacuum cleaner was first designed as shown in Fig. 8–24. The bakelite wheels assembled on plain shafts were separated from

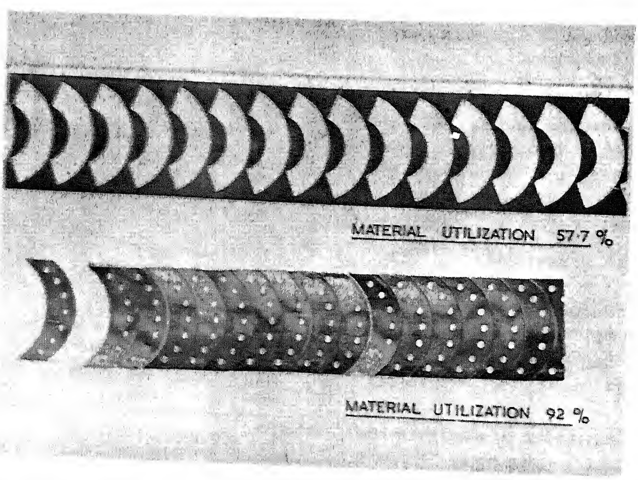


Figure 8–25. ***Design modification of a rotor segment. It is not always imperative to adhere to the concentric circles convention in design.*** (Courtesy of Hoover, Ltd, England)

the "pedestals", in which the shafts were located, by spacers. In the modified design the number of components was reduced from nine to four and positioning of the shaft was facilitated by a wire spring clip.

5. A segment of rotor balance weights is shown in Fig. 8–25. It was first designed in the form of concentric circles, a conventional method of design. In the new design the arcs were no longer parts of concentric circles, and the material utilization increased from 58 to 92 per cent. The disposal of swarf was also greatly simplified.

6. The hinge bolt shown in Fig. 8–26 was originally turned from a mild steel bar having a square cross-section. By changing the design to two separate parts (a head and a shank) joined by copper brazing, 46 per cent saving in material and 55 per cent in total costs were effected. Provision had to be made for venting the air in the hole during the brazing operation, but the saving in labor and machine time was still very marked, as indicated by the reduction in total costs.

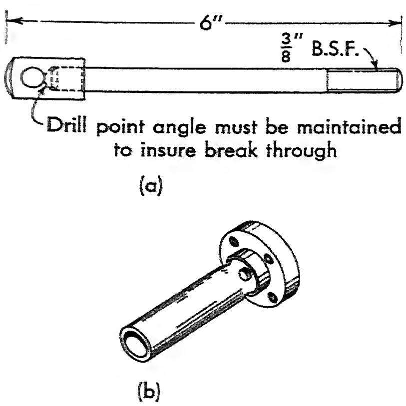


Figure 8–26. ***Dividing a component. (a) A hinge bolt. Originally turned from square mild steel bar; the method was changed to make the head and shank separately and join them by copper brazing. It should be noted that, when brazing in a blind hole, means must be provided for venting it, otherwise the components will be separated by the expansion of the entrapped air. (b) A guide bush. Originally turned from solid bar 2¼ in. diameter; the method was changed to an assembly of head and body joined by copper brazing. (In both examples the cost of production was halved when the revised methods were adopted.)***

7. The ratchet wheel in Fig. 8–27 was originally produced from a solid bar. By adopting a copper brazing assembly the material utilization was improved from 15 to 54 per cent. The shoulder cam illustrated in the figure is a similar example.

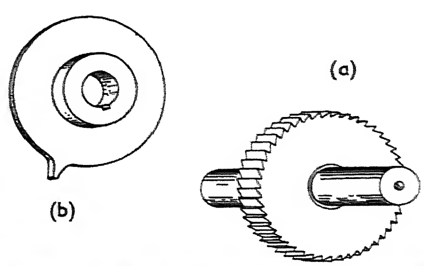


Figure 8–27. ***Dividing a component. (a) A rachet wheel; originally produced from solid bar, the method was changed to a copper brazed assembly. (b) A shoulder cam; now produced by assembling the shoulder portion into the disc and copper brazing, the part is hardened on the periphery only. By this method only 36 in. as much material is required as is needed to make it from the solid. The shoulder portion is an easy fit in the disc and is retained in position during brazing by four center punchings on the opposite side from that shown.***

8. Figure 8–28 shows a generator shaft produced by forging; but when deliveries were delayed, other alternative processes were explored. By changing the method so that a steel casting was shrunk on the end of the shaft made of stock bar, a spectacular saving in total costs could be achieved. It is interesting to note that in this example the labor cost was reduced by 23 per cent, but due to the high cost of forging, which in the new method was eliminated, the cost of the assembly was reduced to a quarter of the original cost.

9. The selector for an automobile gear box, shown in Fig. 8–28, is a typical example of a complex shape that poses tricky chucking problems and involves lengthy machining operations, when these are required. The original method consisted of forging in one piece and subsequent turning of the shaft. The new design included two parts: the shaft, turned and ground from stock bar; and a pressing, the two being welded to a final assembly. Saving of 55 per cent in materials cost and 35 per cent in labor cost were obtained.

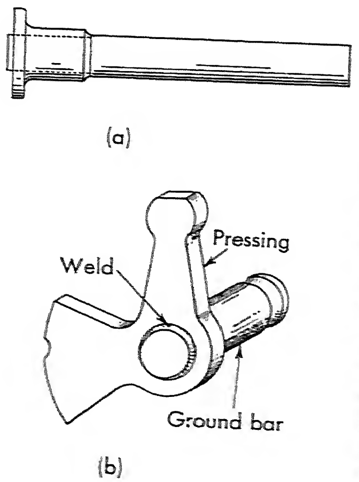


Figure 8–28. ***Dividing a component. (a) A generator shaft; originally produced as a forging: due to the difficulty and delay in obtaining forgings, the method was changed to making from stock bar plus a steel casting shrunk on to the end. (b) A selector for gear box; formerly produced as a forging and has now been changed to a fabrication from a pressing and a shaft welded together. Apart from the saving in material, the production is very much easier as the forging was very awkward to hold for turning. The revised method showed a saving in material cost of 55 per cent and a labor cost of 35 per cent.***

10. The bearing sleeves shown in Fig. 8–29 were first made from brass tubing with bushes fitted in the ends. In the modified design, not only were the tube and two bushings combined to eliminate assembly work but also the rolling operation

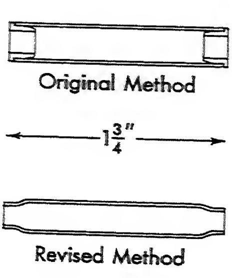


Figure 8–29. ***Modified design of bearing sleeves.*** (Figures 8–26, 8–27, 8–28, and 8–29 are reproduced from "Material Utilization in the Metalworking Industries", Institution of Production Engineers, London, 1955)

to form the internal diameter required at the end was carried out simultaneously with cutting-off in an auto screw machine.

11. The guide bush in Fig. 8–26, made of monel metal, was originally machined from a solid bar. Again, splitting into two components and joining by copper brazing proved to be effective. The machining time was not greatly reduced here (from 82 minutes to 80 minutes), but the saving in material (utilization increased to 46 per cent from 16 per cent!) led to a cost reduction of 45 per cent.

Summary

Design, like any other planning activity, should have as one of its goals cheap and easy production, within the framework of the specified functional qualities and other characteristics that the product is supposed to possess. Naturally this implies that designers should have a knowledge and appreciation of production materials, processes, and methods; otherwise, without being acquainted with recent developments in this field, the designer will be unable to make an objective and realistic evaluation of the available alternatives. A value analysis helps to clarify the essential features of operations, materials qualities and components, and to select the best and cheapest to satisfy given value levels. Surveys in industry indicate that material utilization is usually fairly low and that substantial saving can be realized by careful selection of materials and processes.

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Problems

1. Does the maxim "design for production" impose any limitations on the product development and design departments?
2. Give two examples, one to illustrate design for functional ends and one to show how production considerations were the prime factors in the designer's mind. Can these two objectives be at variance with each other? Illustrate your point by an example.
3. In the ten tests for value analysis, distinguish between those that are primarily concerned with selection of materials and those concerned with production processes. Give examples to show in what order you would prefer these tests to be applied.

4. Analysis of a product shows the following figures:

Part	*Main Process*	*Weight of Used Materials* (lb.)	*Weight of Finished Part* (lb.)	*Production Costs* ($)
1	Casting and machining	2.20	1.60	8.08
2	Casting and machining	1.50	1.05	6.21
3	Casting and machining	0.75	0.42	2.25
4	Casting and machining	0.60	0.40	2.05
5	Press work	0.12	0.09	0.32
6	Press work	0.08	0.07	0.30
7	Machining	0.50	0.38	2.00
8	Machining	0.25	0.18	0.60
9	Machining	0.20	0.18	0.31
10	Standard parts, bought out			0.48
	Total			22.60

(i) Find the product material utilization figure (for bought-out parts assume 100 per cent utilization)

(ii) Select the three parts that should be studied for material utilization and assume that it can be increased by 10 per cent. If the production costs per pound of raw material remains unchanged, by how much would the material utilization be improved for the product?

(iii) Draw the materials utilization polygon before and after the improvement.

5. Take a product or an assembly of four to six parts and find the materials utilization for the whole product. Point out the part that should be investigated first and suggest a new method to produce it. What effect does your suggestion have on the materials utilization of the whole product?

6. Compare the total production time for manufacturing the washer illustrated in Fig. 8–14. Consult a mechanical engineering handbook and press catalogs and select appropriate cutting and blanking speeds for mild steel and aluminium alloy; assume that in the machining operation, a multiple parting-off tool may be employed (to part off, say, four washers in succession per one traverse movement of the cross-slide). Also compare the materials utilization when methods (ii) and (iii) in Fig. 8—22 are used.

7. Select a machine tool shop that produces up to 20 different parts in one week and find the equivalent materials utilization factor for the shop.

8. Select a part that can be produced by (i) sand casting with subsequent machining; (ii) die casting; (iii) machining from the solid. Make a cost analysis and specify for what production volumes these processes are suitable.

9. Summarize the advantages and disadvantages of the following processes: shell molding, investment casting, casting with plaster of Paris, powder metallurgy, argonaut welding, spinning, barreling, shot peening.

9

THE PRODUCTION ORDER

Production planning activities (as suggested by Fig. 1–1 and Fig. 3–1) finally culminate in the issue of the production order. Treatises on production planning and control usually carry elaborate descriptions of forms and sheets that are supposed to constitute the paper-work system associated with production orders, tedious and complicated though it may be. These descriptions are useful, but forms by themselves do not make a system. They are just an expression of the principles of a system, a tool to help in its implementation. In Chapter 2 we discussed some aspects of the types of production, sizes of plant and types of industry, and evidently these have a serious bearing on the system that is eventually selected or developed as the most appropriate for the occasion. This is why such systems can rarely be copied and transferred from one plant to another, without some modifications to satisfy specific requirements. With this qualification in mind, we may proceed now to examine some of the more common features that make up such systems. The purpose of a production order is:

- To pass information to everyone concerned with the product and its specifications, the required output, the intended date of delivery, and the proposed outlined timetable of activities associated with its production
- To authorize the various departments to take appropriate actions at predetermined times
- To start the control system working
- To provide basic data for evaluation of performance when production is completed (schedule-wise, quality-wise and cost-wise)

The actual release of the production order and the activities that follow are production control functions (see Chapter 15). The procedure leading to the formulation of the production order may be summarized as follows:

1. Obtain all specifications (drawings, materials, quality required, annual quantity required)
2. Outline possible alternative methods of production and make a first estimate as to which is the best one to select (bearing in mind the existing available plant facilities)
3. Determine the sequence of operations (using process charts) and plan each in detail, including the use of production and handling aids

4. Work out the operation times required
5. Summarize these data on an operation sheet for each part
6. Work out a production master program and a production program for each component
7. Prepare a route card or a combined operation and route sheet (in which assignments of work to departments or individual machines are made)
8. Check on the availability of machine time and prepare a production schedule
9. Prepare machine loading charts to show how the work will be distributed
10. Prepare the job card, materials requisition card, tools and drawing requisition cards, inspection cards, and all the remainder of the paraphernalia that make up the production order

Let us now examine some of the tools that are often employed in this planning procedure.

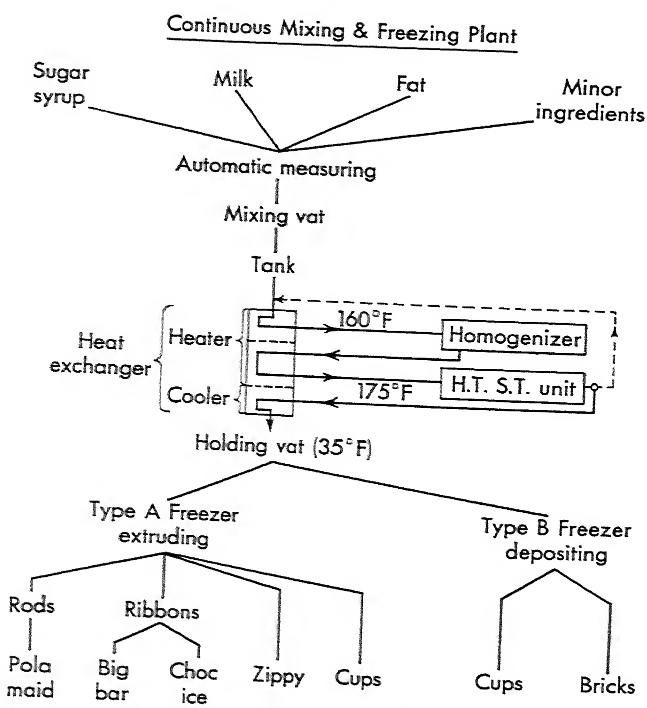


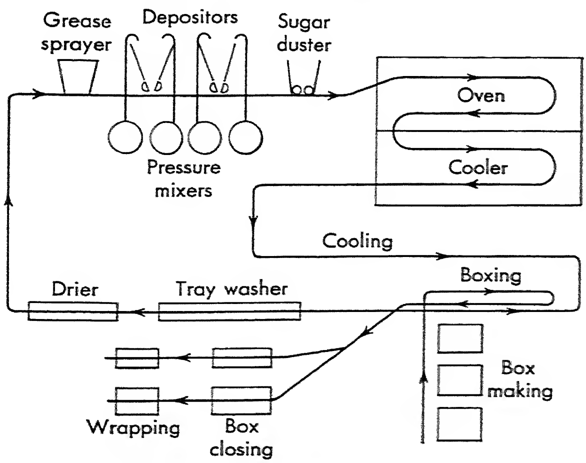
Figure 9–1. ***Diagram of an ice-cream process. A schematic process outline, showing how different products are made from the same ingredients.*** (From "Quality Control in the Food Industry", by J. H. Bushill, J. Institution of Production Engineers, 1959)

Schematic Process Outlines, Process and Activity Charts

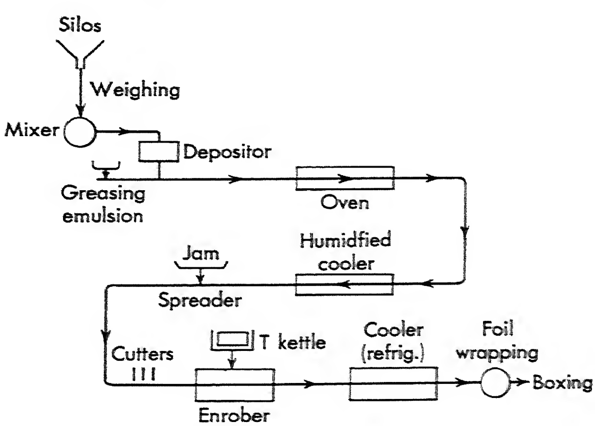
The purpose of all these charts is to represent in a graphical form the sequence of operations and allied activities that take place, in order to provide a useful basis for detailed production planning or for analysis and revision of methods employed hitherto. Let us briefly describe three types of charts commonly used.

Schematic process outlines

There are two types of process outlines. The first shows how different products are made from the same ingredients, indicating the conditions required at the



(a) Diagram of Sponge Cake Process



(b) Diagram of Choc Roll Process

Figure 9–2. ***Two examples of schematic process outlines.*** (From "Quality Control in the Food Industry", by J. H. Bushill, J. Institution of Production Engineers, 1959)

intermediate processing stages. Figure 9–1 is an example of this type of process outline for the manufacture of various ice-cream products. The second type shows the processing sequence, with special reference to flow from one machine or piece of equipment to another. These diagrams are very useful in preliminary layout analyses in processing industries. Two examples are shown in Fig. 9–2, giving schematic process outlines for the manufacture of two popular brands of cake.

Process charts

ASME Standard No. 101 distinguishes between two types of process charts:

1. *Operation process charts*, which constitute a general outline showing the principal operations and inspections, as well as the points at which materials enter the process.
2. *Flow process charts*, which present a more detailed picture, describing the activities associated with materials, men, or machines, and which record the sequence of operations, transportations, inspections, delays, and storages that occur.

Even more detailed process charts have been designed, such as two-handed process charts (to compare the activity of two hands of an operator) or simo charts (to describe the motions of an operator's hands, fingers, or legs, if they are relevant to his performance). These charts, which were originally designed for time study, may also be used as instruction supplements to job cards, while the operation and flow process charts may be considered necessary tools at the very first stages of production planning.

The symbols adopted by ASME, which are now in common use in process charts, are as follows:

○ *Operation*, to describe an action that transforms the material in shape or in other properties (physical or chemical), including assembly work or activities connected with preparing the material for a subsequent operation or for transportation.

⇨ *Transportation*, to describe the movement of an object from one location to another.

□ *Inspection*, to describe an activity which is concerned with checking whether the object conforms with predetermined specifications.

D *Delay*, to indicate that an object is held up and cannot immediately proceed to the next operation.

△ *Storage*, to indicate that an object is deliberately kept at a certain place.

⧈ *Combined activity*, to show that several actions are carried out simultaneously. A square and a circle signify that an operation and an inspection are combined.

Figures 9–3 and 9–4 show examples of process charts. Figure 9–3 gives an outline for the manufacture and assembly of a rocker for a valve mechanism. In addition to the sequence of operations specified for each part, Fig. 9–3 shows at

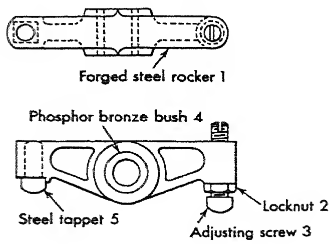
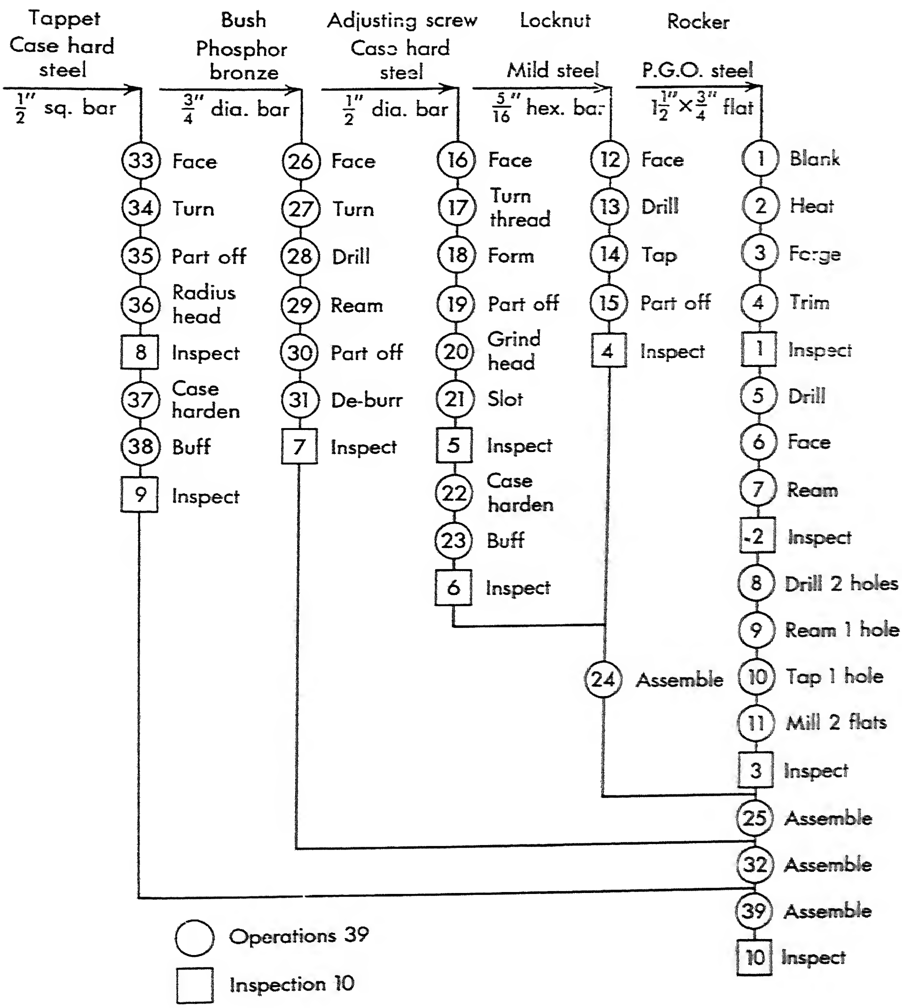


Figure 9–3. ***An example for an outline process chart for the production of a rocker arm.*** (From "Work Study", British Institute of Management, 1955)



SUBJECT CHARTED *RELIEF VALVE BODY* CHART NO. *1021*
DRAWING NO. *A520612* PART NO. *16150* CHART TYPE
CHART BEGINS *Barstock Storage* CHARTED BY *J. Smith*
CHART ENDS *Assembly Department Storeroom* DATE *9-9-43*
SHEET NO. *1* OF *1* SHEETS
COST UNIT *1 Valve Body*

○ OPERATION ⇨ TRANSPORTATION
□ INSPECTION D DELAY ▽ STORAGE

DIST. IN FEET	UNIT TIME IN *Hours*	CHART SYMBOLS	PROCESS DESCRIPTION OF *Proposed* METHOD
		▽ 1	*Stored in bar stock storage until requisioned*
10	.0002	○ 1	*Bars loaded on truck upon receipt of requisition from machine shop (2 men)*
210	.0002	⇨ 1	*Moved to #301 machine*
10	.0002	○ 2	*Bars unloaded to bar stock rack near #301 machine*
	4.000	D 1	*Delayed waiting for operation to begin*
8	.0550	○ 3	*Drill, bore, tap, seat, file, and cut off*
	2.000	D 2	*Delayed awaiting drill press operator*
20	.00002	⇨ 2	*Moved to drill press by operator*
8	.0350	○ 4	*Drill 8 holes*
	2.000	D 3	*Delayed awaiting moveman*
300	.0011	⇨ 3	*Moved to burring department*
	1.500	D 4	*Delayed awaiting burring operation*
6	.0100	○ 5	*Burr*
	2.000	D 5	*Delayed awaiting moveman*
550	.0005	⇨ 4	*Moved to seat lapping machine in detail department*
	6.000	D 6	*Delayed awaiting operator*
6	.1700	O-6 INS-1	*Lap, seat, test, and inspect*
	2.000	D 7	*Delayed awaiting moveman*
400	.0004	⇨ 5	*Moved to paint booth*
	6.000	D 8	*Delayed awaiting painter*
15	.0380	○ 7	*Mask, prime, paint, dry, unmask, and pack in box*
425		⇨ 6	*Sent by conveyor to assembly department storeroom*
	60.00	▽ 2	*Stored until requisitioned*

Figure 9-4. ***An example for a flow process chart.*** (Extracted from "Operation and Flow Process Charts", ASME Standard 101, with permission of the publisher, The American Society of Mechanical Engineers)

what stage these parts are assembled. No time or location is indicated in this example, but this information can naturally be added whenever necessary. Figure 9–4 is an example for a flow process chart that includes details about distances involved in the transportation of the object and times of operations. A similar method of presentation could use the chart in Fig. 9–5.

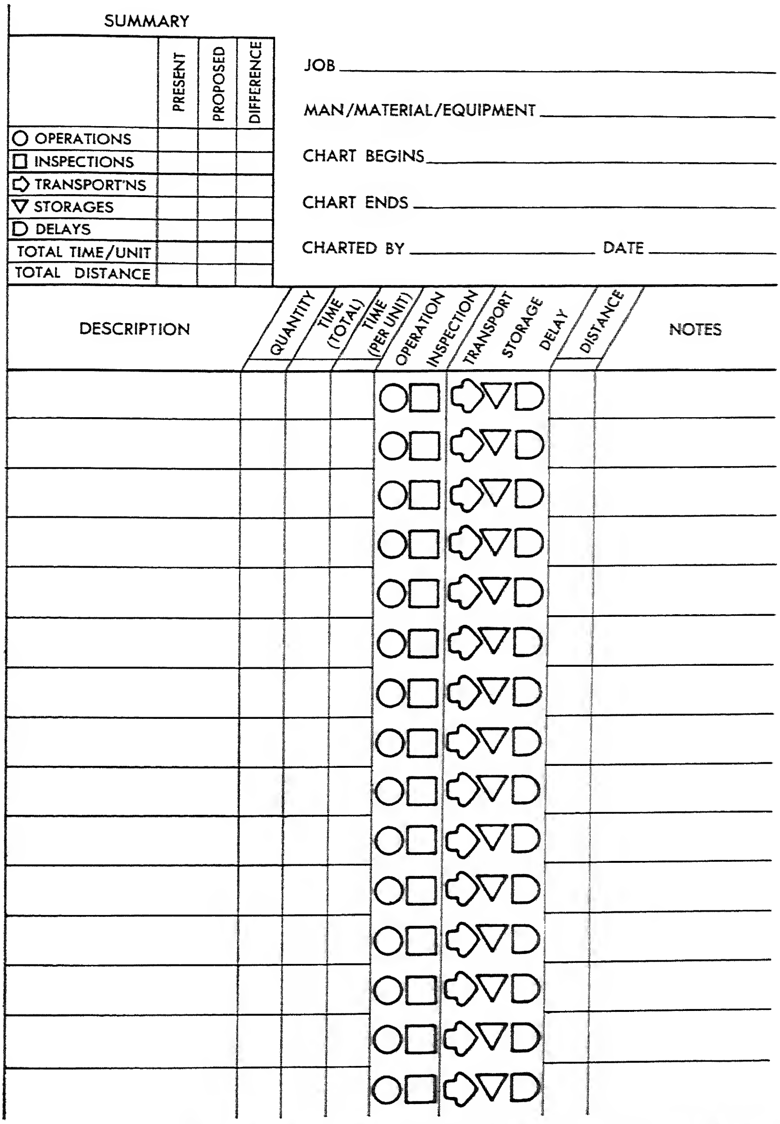


Figure 9–5. ***A flow process chart.*** (From "Work Study", British Institute of Management, 1955)

Activity charts

An activity chart is a graphical presentation of a whole or a portion of a work cycle, which shows the relative periods of activity and idle times of men and machines. It is particularly useful when we want to study the breakdown of a work cycle with the view of shortening or simplifying it. The activity chart indicates the interdependency and sequence of tasks, as well as those that require the simultaneous activities of several men or machines. Take, for example, the work cycle of an operator supervising two machines, the activities being as follows:

1. Operator loads and starts machine 1.
2. Machine 1 performs its task, at the end of which it stops automatically.
3. Operator, in the meantime, unloads the finished product from machine 2.
4. Operator inspects the product.
5. Operator loads and starts machine 2.
6. Machine 2 performs task and stops automatically when task is finished.
7. Operator turns to machine 1 and unloads the product.
8. Operator inspects the product.
9. Operator loads and starts machine 1; etc.

Figure 9–6 shows how these tasks can be presented by a multiactivity chart on a time basis. The chart indicates that while the operator is fully occupied

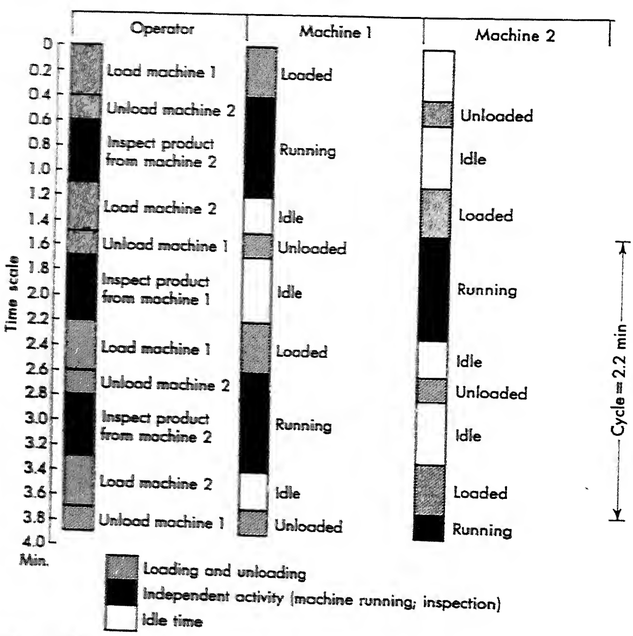


Figure 9–6. *A multi-activity chart (one operator supervising two machines).*

looking after the machines and inspecting the work, each machine is idle on two occasions during the cycle: when the other machine is being loaded and when the product is inspected by the operator. The active time of each machine is 0.4 (loading) + 0.8 (running) + 0.2 (unloading) = 1.4 minutes, so that the efficiency of the machine cycle is

$$\eta = \frac{\text{active time}}{\text{cycle time}} = \frac{1.4}{2.2} = 0.64$$

Having worked out through activity charts the active and idle times in relation to the cycle time and the sequence in which these activities occur, we can seek alternatives that would make the cycle more efficient; for instance, by studying the sequence of operations, reducing the cycle time, adjusting the number of machines an operator should supervise, etc. Some of these problems are further discussed in Chapter 12.

When outlining the proposed sequence of operations some basic principles must be borne in mind, namely:

Saving in effort
Saving in skill
Saving in handling and setting time
Saving in operation time

Hence it is useful to subject the proposed sequence of operations to the following cross-examination:

1. Is the operation really necessary? What does it contribute to the product value?
2. Can two operations be combined or carried out concurrently?
3. Should operations be divided into simpler elements for the purpose of work standardization (using similar machines and skills, identical instruction sheets, and facilitating more accurate estimating)?
4. Can the sequence of operations be changed to advantage?
5. Is it worth while to install mechanical aids for loading, setting, and unloading of the work?
6. Is it economical to install mechanical handling devices to transfer the work from one operation to the other?
7. Should inspection be carried out by the operator? Could inspection and handling between operations be combined (by incorporating an inspection mechanism into the handling system, thereby letting through to the next operation only work that is up to standard)?
8. Are the operator, the machine, and the tools the right ones for the job?

Production Master Programs

Operation process charts with the final sequence of operations form a useful basis for a production master program. Figure 9–7 shows an example of such a program, involving the manufacture of a product made of eight parts. After the parts have been produced, some are put together into sub-assemblies, which have to undergo various operations prior to the final assembly. If the date on which the product is to be finished is denoted by zero, the scheduling for the latest

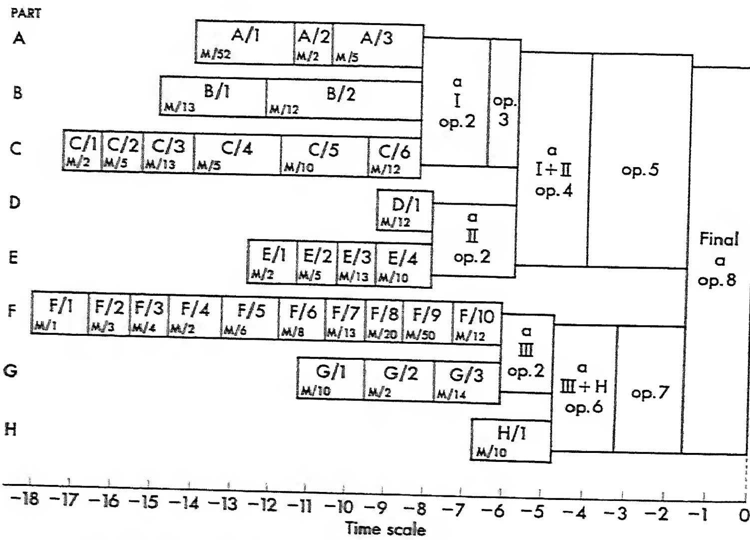


Figure 9–7. *A* ***production master program.*** (A schematic example illustrating the effect of sequence of operations on scheduling)

A product is made of 8 parts: A, B, C, D, E, F, G, H.

Sequence of operations:

1. All parts are produced.
2. Parts are grouped in the following sub-assemblies:
   - I Parts A+B+C
   - II Parts D+E
   - III Parts F+G
3. Sub-assembly I is subject to an operation before further assembly work.
4. I and II are assembled to form major-assembly (I+II).
5. Major-assembly (I+II) is subject to an operation (e.g. painting by a spray-gun) after which it is ready for final assembly.
6. In the meantime, III and H are assembled to form major-assembly (III+H).
7. Major-assembly (III+H) is subject to an operation, after which it is ready for final assembly.
8. Final assembly takes place.

Notation:

A/1, A/2, etc.	Operations required to produce part A.
M/52	Machine on which operation A/1 is made.
a	Refers to an assembly operation.
op	Refers to one of the operations listed in the above sequence.

date on which production of the parts has to start is shown by the tails of the individual components on the chart. Work on part A will have to begin about 14 time units (days, weeks, etc.) before "zero day"; part B, about 14.8 time units, etc. The longest total operation time is connected with part F, which will have to start at −18 time units.

Required data

This master program includes information about the type of equipment or machine required to perform the operation in question; for example:

Part A: operation 1 will require machine M/52.
operation 2 will require machine M/2.
operation 3 will require machine M/5, etc.

This information is important because at a glance we can verify whether there is no overlap of time for the same machine. We see in Fig. 9–7, for instance, that operations $C/3$ and $B/1$ are to be performed on M/13, so that unless we have more than one machine of this type, we have to shift forward in time operation $B/1$ or $C/3$ in order to avoid any clash in timing. Similarly, operations $B/2$ and $C/6$ overlap in time. Thus, although at first glance part F requires the longest time, we may find that either parts B or C have to be produced first. One possible sequence is shown in Fig. 9–8, but obviously the number of possible

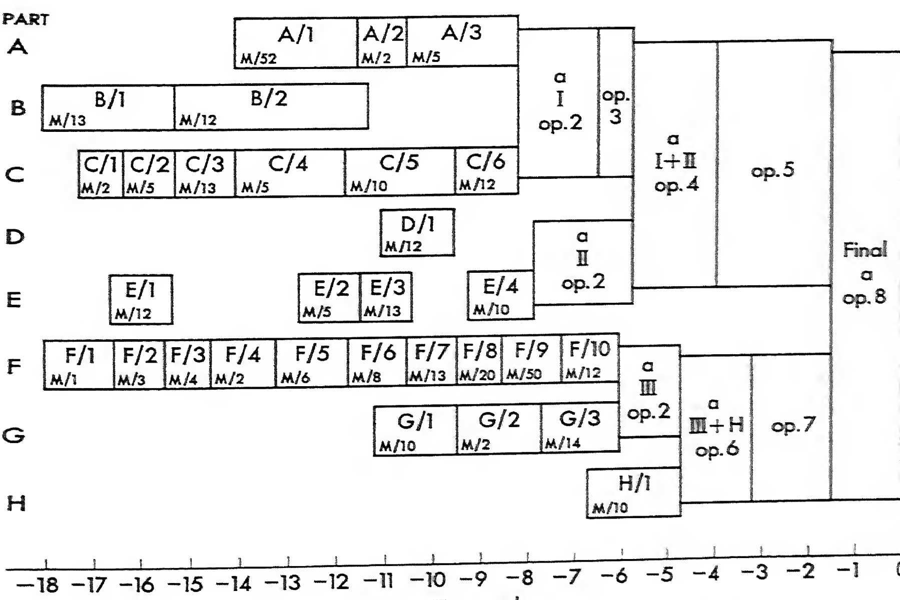


Figure 9–8. ***A feasible solution for a production master program.*** (See Figure 9–7)

alternatives increases very rapidly with the number of overlaps, and they have to be carefully studied to ensure that the one selected yields the shortest total time span between start and finish. Figure 9–8 is a feasible solution for this particular product, but naturally the final schedule can be set up only after individual machine commitments for other products are also accounted for.

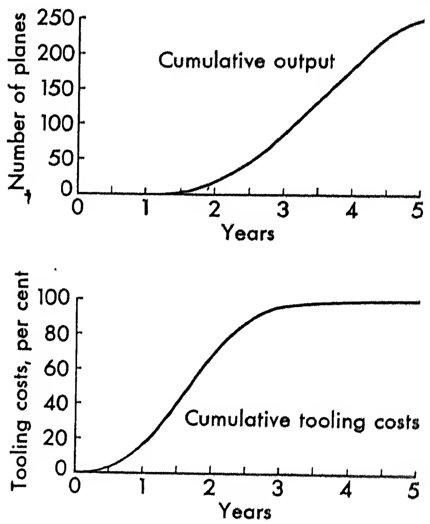


Figure 9–9. ***Phasing tooling and production of airplanes.***

Long-term planning

When producing batches of large units (houses, ships, large generators, planes, etc.), which are spread over a period of several months or years, and when special tooling is required, it is important that tooling expenditure should lead the production program by a suitable phasing period.

Figure 9–9 is an example for a production program of 250 airplanes, to be undertaken in five years. A preliminary period of about 18 months is allowed for preparation for production, after which the first plane is scheduled to come out of the plant. The rate of output is then increased gradually, reaching its maximum value in the fourth year. The tooling program, however, starts almost immediately, so that by the time the plant is expected to yield maximum capacity, most of the tooling outlay has already been spent, leaving only a very low margin for replacement of minor items during the last two years of the program. Each of the units scheduled for production is a big enough undertaking to require detailed planning of production progress, and a general outline can be indicated on the production program as, for example, in Figs. 9–10 and 9–11.

The limiting factor in planning may be governed (among other things) by the number of berths in a shipyard or in a plane construction plant, the number of rail lines in a locomotive repair shop, the number of cranes in a loading or

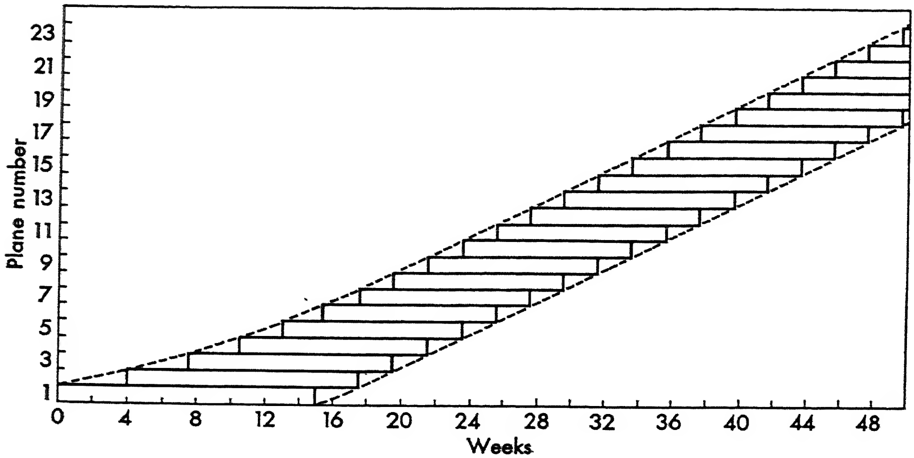


Figure 9–10. ***Phasing in an airplane production schedule.***

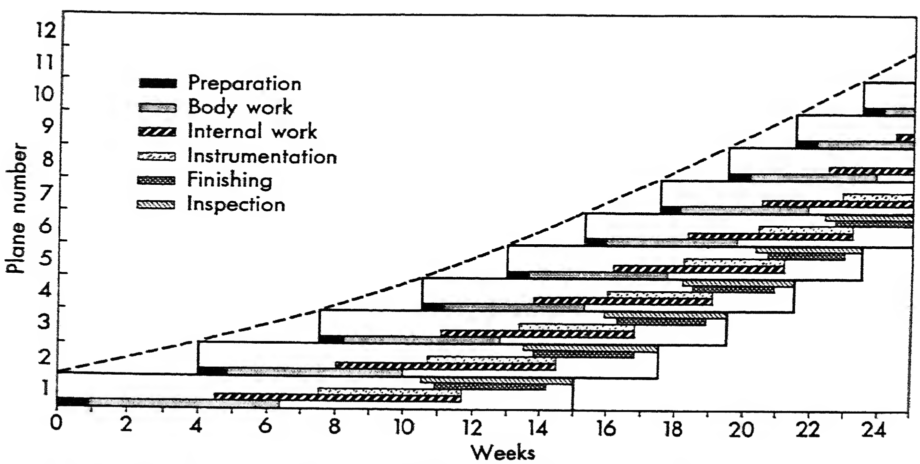


Figure 9–11. ***A 25-week production program for airplanes*** (See Figure 9–10)

unloading operation, etc. Figures 9–10 and 9–11 were drawn out on the assumption that no more than five planes could be constructed concurrently. As this number is the limiting factor for production output, no time should be lost between the completion of one plane and the beginning of production of the next one on the line. To allow specialized teams to proceed from one plane to another, construction does not start on all planes on the same date, but work is carefully phased to allow for the capacities of the machines and the teams to be utilized to the maximum. This problem will be discussed further in Chapter 12.

Programing component production

Following the completion of the master program, a production program for each component can be outlined (Fig. 9–12), with copies sent to the appropriate departments so that everybody concerned will be duly forewarned about the schedule. The program must, naturally, allow for transportation time within the plant, inspection time and delays, particularly when the components are produced for the first time and various production difficulties may have to be overcome (notice, for example, that the production time for the first plane in Fig. 9–10 is longest). Very often, however, production planning departments tend to be overcautious and allow too large a safety margin, with the result that the shop may be choking with components in process, waiting for issue to the assembly line. Frequent studies of the causes for delays and deviations from the outlined program—and subsequent removal of such causes (whenever possible)—help to plan a balanced schedule with properly phased operations.

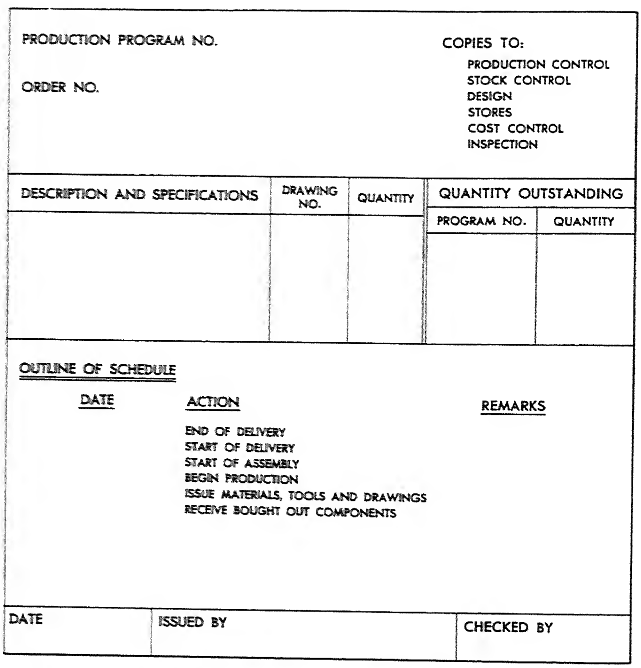


Figure 9–12. *A component production program.*

Operation and Route Sheets

The final sequence of operations is set out in an operation sheet, which should include the following details: part number and name (and if possible a sketch as well), materials specifications, description of the operations and the type of machines they should be carried on, tools to be used, and time allowed for setting

and performing the operations. The operation card, therefore, constitutes a general statement about how, in the view of the production planning department, the production sequence should be carried out. It indicates the type of machines that could be used, but it does not normally specify precisely to which machine in the plant the work should be allocated. In other words, the operation sheet is one step prior to machine loading. When allocation of work is carried out, a route sheet should be prepared, detailing the machines and departments to which the work will flow and again listing the sequence of operations, each operation described in conjunction with the corresponding machine. The operation sheet is filed in the records of the planning department and can be used again when the order is repeated. The route sheet, on the other hand, may have to be revised if some of the machines have been committed to other jobs.

This subtle difference between an operation and a route sheet (apart from the routing details, they contain practically the same information) often leads production planning departments to adopt a combined sheet, as shown in Fig. 9–13. When the form is to be used for the records as an operation sheet, the routing columns just indicate the type of machines that could be used for the purpose, and no details of "quantity" and "to be finished on" need be specified. Copies of this form are used as route cards.

OPERATION AND ROUTE SHEET		NO.
PART NO.	DRAWING	
NAME		
MATERIAL SPEC.	QUANTITY	TO BE FINISHED ON

ROUTING		OP. NO.	DESCRIPTION	TOOLS	JIGS, FIXTURES	TIME (MIN.)		
DEPT	M/C					SET-UP	OP. TIME	TOTAL

Figure 9–13. ***A combined operation and route sheet.***

The operation sheet is very useful for re-evaluation of the decisions taken so far. It includes operation times so that a cost estimate can be made and the economic production quantities can be computed; these two figures, the cost per unit and the quantity, can then be analyzed and compared against other alternatives that may be open (e.g., using an entirely different process or sequence, or buying the components from the outside).

29 4 57	29 5 57	1000	500	2345	REMARKS
DATE OF ISSUE	DATE REQUIRED	TOTAL QTY	BATCH QTY	WORKS ORDER NO.	DATE STARTED / DATE FINISHED
PART NAME Center Wheel		DRAWING NO. A.3737	PART NO. AW 3737/24		LABOUR SCRAP
MATERIAL SPECIFICATION Brass strip 14 G x 1½		UNIT QUANTITY 68 feet per 500		CHECKED BY	TOTAL SCRAP / TOTAL GOOD

CHECK NO.	OP. NO.	OPERATION DETAILS	M/C No. or GROUP	JIG TOOL & GAUGE Nos.	SETTING TIME	OP. TIME EACH HRS.	OP. TIME EACH MINS.	SCRAP MAT'L	SCRAP MAN'S	No. for RECT'N	No. GOOD	VIEW DATE
	1	Rough Grind	73/80	J 45			.3					
	2	Cut		42			.6					
	3	Keyway		41			.2					
	4	Bore		22			.3					
	5	Drill 4 holes 2 BA		43			.6					
	6	Finish Grind		23			.2					
	7	Inspection										

Figure 9–14. ***A master card for production order breakdown.*** (Courtesy Bulmer's Calculators Ltd.)

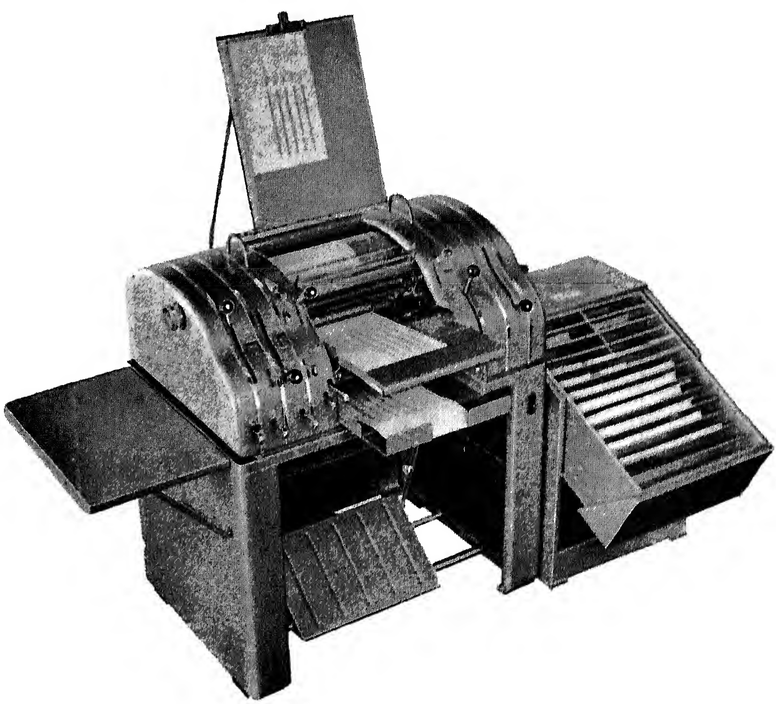


Figure 9–15. ***The orlid super universal for preparation of production planning documents.*** (Courtesy Bulmer's Calculators Ltd.)

Breakdown of the Production Order

The operation and route sheet can serve as a production order, from which detailed job cards for each operation have to be prepared. Other ancilliary documents include: a materials requisition form, an inspection card, an identification card (which is attached to the work throughout its travel in the plant), a delivery note of the finished product to the store, etc. One method for preparing all these documents is shown in Figs. 9–14, 9–15, and 9–16. First a master card is made to include all the details on the operation and route sheet, and to allow for further details on the current production run (date of issue, date required, quantity, etc.) and for information to be inserted after production (mainly on inspection results). By using a reproduction process, such as offered by the

29 4 57	29 5 57	1000	500	2345		
DATE OF ISSUE	DATE REQUIRED	TOTAL QTY.	BATCH QTY.	WORKS ORDER NO.		
PART NAME	Center Wheel	DRAWING NO. A.3737		PART NO. AW 3737/24		
MATERIAL SPECIFICATION	Brass strip 14 G x $1\frac{1}{2}$	UNIT QUANTITY 68 feet per 500			CHECKED BY	
OP. NO.	OPERATION DETAILS	M/C No. or GROUP	JIG TOOL & GAUGE Nos.	SETTING TIME	OP. TIME EACH HRS.	MINS.
1	Rough Grind	73/80	J 45			.3

29 4 57	29 5 57	1000	500	2345		
DATE OF ISSUE	DATE REQUIRED	TOTAL QTY.	BATCH QTY.	WORKS ORDER NO.		
PART NAME	Center Wheel	DRAWING NO. A.3737		PART NO. AW 3737/24		
MATERIAL SPECIFICATION	Brass strip 14 G x $1\frac{1}{2}$	UNIT QUANTITY 68 feet per 500			CHECKED BY	
OP. NO.	OPERATION DETAILS	M/C No. or GROUP	JIG TOOL & GAUGE Nos.	SETTING TIME	OP. TIME EACH HRS.	MINS.
2	Cut		42			.6

29 4 57	29 5 57	1000	500	2345		
DATE OF ISSUE	DATE REQUIRED	TOTAL QTY.	BATCH QTY.	WORKS ORDER NO.		
PART NAME	Center Wheel	DRAWING NO. A.3737		PART NO. AW 3737/24		
MATERIAL SPECIFICATION	Brass strip 14 G x $1\frac{1}{2}$	UNIT QUANTITY 68 feet per 500			CHECKED BY	
OP. NO.	OPERATION DETAILS	M/C No. or GROUP	JIG TOOL & GAUGE Nos.	SETTING TIME	OP. TIME EACH HRS.	MINS.
3	Keyway		41			.2

29 4 57	29 5 57		1000		500	2345				
DATE OF ISSUE	DATE REQUIRED		TOTAL QTY.		BATCH QTY.	WORKS ORDER NO.				
PART NAME	Center Wheel				DRAWING NO. A.3737		PART NO. AW. 3737/24			
MATERIAL SPECIFICATION	Brass strip 14 G x $1\frac{1}{2}$				UNIT QUANTITY 68 feet per 500			CHECKED BY		
OP. NO.	OPERATION DETAILS				M/C No. or GROUP	JIG TOOL & GAUGE Nos.	SETTING TIME	OP. TIME EACH HRS.		MINS.
4 5 6	Bore Drill 4 Holes 2 BA Finish Grind **JOB CARD**					22 43 23				.3 .6 .2
CHECK NO.	NAME	DATE	QTY FIN'D	MTL.SCRAP	MAN'S SCRAP	GOOD	VIEWER	£	s.	d.

Figure 9–16. ***Breakdown into job cards.*** (Courtesy Bulmer's Calculators Ltd.)

machine in Fig. 9–15, the list on the master card can be broken down so that the heading plus any lines singled out from the list are reproduced on separate cards. The breakdown into job cards is shown in Fig. 9–16, and similarly a materials requisition and other documents can be made out. The machine in Fig. 9–15 is a spirit duplicator; another method uses a printing plate for the heading of all documents, the plate being similar to those used in addressing machines.

Stock Planning

Although the actual issue of materials to the shop floor is performed when the materials requisition orders are released through the dispatching function, the availability of required materials in stock must be ensured some time in advance, to allow for delivery lead times in case some materials have to be purchased. The functions of stock planning would, therefore, be:

On receipt of lists of materials and component requirements, the quantities held in stock have to be checked and the appropriate amounts allocated for issue.

The rate and pattern of demand for materials from the store should be studied so that the optimal policy of replenishment can be worked out.

Orders for replenishment are issued to the purchasing department.

When goods are received, they have to be checked for
 (i) quantity (to allow for claims to be made in cases of deficiencies);
 (ii) quality (to ensure that specifications are complied with).

Reports on receipt of consignments are made to enable the purchasing department to reassess the efficiency and reliability of the vendors and to allow the accounting department to proceed with any appropriate action that may be required.

The consignment has to be duly recorded so that up-to-date information about stock levels is always available.

A suitable classification system for materials and components held in stock should be in operation and reviewed from time to time to ensure that artificial variety, caused by having items under different names, is avoided, and to provide a basis for a further simplification program (see, for example, Fig. 9–17).

In short, stock planning is associated with two main responsibilities: that of recording events and that of formulating the best replenishment policy. The latter is closely concerned with how much to order, when to order, and what limits of stock levels seem desirable for effective management of the stores. These problems will be further discussed in Chapters 10 and 11, where optimal batch quantities for production are analyzed, and in Chapters 17 and 18, where some specific problems in stock control are dealt with.

The necessity for keeping stock records and accounting for demand requirements is self-evident. Without data, no analysis can be carried out and no policy (let alone an optimal policy) can be defined. The question is, however: How

1. The same material Polyvinyl Chloride has been found to be commercially available under the following proprietary names:

BX PVC	Everflex	Portex Vinyl
Capovin	Flexatex	Rilene
Cap-Plastube	Koroseal	Tenaduct
Chlorovene	Kenutuf	Tygan
Clorcom	Li-Lolastic	Vinatex
Corvic	Lorival	Vinylite
Craylene	Marvinol	Vynan
Duraplex	Mipolam	Welvic
Erinoid	Periflex	

2. The same type of component was separately designed in one firm, produced and stored as:

Arbor	Plug
Axle	Plunger
Bar	Post
Bolt	Rivet
Boss	Rod
Button	Roller
Column	Shaft
Cotter	Shank
Dowel	Spigot
Mandrel	Spindle
Peg	Stud
Pillar	Tappet
Pin	Trunnion
Piston	Valve
Pivot	

It was also found under composite names, such as:

Pin anchor	Pin link
Pin crank	Pin linge
Pin push lever	
Pin clamp plate	
Clutch toggle pin	
Governor drive coupling pin	
Distributor drive plain pin	

3. The same pipes and pipe fittings were stored in three different locations according to their use:

(*a*) Factory Maintenance Stock:
Piping for gas, water, compressed air

(*b*) Production Stock:
Parts used in construction of products

(*c*) Plant Spares:
Parts serving machine tools

Figure 9–17. ***Three examples of artificial variety*** (from "Maximum et Minimo" by E. G. Brifsh, J. Institution of Production Engineers, June, 1954).

much data should we collect? If we record information that is never used for any evaluation or planning purpose, the system may become unnecessarily cumbersome. Sometimes too little information is recorded. Figure 9–18, for instance, depicts a very conventional stock or bin card, which shows the amounts received, amounts issued, and the balance on hand, but gives no information about quantities allocated for issue or about quantities on order. Needless to say, when lag times between stock allocation and stock issue or between order and

physical replenishment are comparatively long, the lack of such information may put the stores management at a serious disadvantage. A modified stock record card, such as in Fig. 9–19, would include special columns to indicate the amounts allocated and amounts expected to come in, as well as balance on hand and the "effective balance" (= actual stock level + orders − allocations).

BIN NO.................................... CODE OR SPEC. NO...................

CARD NO....................................

DESCRIPTION ....

DATE	REFERENCE	IN	OUT	BALANCE
	BROUGHT FORWARD			
TOTALS	BALANCE CARRIED FWD			

Figure 9–18. ***A conventional bin card.***

BIN NO. SPEC. NO.

CARD NO.

DESCRIPTION

				+		−		+	=
DATE	REFERENCE	IN	OUT	ACTUAL BALANCE	ALLO-CATED	TOTAL ALLO-CATED	OR-DERED	TOTAL ON ORDER	EFFECTIVE BALANCE
	BROUGHT FORWARD			2000		–		1400	3400
16.10	P.O. 620			2000	800	800		1400	2600
17.10	P.O. 621			2000	400	1200		1400	2200
19.10	PUR 92	1200		3200		1200		200	2200
20.10	P.O. 620		800	2400		400		200	2200
1.11	P.O. 623		1200	1200		400		200	1000
5.11	PUR 101	200		1400		400		–	1000
TOTALS									

Figure 9–19. ***A detailed stock card.***

Basic Production Planning Problems

Having worked out the sequence of operations, and having obtained information about orders and demand for the product, we now possess sufficient data on which production scheduling can be based. On the one hand there are the rates of demand that have to be satisfied, the operations that have to be performed, the kind of accuracy and skill they require, and the time it will take to carry them out. On the other hand we have the facilities, namely: the machines and equipment, the personnel and the time available for production. The final and crucial task of production planning is to match these two in such a manner that the objective is achieved with maximum utilization of these facilities. Basically, therefore, there are three kinds of production planning problems:

1. *Problems of quantity:* how to determine the quantities that should be produced of each product or component during a given period of time
2. *Problems of allocation:* how to allocate the work to the machines and how to assign the manpower to a given array of tasks
3. *Problems of scheduling:* how to arrange the sequence of operations on the machines, so that the sequence for any individual component or product is not interfered with

All these problems are, naturally, interconnected. Allocation of machine time depends on machine capacity and on its other commitments, while an optimal schedule can be constructed only after these allocations have been made. Furthermore, the system is often dynamic, with batches of operations arriving from time to time to be allocated and scheduled, so that not only does the schedule change continuously, but an existing schedule and prior commitments may have to be reviewed each time a new batch of operations arrives. Delays in supplies and machine breakdowns, changes in forecasts on market demand, and trends in future sales, all these may also interfere with the schedule to an extent where a review becomes necessary. Problems associated with determination of manufacturing quantities will be analyzed in Chapter 10, some aspects of allocation and scheduling will be discussed in Chapters 12 and 13, and interrelations between quantities in batch production and scheduling will be considered in Chapter 14.

Summary

The main purpose of the production order and the paper work associated with it is to convey the necessary information to everybody concerned and to provide a basis on which a control system can effectively be supported. Tools frequently used in production planning include: process and activity charts (to analyze the outline of manufacture and the sequence of operations), production master programs (to give a general picture of the production timetable), operation and route sheets (to record detailed data on operations, methods, and times), and the various parts of the production order: job cards, materials requisition cards, etc.

References

Barnes, R. M.: *Motion and Time Study*, Chapters 4–7 (John Wiley & Sons, Inc., 1958). ASME Standard No. 101.

Carson, G. B. (ed.): *Production Handbook*, sections 2, 3 (The Ronald Press Co., 1957).

Maynard, H. B. (ed.): *Industrial Engineering Handbook*, section 2, chapter 3 (McGraw-Hill Book Co., 1956).

Problems

1. Construct an operation chart for a selected product and discuss alternatives to your suggested sequence of operations.
2. Give an example for each question in the cross-examination for any proposed sequence of operations (page 211) to show how production methods could be improved.
3. Visit a plant with the purpose of:
   (i) Studying a specific product and drawing a master production program for it
   (ii) Outlining the flow of documents in production control
   (iii) Discussing which documents are absolutely essential to satisfy the basic requirements of a production order
4. For the example given in Fig. 9–4, draw an activity chart for:
   (i) One operator and one machine
   (ii) One operator supervising two machines but relieved of the task of inspection
   (iii) One operator supervising three machines but relieved of the task of inspection

   Of the three alternatives, which yields the highest machine efficiency?
5. A gang of four operators are engaged in producing a product which consists of four parts (A, B, C and D). The work is divided between the men as follows:
   (i) Operator 1 brings a batch of 30 pieces of each part (this number is determined by the size of the cart; time 0.5 min.). Then operators 2, 3, 4 take parts *A*, *B*, *C* and prepare them on their work benches (time for 30 pieces of part *A* is 1.0 min.; same time applies to parts *B*, *C*). Operator 1 works on pieces of part *D* (0.5 min.). When all the parts have been finished, operator 4 assembles them (time, 2.25 min. for 30 assemblies) while operators 1, 2, 3 prepare packing boxes (time, 0.4 min. for each operator to prepare ten boxes). When the assembly is finished, each one of operators 2, 3, 4 takes ten assemblies to a machine for a final operation (assume no time is needed for this handling operation).
   (ii) Operator 4 works on the machine with the help of operator 1 (time for ten assemblies, 0.25 min.) while the other two wait. Then operator 3 works on the machine with the help of operator 1, while operator 4 packs his finished assemblies in the boxes (time, 0.5 min.), and finally operator 2 operates the machine with the help of operator 1 and then packs his ten assemblies. When all assemblies are packed, operator 1 takes them away (time, 0.5 min.) and brings fresh parts.

Your task:

(i) Draw a gang activity chart of the existing process.
(ii) Discuss this process and suggest possible alterations.
(iii) Draw an improved recommended process.

(Assume all operators to be equal in skill and effort.)

(iv) Calculate in percentage the efficiency of utilization of the machine in the initial and recommended processes.
(v) If maximum utilization of the machine is required, what changes would you introduce? Draw the gang chart for this case.

6. An operator uses a fixture to machine a component on a milling machine. The operations involved are:

Load fixture	4.0 min.
Clamp fixture to m/c	2.0 min.
Start machine	0.05 min.
Adjust flow of lubricant	0.05 min.
Machine	8.3 min.
Stop machine	0.05 min.
Clean part	0.2 min.
Unclamp fixture	0.2 min.
Unclamp part	0.2 min.
Inspect	0.2 min.
Pack in box	0.2 min.

(i) Find percentage of utilized time of operator and of machine.
(ii) Rearrange the sequence of operations to achieve a higher percentage of utilization.
(iii) Draw a man and machine chart for present and suggested sequence.

10

QUANTITIES IN BATCH PRODUCTION

Batch production is necessary when the demand for a commodity is limited or when the rate of production is so high, compared with the rate of consumption, that they cannot be geared to each other by utilizing continuous production. The product is therefore manufactured periodically in a quantity that will be sufficient to satisfy the demand for some time until manufacture of this product is resumed. In the interval between two production periods, the plant and equipment can be used for the manufacture of a variety of goods in a similar fashion. A complete production cycle can thus be organized, in which each product in turn is manufactured in quantities corresponding to the total demand for it throughout the cycle time.

Two basic problems immediately present themselves. First, if each product were to be analyzed separately, disregarding any effects other products might have on the schedule, what is the quantity (or batch size) that should be made each time the product is manufactured? Secondly, once the "best" batch sizes for each product have been determined, how can a satisfactory master production schedule be set up to take account of the plant capacity and the effects of these batch sizes on the cycle time? The question of batch sizes will be dealt with in this chapter and that of the effect on scheduling in Chapter 14.

Stock Control

Batch production in its simplest form is shown in Fig. 10–1, where the variations in the level of the stock (Q) are plotted against time (T). The line 0–1 represents the production process, in which the stock is built up at the uniform rate of a_p units per unit time. After the production period T_p, the level of stock reaches a point Q_1, when production stops. Line 1–2 represents the variations in stock due to consumption at the uniform rate of a_c units per unit time, and after the consumption period T_c, no stock is left and production starts again at point 2. The same cycle can now be repeated, unless it is decided to produce a new lot, Q_3, different in size from Q_1, in which case the length of the production and the consumption periods will differ, by the same proportion, from the

corresponding values in the preceding cycle. During the consumption period T_c the equipment and the plant can be utilized for manufacture of other products.

From triangle 0–1–2 in Fig. 10–1 it can be shown that

$$Q_1 = a_p T_p \tag{10–1}$$

and similarly,

$$Q_1 = a_c T_c$$

Hence

$$\frac{T_p}{T_c} = \frac{a_c}{a_p} = \gamma \tag{10–2}$$

The ratio of production to consumption periods is determined by the ratio of consumption to production rates (denoted by γ). This ratio is a measure of the amount of time spent in any production cycle on one product, a high value of γ implying that a high proportion of the available time is spent on the manufacture of the product in question. The highest value of γ is 1, when rates of production

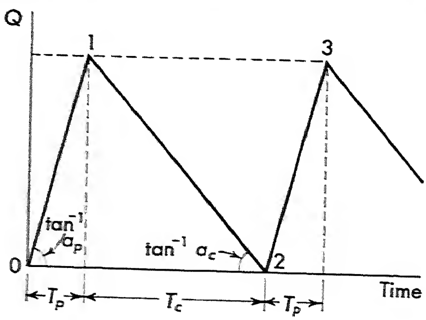


Figure 10–1. ***A simple stock control model.***

and consumption are equal and production ceases to be of the batch type and becomes continuous, the production being geared to the demand for the product. Sometimes when $\gamma = 1$, production cannot keep up with the potential demand and no stock can be built. The market may then rely on other sources of supply and a queue of orders is often formed. The lowest value for γ is 0, which means that no time is required for production and the problem becomes an inventory one. In the latter case provision must be made not for a production period as such but for a delivery period during which the order given by the stores is executed.

The pattern shown in Fig. 10–1 completely segregates the production and consumption periods and is, therefore, oversimplified. First, no account is taken of the fact that consumption also takes place during the production period. Secondly, at point 2 when the stock dwindles to zero, no safety margin is left, and supply from the stores is likely to stop unless production starts promptly at this point. The warning light to start production should be given not when the stock is zero but at such a point that will allow building up of the stock to a predetermined level at the completion of the production period.

When these facts are considered, the change in stock follows the pattern shown in Fig. 10–2. Instead of restarting production at point 2, when the old stock is fully consumed, a new stock level at point 3 is required. Production must start at point 2′ (the distance 2′–2 being the time T_p required for production), allowing for accumulation of stock as denoted by point 3. At point 2′ the stock level is Q_0, which may be regarded as a safety stock (representing the minimum number of pieces) below which the stock should not normally fall. Actual preparations for the production run will have to start at an even earlier stage to allow setup of the machines to be finished at point 2′. If additional delays are normally expected, or if machines are not always available, production orders should be issued at point 2″, accounting for this delay interval, T_d. The stock level at order point is then Q'_0.

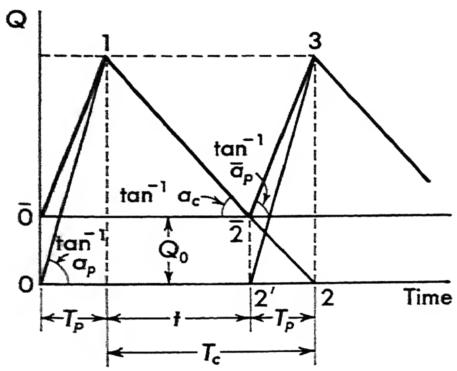


Figure 10–2. ***Stock control with a buffer stock.***

During the production period the stock of Q_0 continues to dwindle at a rate of a_c units per unit time, and a new stock (represented by the line 2′–3) is built up at a rate of a_p units per unit time. The net increase in stock level is given by the difference of these rates, and since both are assumed here to be linear, the net increase is also linear, as shown by the line $\bar{2}$–3. The stock changes are thus represented by the lines 1–$\bar{2}$, $\bar{2}$–3, etc. This pattern is similar to that of Fig. 10–1 except that the horizontal axis is shifted upward by a distance of Q_0 from 0–T. The consumption rate remains as before, while the rate of increase in stock during the production period is given by

$$\bar{a}_p = a_p - a_c$$

The interval time available for utilizing the equipment for other purposes is

$$t = T_c - T_p - t_s$$

t_s being the setting-up time required for preparing the machines before the production period T_p. By using the relation 10–2, this interval becomes

$$t = T_c\left(1 - \frac{T_p}{T_c}\right) - t_s = T_c\,(1 - \gamma) - t_s \tag{10–3}$$

The safety stock Q_0 can be determined from the triangle $\bar{2}$–2′–2 in Fig. 10–2 as

$$Q_0 = T_p a_c$$

or, if this equation is divided by one of the expressions of Eq. 10–1,

$$\frac{Q_0}{Q} = \frac{a_c}{a_p} = \gamma \tag{10–4}$$

As already mentioned, the actual stock level at which the order for production is given is higher than Q_0, and from Fig. 10–2 it can be shown that

$$\frac{Q'_0}{Q_0} = \frac{T_p + T_d}{T_p} = 1 + \frac{T_d}{T_p} \tag{10–5}$$

If the delay period is negligible compared with the production period, the stock at order point will be the same as the safety stock Q_0.

Definition of Batch Sizes

In continuous production the average rates of production and consumption are virtually the same. A buffer stock is built to take care of possible stoppages or breakdowns. If the production rate remains constant, the stock level will vary according to fluctuations in demand. Alternatively, the stock may be kept more or less at a constant level, while production rate is regulated to follow trends or changes in demand, in order to avoid excessive increase of stocks.

In batch production, however, the stock of each product has inevitably to be raised to a predetermined value before the production of another article is undertaken by the plant. If the stock level is kept too low, that is, a small batch is produced, the setup costs per piece and the setup time would be high. From the production point of view, long runs are desirable; however, if too large a batch is produced and stocked, high carrying costs are incurred (in the form of interest charges on the capital invested and in storage, maintenance, and deterioration costs while the goods are in the stores). The main factors affecting the selection of batch sizes may be summarized as follows:

1. Setup costs of machines and other costs of preparation for the run
2. Consumption rate
3. Production rate
4. Interest charges per piece per unit time
5. Average storage costs (including maintenance of products in stores and deterioration)
6. Sales price

The selection of a batch size is evidently a question of determining an optimum value that will yield the best possible results, when the criterion for judging these results has been adequately defined. This problem has been known since the

early 1920's and numerous attempts at a solution are recorded in literature. Most of the suggested solutions were concerned with finding the batch size that results in minimum total production costs per piece, the assumption being that minimum production costs inevitably result in maximum profit and yield the best performance. Under circumstances of severe competition, the sales price per unit may have to be reduced to such an extent that production at minimum costs, leaving only marginal profits, is an inevitable policy. But when manufacturers are not forced to produce at minimum costs, other factors must be considered so as to compute the optimal batch size for the prevailing circumstances.

How is an optimal batch size to be defined? What is the criterion by which its effectiveness can be measured and what are the aims that should be set to guide the policy of selecting batch sizes? Let us examine the following four criteria:

1. Minimum costs per piece
2. Maximum profit for the batch
3. Maximum ratio of profit to cost of production (to be called hence maximum return)
4. Maximum rate of return per unit time

On the face of it, the desirability of achieving each one of these goals would seem self-evident; but are they compatible with each other, i.e., can all these goals be attained at one and the same time? If not, how should prevailing circumstances affect the selection of policy?

Minimum-cost Batch Size

The production costs per piece consist mainly of four factors:

1. Constant costs per piece, c, which include materials m, labor l, and some overheads o: $c = m + l + o$.

2. Preparation costs, s, per batch, which include drawings, planning, setting-up of machines and equipment, etc. The costs per piece are s/Q.

3. Interest carrying costs paid on the money invested in articles kept in stock: $I = ic$ for one piece per unit time. The average level of the stock during the consumption period T_c is $\frac{1}{2}(Q + Q_0)$ (see Fig. 10–2); thus the carrying costs for the stock for this period are

$$I \frac{Q + Q_0}{2} T_c$$

Substituting $T_c = Q/a_c$ and dividing by Q, the carrying costs per piece are

$$\frac{I}{2a_c} Q (1 + \gamma)$$

4. Storage carrying costs, including charges for space and personnel, maintenance while in stock, and deterioration: if the average storage costs per piece are B per unit time, then for the consumption period T_c they are

$$BT_c \quad \text{or} \quad \frac{BQ}{a_c}$$

These factors are shown in Fig. 10–3, the total costs per piece being

$$Y = c + \frac{s}{Q} + \frac{Q}{2a_c}\left[I\,(1+\gamma) + 2B\right] \tag{10–6}$$

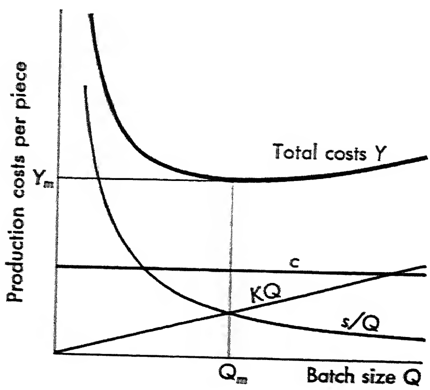


Figure 10–3. ***Production costs per piece.***

The last two factors can be combined into total carrying costs and denoted by KQ (where K is the carrying costs factor), so that

$$K = \frac{I(1+\gamma) + 2B}{2a_c} \tag{10–7}$$

and

$$Y = \underbrace{c}_{\substack{\text{constant}\\\text{costs}}} + \underbrace{\frac{s}{Q} + KQ}_{\substack{\text{variable}\\\text{costs}}} \tag{10–8}$$

The point where the costs per piece are minimum is found by

$$\frac{dY}{dQ} = 0$$

or

$$-\frac{s}{Q^2} + K = 0$$

$\therefore$

$$Q_m = \sqrt{\frac{s}{K}} = \sqrt{\frac{2a_c s}{I(1+\gamma) + 2B}} \tag{10–9}$$

At the point of minimum costs it is evident from Eq. 10–9 that

$$\frac{s}{Q_m} = KQ_m \tag{10–10}$$

i.e., the total carrying costs per piece and the preparation costs per piece are equal, and the batch Q_m corresponds to the point of intersection of the two variable costs curves in Fig. 10–3. The minimum total production costs per piece are

$$\begin{aligned} Y_m &= c + \frac{s}{Q_m} + KQ_m \\ &= c + \frac{2s}{Q_m} \\ &= c + 2KQ_m \end{aligned} \tag{10–11}$$

Example 1

A product is sold at a rate of 500 pieces a day and is manufactured at a rate of 2,500 pieces a day. The setup costs of the machines are \$1,000 and the storage costs are found to be 1.5×10^{-3} dollars per piece per day. Labor charges are \$3.20, materials \$2.10, and overhead \$4.10 per piece. If the interest charges are 8 per cent, find the minimum-cost batch size and the costs of the production run.

Solution

In this case we have

$$\begin{aligned} s &= \$1{,}000 \\ B &= \$0.0015 \text{ per piece per day} \\ a_c &= 500 \text{ pieces per day} \\ a_p &= 2{,}500 \text{ pieces per day} \end{aligned}$$

Hence
$$\gamma = \frac{a_c}{a_p} = \frac{500}{2{,}500} = 0.2$$

As the time unit taken here is one day, it is necessary to calculate the interest charges per day. These are expressed in a decimal form. Assume 300 working days in a year; then

$$i = \frac{8}{100}\,\frac{1}{300} = \$2.67 \times 10^{-4} \text{ per day}$$

and the constant costs per piece are

$$c = 3.20 + 2.10 + 4.10 = \$9.40 \text{ per piece}$$

$$\therefore \quad I = ic = 2.67 \times 10^{-4} \times 9.40 = \$2.5 \times 10^{-3} \text{ per piece per day}$$

The minimum-cost batch size, by Eq. 10–9, is

$$Q_m = \sqrt{\frac{2 \times 500 \times 1{,}000}{2.5 \times 10^{-3} \times 1.2 + 2 \times 1.5 \times 10^{-3}}} = 13{,}000 \text{ pieces}$$

The total production costs per piece, by Eq. 10–11, are

$$Y_m = c + \frac{2s}{Q_m}$$

$$= 9.40 + \frac{2{,}000}{13{,}000} = \$9.55 \text{ per piece}$$

The production run will involve the sum of

$$Q_m Y_m = 13{,}000 \times 9.55 = \$124{,}000$$

Special cases

1. When the production period is comparatively short and the storage charges small, a simplified version can be obtained for Eq. 10–9. These conditions may be expressed by

$$B \ll I$$

$$\gamma \ll 1$$

making the carrying costs factor K (from Eq. 10–7)

$$K = \frac{I}{2a_c} \tag{10–7a}$$

and the batch size for minimum costs is then

$$Q_m = \sqrt{\frac{2a_c s}{I}} \tag{10–9a}$$

This formula was first suggested by Harris (1915) and later by Camp (1922), and is often quoted in literature. Under the circumstances described above, it will yield quite satisfactory results, but when γ and B are not negligible, its use is not justified, as illustrated in Example 2.

2. When the production period is either relatively short (i.e., $\gamma \ll 1$) or irrelevant (as in inventory problems), the modified form of Eq. 10–9 becomes

$$Q_m = \sqrt{\frac{2a_c s}{I + 2B}} \tag{10–9b}$$

This is called the *inventory formula* for optimal batch sizes. The ratio γ does not figure in this formula, which may sometimes be successfully used in problems of batch sizes in manufacture because it is obviously a better approximation than Camp's formula.

Example 2

Machine components supplied to the assembly shop are produced in a plant at a rate of 100 pieces a day. A cost analysis showed that the constant production costs per piece, including labor, materials, and overhead, amount to \$2.40 per piece, and the storage costs are \$0.0005 per piece per day (this figure includes

the maintenance charges). If the preparation and machine setup costs for a production run amount to \$500 and the assembly bay is using 40 pieces per day, find the minimum-cost batch size and the length of the production run (assume interest charges are 12 per cent). Compare these results with those obtained by Camp's formula and the inventory formula.

Solution

From the given data

$$\begin{aligned} c &= \$2.40 \text{ per piece} \\ s &= \$500 \\ B &= \$0.0005 \text{ per piece per day} \\ a_c &= 40 \text{ pieces per day} \\ a_p &= 100 \text{ pieces per day} \end{aligned}$$

Assume 300 working days in a year; hence the interest charges per day are

$$i = \frac{12}{100} \times \frac{1}{300} = 4.0 \times 10^{-4} \text{ per day}$$

or $$I = ic = 2.40 \times 4.0 \times 10^{-4} = \$9.6 \times 10^{-4} \text{ per piece per day}$$

and $$\gamma = \frac{a_c}{a_p} = \frac{40}{100} = 0.4$$

Computations of the minimum-cost batch size by expression 10–9 give

$$Q_m = \sqrt{\frac{2 \times 500 \times 40}{9.6 \times 10^{-4} \times 1.4 + 10^{-3}}} = 4{,}140 \text{ pieces}$$

The length of the production run (excluding setup time) is

$$T_p = \frac{Q_m}{a_p} = \frac{4{,}140}{100} = 41.4 \text{ days}$$

If Camp's formula or the modified inventory formula are used, the results for Q_m will be

By Camp: $$Q_m = \sqrt{\frac{2 \times 500 \times 40}{9.6 \times 10^{-4}}}$$

$$= 6{,}430 \text{ pieces involving 64.3 days' production}$$

By the inventory formula:

$$Q_m = \sqrt{\frac{2 \times 500 \times 40}{9.6 \times 10^{-4} + 10^{-3}}}$$

$$= 4{,}520 \text{ pieces corresponding to 45.2 days' production}$$

In this case Camp's formula obviously does not yield satisfactory results because it ignores the marked effect of storage costs. Even the inventory formula gives a batch size that is larger by 9.2 per cent than the quantity suggested by formula 10–9.

Other formulae

Apart from the three formulae given by 10–9, 10–9a, and 10–9b, numerous other formulae are to be found in literature. Essentially, the difference between the various published expressions lies in the evaluation of the factor K and in the importance attributed to some variables likely to affect it. We shall mention here three of the most important attempts in this direction:

Raymond's formula

In the late 1920's F. E. Raymond carried out an extensive study of variables affecting batch size determination. He was mainly concerned with the effect of dividing a batch into a number of lots and with the time involved in waiting between operations. To account for these and other factors, he suggested several expressions, of which the "simplified" formula (published in 1930) is as follows:

$$Q_m = \sqrt{\frac{2a_c s}{I\left[2F - \gamma\left(1 - \frac{1}{N}\right)\right] + I\gamma\frac{2k}{A} + \frac{2bv}{h}\left[1 - \gamma\left(1 - \frac{1}{N}\right)\right]}} \qquad (10\text{-}12)$$

where F = a stock coefficient, being the ratio of the average number of articles in the stock above Q_0 to the maximum number above Q_0; for uniform consumption $F = \frac{1}{2}$.

k = a factor to take account of "work in process"

$$= \frac{c'}{c} = \frac{1}{2}\frac{m + c}{c}.$$

c' = cost of piece in process $= m + \frac{1}{2}(l + o)$.

v = volume storage space per piece.

b = storage costs per 1 sq. ft. per unit time.

h = average height to which storage is permitted.

N = number of lots in the batch.

A = time factor

$$= 1 + (N - 1)\frac{\text{(time for first operation in the batch)}}{\text{(total process time for first lot)}}$$

Example 3

A component is supplied to the store at the rate of 200 pieces a day, when scheduled for production, the rate of consumption being 20 pieces a day. The preparation costs are \$1,200 per batch, but it has been decided to divide the batch into three lots, to be produced in succession. The volume of the component is 0.01 cu. ft. and the storage costs are 1c. per cubic foot per day. The constant production costs per piece are \$2.50, but the costs per piece in progress are \$2.00. If the time for the first operation in the batch is 8 per cent of the total process time for the first lot, and the interest charges are 12 per cent, find the minimum-cost batch size.

Solution

From the data given above,

$$s = \$1{,}200$$
$$c = \$2.50 \text{ per piece}$$
$$c' = \$2.00 \text{ per piece}$$

Hence

$$k = \frac{c'}{c} = \frac{2.00}{2.50} = 0.8$$
$$F = 0.5$$
$$i = 4 \times 10^{-4} \text{ per day}$$
$$I = ic = 4 \times 10^{-4} \times 2.5 = \$0.001 \text{ per piece per day}$$
$$v = 0.01 \text{ cu. ft.}; \qquad b = \$0.01 \text{ per cu. ft. per day}$$
$$N = 3$$
$$A = 1 + 0.08\,(3 - 1) = 1.16$$
$$a_c = 20 \text{ pieces per day}$$
$$a_p = 200 \text{ pieces per day}$$

Hence $$\gamma = \frac{a_c}{a_p} = \frac{20}{200} = 0.1$$

The batch size computed by Raymond's formula is

$$Q_m = \sqrt{\frac{2 \times 20 \times 1{,}200}{10^{-3}\left[1 - 0.1\left(1 - \frac{1}{3}\right)\right] + 10^{-3} \times 0.1 \times \frac{1.6}{1.16} + 2 \times 10^{-2} \times 10^{-2}\left[1 - 0.1\left(1 - \frac{1}{3}\right)\right]}}$$
$$= 6{,}120 \text{ pieces}$$

If the simpler expression, 10–9, is used,

$$Q_m = \sqrt{\frac{2 \times 20 \times 1{,}200}{10^{-3} \times 1.1 + 2 \times 10^{-4}}} = 6{,}070 \text{ pieces}$$

This result differs from that obtained by Raymond's formula by less than 1 per cent. The discrepancy will obviously increase with N and γ, but in most cases the difference is so small that many of the factors included by Raymond can be ignored and formula 10–9 may be satisfactorily applied.

This argument may well be illustrated by calculating the total production costs per piece for these two batch sizes, using expression 10–11: For $Q_m = 6{,}120$ (obtained by Raymond's formula),

$$Y_m = c + \frac{2s}{Q_m} = 2.50 + \frac{2{,}400}{6{,}120} = \$2.89 \text{ per piece}$$

while, for $Q_m = 6{,}070$ (obtained by expression 10–9),

$$Y_m = 2.50 + \frac{2{,}400}{6{,}070} = \$2.90 \text{ per piece}$$

the difference being actually one-third of a cent between the two figures (when accurately calculated), which amounts to about 0.1 per cent. There is clearly no merit in employing elaborate formulae in cases of this kind.

Lehoczky's formula

This formula can be expressed as

$$X = \sqrt{\frac{M}{2L}(m' + \gamma - m'\gamma)} \tag{10-13}$$

where X = number of batches to be produced per year.
M = interest on raw materials purchased once a year.
L = setup charges of machines including preparation and costs such as drawing, planning, ordering, and tooling.

$$m' = \frac{\text{cost of finished product}}{\text{cost of raw material}}$$

Davis' formula

This formula is

$$Q_m = \sqrt{\frac{2a_c s}{(I + 2B)(1 + k\gamma)}} \tag{10-14}$$

Here

$$I = rc$$

where r = the desired rate of profit on the working profit, expressed as a decimal value.
k = a batch factor, expressing the influence of batch production on the economy of manufacturing, given by $k = 2F - 1$.

$$F = \frac{\text{quantity at order point}}{\text{quantity used while awaiting delivery}}.$$

Methods for computing Q_m

Use of formula

As already mentioned, this method has been a popular one expounded by many writers. Although attempts have been made at formulating very elaborate expressions (some of which were listed above) which pertain to provide very great accuracy, those have not been very popular, mainly for two reasons. First, the underlying assumption on which this theory is based states that the costs function is a quadratic one of the form given by expression 10–8, having linear terms for the carrying costs and hyperbolic terms for the preparation costs as the only variable features of the function. Such an assumption, as a prelude to a very strict mathematical treatment and a claim for high accuracy, is hardly justified. The production costs function, expression 10–8, is at best a convenient model that represents the problem under consideration within certain

limitations. The advantage of this model is its simplicity; it does not, however, resemble reality accurately enough to warrant very elaborate formulae. Secondly, these complex expressions are cumbersome to handle and require accurate determination of a large number of variables, a task which—if not sometimes impossible—may at least be a difficult and lengthy affair. In many cases the use of complex formulae may even be unjustified, as demonstrated in Example 3.

Expression 10–9 and its approximations, 10–9a and 10–9b, involve a smaller number of variables and are by far simpler to use. Moreover, batch production is a dynamic affair; circumstances may rapidly change and a handy tool is required to assess the effect of changing factors.

Use of a nomograph

A nomograph facilitates rapid calculations, and if drawn to a suitable scale, it yields quite satisfactory results. The nomograph in Fig. 10–4 is based on the formula 10–9. The left-hand half is concerned with the variables I (ordinate), γ (oblique lines), B (abscissa), and a_c (oblique lines). The right-hand half contains three variables: K (ordinate, obtained from left-hand half), s (oblique lines), and Q_m (abscissa).

Example 4

Take the data given in Example 1. Use the nomograph as follows: Mark $I = 2.5 \times 10^{-3}$ on ordinate on the left-hand half. Draw a horizontal line to intersect the γ lines at $\gamma = 0.2$. The abscissa now gives the interest charges, corrected to account for the effect of the production period: $\frac{1}{2}I(1 + \gamma)$. Add the storage charges: $1.5 \times 10^{-3} + 1.5 \times 10^{-3} = 3.0 \times 10^{-3}$. Draw a vertical line to intersect a_c lines at $a_c = 500$. The ordinate gives the factor K $(= 6 \times 10^{-6})$. Extend the horizontal line to intersect s at $s = 1{,}000$, and the minimum-cost batch size is given by the abscissa at the point of intersection; in this case, $Q_m = 13{,}000$ pieces.

When the ratio γ is very small or when batch sizes have to be computed for inventory purposes, the procedure of intersecting the γ lines can be bypassed, either by using $\gamma = 0$ or by directly marking $I/2$ on the abscissa on the left.

The effect of using the simple Camp relation is also illustrated on the nomograph. No use is made of the parameter γ, and the left-hand abscissa is not corrected for storage charges, In our example this would lead to $Q_m = 20{,}000$ pieces. It is obvious from the nomograph that only when γ and B are negligible will the error resulting from Camp's formula be small.

A graphical method

This method can also be called *learning from bitter experience.* If a product has been manufactured several times in various quantities, the curve of costs per piece versus batch size can be plotted (Fig. 10–5) and the point of minimum costs obtained from it.

The main disadvantages of this method are: First, a large number of points

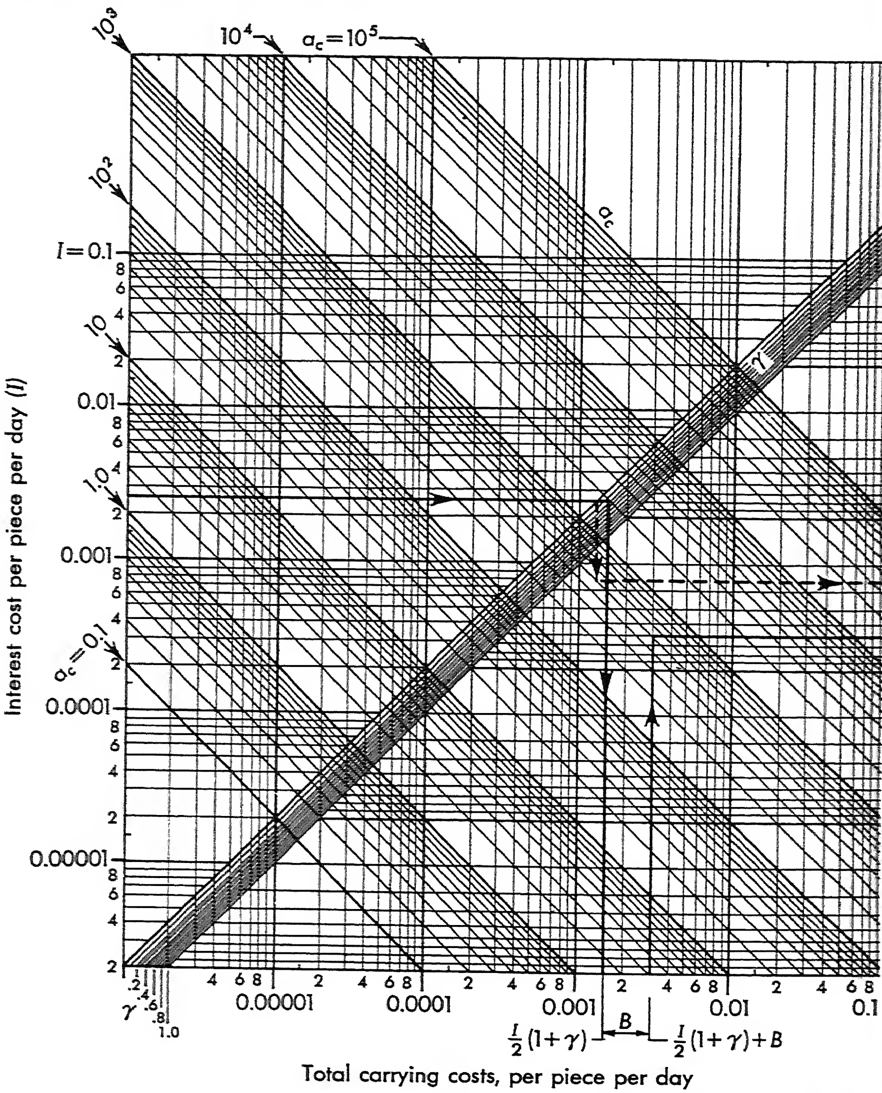
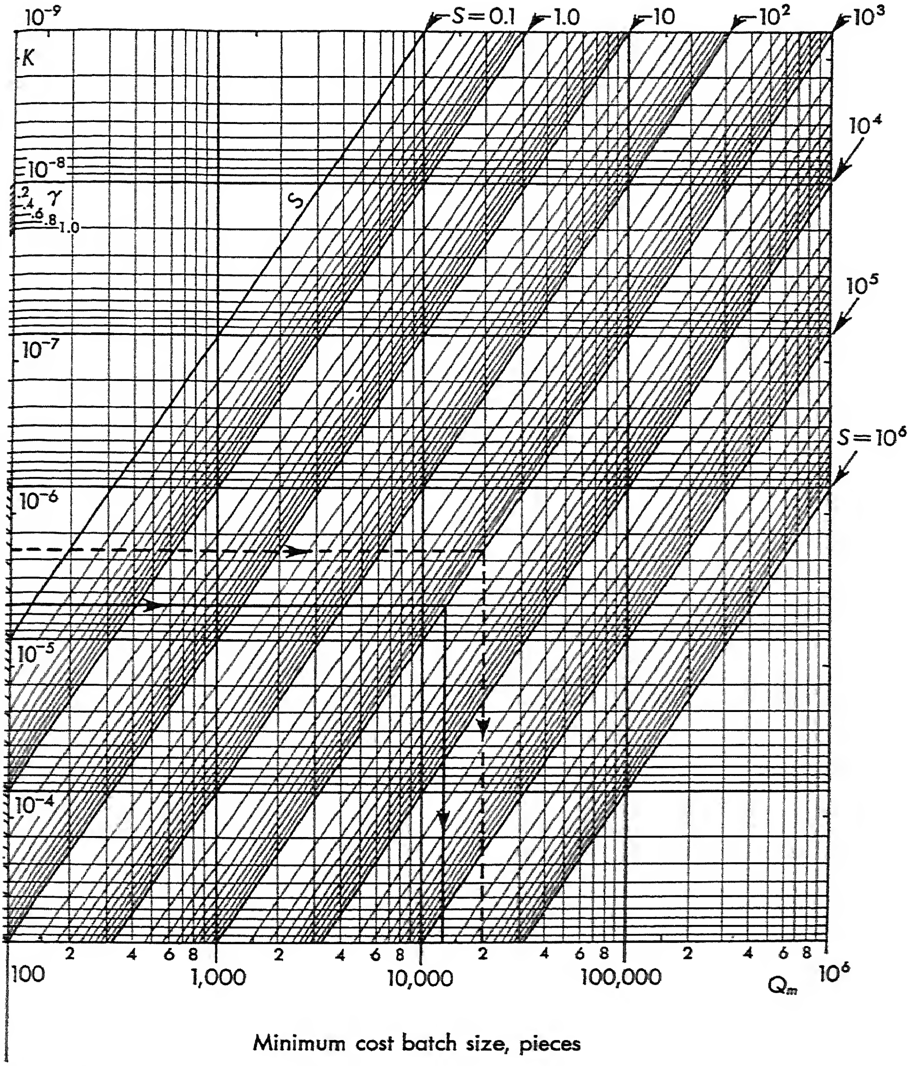


Figure 10–4. ***A nomograph for computing***

is required to facilitate the construction of the curve, all relating to circumstances in which the constant costs per piece are the same. If this part of the cost function undergoes a change, the curve is no longer valid. Secondly, because the curve is flat at the minimum point, it may be difficult to establish where this point is.

This may not be considered a practical method for current evaluation of Q_m, but it can provide a useful tool for management when trying to assess whether a



the minimum-cost batch size.

product with a history behind it is sensitive to variable costs, in which case a fuller analysis is justified.

A modified graphical method

In this method it is not necessary to plot the curve or to have a large number of points on it. It is sufficient to locate two points on either side of Q_m which have the same ordinate; i.e., the same production costs per piece, corresponding

to two batch sizes Q_{I} and Q_{II} (Fig. 10–5). It can be shown (see Eq. 10–20) that the relation between these batches and Q_m is

$$Q_m = \frac{Q_{\mathrm{I}} + Q_{\mathrm{II}}}{2p}$$

where the factor p is the variable costs per piece incurred at Q_{I} and Q_{II} compared with those incurred when Q_m is produced (see definition Eq. 10–15). The fact that it is necessary to assess the value of p by the available data may introduce errors in the calculations, but the accuracy of Q_m mainly depends on the accuracy with which Q_{I} and Q_{II} can be determined.

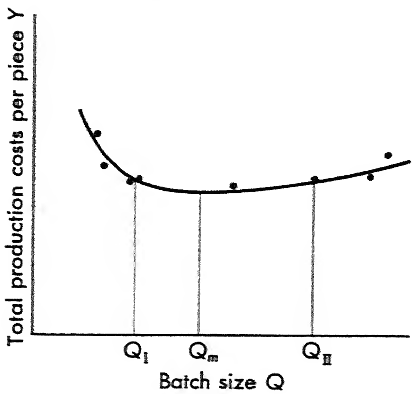


Figure 10–5. ***A graphical method for finding the minimum-cost batch.***

Equating the variable costs

At the point of minimum costs, the two terms of variable costs are equal:

$$KQ_m = \frac{s}{Q_m}$$

This fact provides a good method for finding the point of minimum. Once the values of s and K have been determined, the curves of the two variables are plotted (Fig. 10–6), and Q_m is given by the abscissa of the point of intersection,

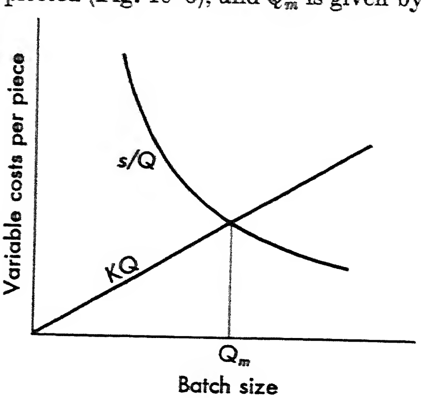


Figure 10–6. ***Finding Q_m by equating the variable costs.***

which is more clearly definable than the determination of a minimum point on a shallow portion of a curve.

The Production Range

A production range may be likened to dimensional tolerances on engineering components. A certain batch size may be considered the best target that should be aimed at, but in practice it may be impossible to obtain a satisfactory production schedule consisting of ideal batches only.

Determination of the production range

Any increase in total production costs above the minimum total costs corresponds to two batch sizes on either side of Q_m, as shown in Fig. 10–7. If such an increase in costs is allowed, it is possible to deviate from Q_m and select any convenient batch size that lies between the two limits without causing an increase in production costs above a predetermined value. The range between the two limits, called the *production range*, offers the desired flexibility when scheduling of the batch is attempted.

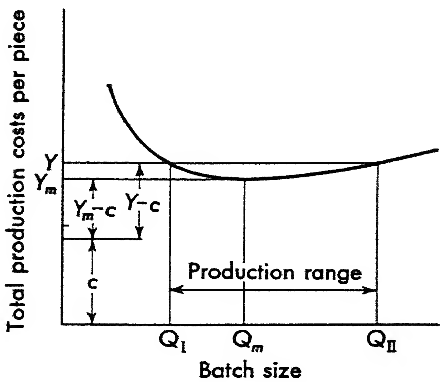


Figure 10–7. ***The production range.***

In order to find the numerical value of the two limits Q_I and Q_{II} (see Fig. 10–7), we shall first define a nondimensional ratio p as follows:

$$p = \frac{Y - c}{Y_m - c} = \frac{\text{variable costs}}{\text{minimum variable costs}} \tag{10–15}$$

Since deviations from the minimum-cost batch size result in a change in the variable terms of the total production costs function (expression 10–8), the ratio p is a convenient measure of increase in costs above the minimum. The value of p is higher than 1 and the costs function expressed by 10–8 may be written as

$$\begin{aligned} Y &= c + p(Y_m - c) \\ &= c + 2KQ_m p \\ &= c + 2\frac{s}{Q_m}p \end{aligned} \tag{10–16}$$

To find Q_{I} and Q_{II}, the limits of the production range (Fig. 10–7) substitute expressions 10–8 and 10–11

$$Y - c = \frac{s}{Q} + KQ$$

$$Y_m - c = 2KQ_m$$

to the definition 10–15 of p; hence

$$p = \frac{(s/Q) + KQ}{2KQ_m} = \frac{1}{2}\left(\frac{s}{K}\frac{1}{QQ_m} + \frac{Q}{Q_m}\right)$$

By substituting now $s/K = Q^2{}_m$ (by 10–9) and a nondimensional ratio

$$q = \frac{Q}{Q_m} \tag{10–17}$$

we get

$$\begin{aligned} p &= \frac{1}{2}\left(\frac{Q_m}{Q} + \frac{Q}{Q_m}\right) \\ &= \frac{1}{2}\left(\frac{1}{q} + q\right) \end{aligned} \tag{10–18}$$

By solving this equation for q, when p is known, we have

$$q = p \pm \sqrt{p^2 - 1} \tag{10–19}$$

or

$$Q_{\text{I}} = Q_m(p - \sqrt{p^2 - 1})$$

and

$$Q_{\text{II}} = Q_m(p + \sqrt{p^2 - 1}) \tag{10–20}$$

It is significant that Eq. 10–19 is a nondimensional expression, the tolerance on the production range being dependent on the allowable increase in variable costs. If this increase is by 1 per cent (i.e., $p = 1.01$), the limits of the production range would be

$$q = 1.01 \pm \sqrt{(1.01)^2 - 1} = 1.01 \pm 0.142$$

or

$$q_{\text{I}} = 0.868$$

$$q_{\text{II}} = 1.152$$

In other words, the limits of the production range are 13.2 per cent below and 15.2 per cent above the minimum-cost batch size. These figures indicate that the total production-costs function is very "flat" near the point of minimum, being more flat in the region where $Q > Q_m$ than when $Q < Q_m$. Another significant feature of this function is the amount of flexibility that results when a comparatively small increase in variable costs is allowed, and one must not forget that the increase in total production costs due to a deviation from Q_m would be even smaller. If the variable costs are allowed to increase by 5 per

cent (i.e., $p = 1.05$), the range is -27 to $+37$ per cent, and when 10 per cent is allowed (or $p = 1.10$), it becomes -36 to $+56$ per cent. The relation 10–18 is plotted in Fig. 10–8, from which the limits of the production range are readily obtained once the allowable increase in variable costs has been determined.

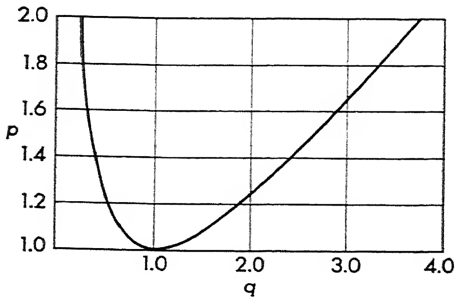


Figure 10–8. ***Determination of the production range.***

Conversely, when scheduling of a number of products is attempted, the desirable degree of flexibility in selecting batch sizes can be ascertained, from which the likely maximum increase in variable costs can be obtained by Fig. 10–8. It is then the responsibility of management to decide whether these variations in production costs can be tolerated.

Effect on production costs

The effect of p on the total costs Y evidently depends on the ratio of constant costs per piece to variable costs per piece; the larger this ratio, the less is Y affected by p. The total costs may be expressed in a nondimensional form. From Eq. 10–16,

$$\frac{Y}{c} = 1 + \frac{2KQ_m}{c} p$$

KQ_m represents the carrying costs per piece when Q_m is produced; hence the ratio KQ_m/c is defined for a particular product under given circumstances. Substituting the ratio

$$u = \frac{c}{s/Q_m} = \frac{c}{KQ_m} \tag{10–21}$$

The cost function becomes

$$\frac{Y}{c} = 1 + \frac{2p}{u} \tag{10–22}$$

The minimum total costs are obtained when $p = 1$:

$$\frac{Y_m}{c} = 1 + \frac{2}{u} \tag{10–23}$$

Hence the increase in costs due to a deviation from Q_m is given by

$$\frac{\Delta Y}{c} = \frac{Y}{c} - \frac{Y_m}{c} = \frac{p-1}{\frac{1}{2}u}$$

If expressed in a ratio form, it can be shown that

$$\xi = \frac{\Delta Y}{Y_m} = \frac{p-1}{\frac{1}{2}u+1} \tag{10–24}$$

This expression demonstrates clearly the effect of p. When the ratio u is large, a slight increase in p is unlikely to cause a noticeable change in the total production costs, thus leaving room for a wide production range to be defined, while when the ratio u is small, the reverse is the case. The relation between ξ and u with p as a parameter, is plotted in Fig. 10–9.

The futility of seeking a very accurate mathematical method to calculate

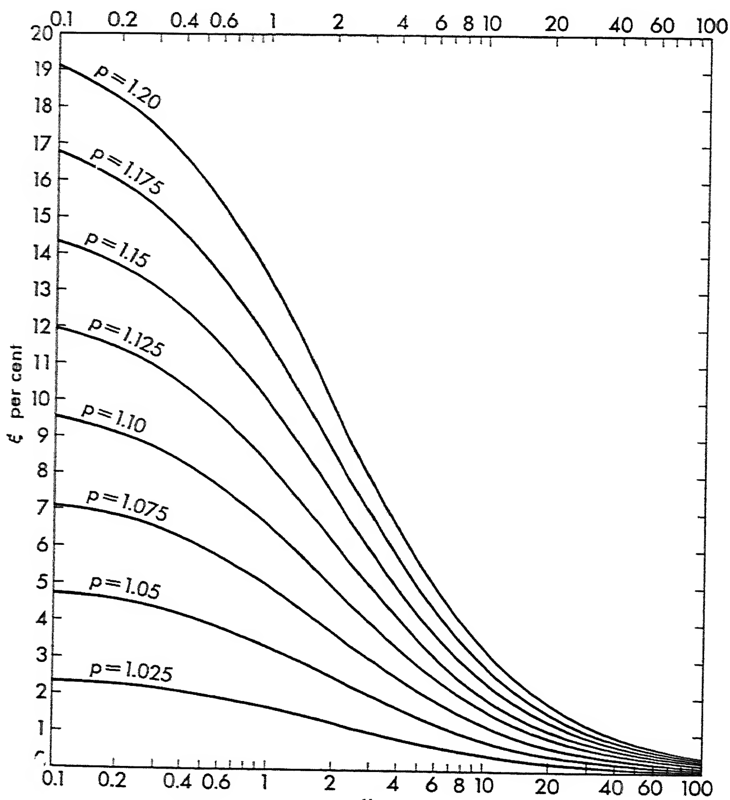


Figure 10–9. ***Increase in costs per piece.***

Q_m is once more indicated by the preceding expression 10–24. Even if comparatively large errors are incurred in determining Q_m, the total costs are affected very little.

Example 5

When the minimum-cost batch size is produced, it is known that the variable costs constitute 25 per cent of the total production costs. If Q_m is increased by 20 per cent, what increase in production costs can be expected?

Solution

It is known that $2KQ_m/Y_m = 0.25$. Hence

$$\frac{Y_m}{2KQ_m} = \frac{c + 2KQ_m}{2KQ_m} = \tfrac{1}{2}u + 1 = \frac{1}{0.25}$$

or

$$u = 6.0$$

Q_m is increased by 20 per cent, or

$$q = \frac{Q}{Q_m} = 1.20$$

$$p = \frac{1}{2}\left(\frac{1}{q} + q\right) = \frac{1}{2}\left(\frac{1}{1.20} + 1.20\right) = 1.017$$

The increase in production costs would be

$$\xi = \frac{p - 1}{\tfrac{1}{2}u + 1} = \frac{1.017 - 1}{3.0 + 1} = 0.0043, \quad \text{or by } 0.4\%$$

Example 6

It is required to establish the production range for the following data:

Setup costs	$s = \$1{,}000$
Carrying charges factor	$K = \$0.25 \times 10^{-3}$ per unit per day
Constant costs per piece	$c = \$2.0$
Allowable increase in costs per piece	$\xi = 2.5\%$

Solution

First find the minimum-cost batch size:

$$Q_m = \sqrt{\frac{s}{K}} = 2{,}000 \text{ pieces}$$

The ratio

$$u = \frac{c}{s/Q_m} = \frac{2}{1{,}000/2{,}000} = 4.0$$

The factor p can be now be found, either from Fig. 10–9 or by Eq. 10–24:

$$0.025 = \frac{p - 1}{2.0 + 1}$$

or

$$p = 1.075$$

And from Fig. 10–8 this relaxation corresponds to

$$q_{\mathrm{I}} = 0.68$$
$$q_{\mathrm{II}} = 1.47$$

the production range being between 1,360 and 2,940 pieces.

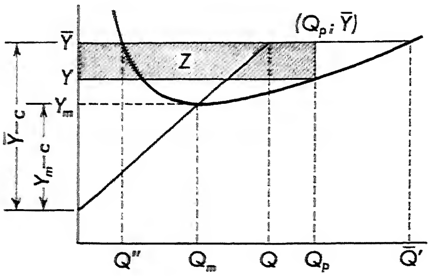


Figure 10–10. ***Batch profit, shown by shaded rectangle.***

Maximum-profit Batch Size[1]

We have seen that when minimum production costs per piece are sought, the computed batch size is unrelated to the sales price of the article. Turning to another criterion by which success is often measured, we shall try now to find the batch size that yields maximum absolute profit.

To conclude without further analysis that production at minimum costs also results in maximum profit for the batch would be fallacious. It is true that the profit per piece under these circumstances would be maximum, but the profit for the whole batch is also dependent on the number of pieces in the batch. As the total costs per piece rise very slowly beyond Q_m, it is reasonable to conclude that when a batch size larger than Q_m is produced, the total profit will rise. The batch size cannot, however, be increased too much; otherwise the production costs will rise to such an extent as to leave a very marginal profit per piece. In the extreme, too large a batch size will involve such high carrying costs that production costs would be equal or even higher than the sales price, leaving no profit at all.

First it is necessary to express the profit in terms of the batch size. If Q pieces are produced at the costs of Y per piece and the sales price is Y' (see Fig. 10–10), the profit per piece is $Y' - Y$, and the profit for the batch is

$$Z = Q(Y' - Y) \tag{10–25}$$

This profit is denoted in Fig. 10–10 by the shaded rectangle, being the difference between the money earned (rectangle $Y'Q$) and the cost of production (rectangle YQ). Maximum profit is achieved when

$$\frac{dZ}{dQ} = 0$$

[1] This section may be omitted at first reading.

or
$$Y' - \frac{d(QY)}{dQ} = 0$$

Substitute
$$Y' = c + \frac{s}{Q'} + KQ'$$

and
$$\frac{d(QY)}{dQ} = \frac{d}{dQ}\left[Q\left(c + \frac{s}{Q} + KQ\right)\right]$$
$$= c + 2KQ$$

Hence
$$\frac{s}{Q'} + KQ' - 2KQ_p = 0$$

where Q_p is the batch size yielding maximum profit. Therefore

$$Q_p = \frac{1}{2}\left(\frac{s}{KQ'} + Q'\right)$$

When $s/K = Q_m{}^2$ is substituted, we get

$$Q_p = \frac{Q_m}{2}\left(\frac{Q_m}{Q'} + \frac{Q'}{Q_m}\right)$$

But by Eq. 10–19

$$\frac{1}{2}\left(\frac{Q_m}{Q'} + \frac{Q'}{Q_m}\right) = p'$$

where
$$p' = \frac{Y' - c}{Y_m - c} \tag{10–26}$$

This expression is similar to 10–15, and p' gives a relation between the sales price and the minimum production costs per piece. Hence

$$Q_p = p'Q_m$$

or
$$q = \frac{Q_p}{Q_m} = p' \tag{10–27}$$

As $p' > 1$, the batch size giving maximum profit is larger than Q_m, a conclusion already mentioned above.

Example 7

For the data given in Example 1, find the maximum-profit batch size, if the sales price has been fixed to \$10.50 per piece. What profit per piece is envisaged?

Solution

$$p' = \frac{Y' - c}{Y_m - c} = \frac{10.50 - 9.40}{9.55 - 9.40} = 7.35$$

It has already been found that $Q_m = 13{,}000$ pieces. Therefore

$$Q_p = 7.35 \times 13{,}000 = 95{,}500 \text{ pieces}$$

The production of Q_p involves an increase of variable costs by

$$p = \frac{1}{2}\left(\frac{Q_p}{Q_m} + \frac{Q_m}{Q_p}\right)$$
$$= \frac{1}{2}\left(p' + \frac{1}{p'}\right)$$
$$= \frac{1}{2}\left(7.35 + \frac{1}{7.35}\right) = 3.74$$

The production costs by Eq. 10–16 are

$$Y = c + 2KQ_m p = 9.40 + 0.15 \times 3.74 = \$9.96 \text{ per piece}$$

and the profit per piece is

$$Y' - Y = 10.50 - 9.96 = \$0.54$$

It is also possible to arrive at the maximum-profit batch size Q_p by a graphical construction based on Eq. 10–27. The minimum-cost batch size and the minimum costs per piece are first computed, and these coordinates describe the point A $(Q_m; Y_m)$ in Fig. 10–11. The constant cost per piece, c, is marked on the ordinate (point B), and the sales price is indicated by a horizontal line. Points A and B are joined and the batch Q_p is given by the point of intersection of this line with the price line. Simple geometric considerations will prove that this construction is in accordance with Eq. 10–27.

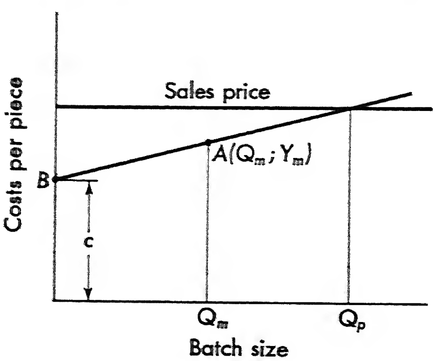


Figure 10–11. ***Graphical method for finding the maximum-profit batch size.***

As already mentioned, it is necessary to produce large batches in order to achieve maximum batch profit. The graphical method clearly illustrates that these batches can be very large indeed when the constant cost content is relatively high. When compared with the minimum-cost batch size the maximum-profit batch has several noteworthy implications:

1. Large batches have various technical advantages: The setup time per piece is reduced and hence the equipment can be more productively used; the

schedule becomes smooth and uninterrupted for a long time and gives more scope for improving production methods. In many cases a detailed method analysis in short runs would be impossible, whereas in long runs such a study may lead to saving in labor, materials, and overhead costs, thus reducing the total production costs per piece.

2. In cases where the length of the production run is not linked with change in methods, the production cost function remains unchanged and higher costs per piece will be incurred when Q_p is produced. If Q_p does not greatly deviate from Q_m, the increase in price may not be too serious, as it was shown above that the total costs curve is usually quite shallow beyond the point Q_m. Sometimes, however, such an increase may be undesirable if there is a likelihood of a change in pricing policy. In Example 7 the deviation from Q_m is very marked, leading to an increase of 41 cents in production costs per piece (4.3 per cent) and a profit margin of only 54 cents per piece. If market conditions or competition force the manufacturer to reduce his sales price by only 5 per cent, he will scarcely make any profit.

3. The product will remain a longer time in stock, leading to slower turnover of the capital. This slow turnover may greatly impede the activities of the organization, especially if its resources are limited.

4. Last but not least, this criterion is misleading. True, the whole concept of profit per batch is largest when Q_p is produced, but then (i) more money is required to produce the batch, and (ii) it takes longer for the profit to be realized. In short, as the consumption volume per annum is fixed, the total profit per annum is a maximum when Q_m is produced and declines if Q_p is produced; therefore both profit and capital turnover decline, and the realization of maximum profit per batch would appear to have little advantage (apart from the technical advantages associated with long runs).

Maximum Return[2]

We have seen that the policy of maximum profit, which actually involves an increase in absolute profit at the expense of even a larger increase in the investment required to execute the production run, may be regarded a poor way to evaluate performance. It would seem that profit should be measured not as an absolute figure, but at least as one in relation to the amount invested in producing the batch. This approach leads us to the third criterion: the ratio of profit to the cost of the production run, or "return" η. The cost to produce the batch is QY, and the return can be expressed as:

$$\eta = \frac{Z}{QY} = \frac{QY' - QY}{QY} \tag{10–28}$$

$$= \frac{Y'}{Y} - 1$$

[2] This section may be omitted at first reading.

In order to attain maximum return, it is necessary to have maximum Y'/Y; i.e., a minimum value for Y. But minimum Y means that the batch is produced at minimum production costs per piece. Hence, in order to have maximum return, Q_m has to be produced, and it may therefore be said that optimal conditions as measured by these two criteria are achieved simultaneously.

If Y and Y' are substituted by Eq. 10–22, the return of η becomes

$$\eta = \frac{2p' + u}{2p + u} - 1 \tag{10–29}$$

$$\therefore \qquad \eta = \frac{p' - p}{p + \frac{1}{2}u}$$

The return is maximum when $p = 1$:

$$\eta_m = \frac{p' - 1}{1 + \frac{1}{2}u}$$

Performance can be now measured by comparing the actual return with the maximum return when a batch size Q is produced, and this measure becomes a kind of efficiency index. By using the above computations, this ratio is

$$\frac{\eta}{\eta_m} = \frac{p' - p}{p' - 1} \cdot \frac{1 + \frac{1}{2}u}{p + \frac{1}{2}u} \tag{10–30}$$

What efficiency index can be expected when deviations from Q_m are made? We have seen that the effect of such deviations on the production costs depends on the value of the factor p, which provides the basis for defining the production range. The departure of η from the point of maximum depends on three factors: p, p', and u. The return, η, is mainly sensitive to $(p' - p)$, while the ratio u has a far lesser effect, especially when u is large.

Maximum Rate of Return[3]

So far no account has been taken of the time during which sales and profits are effected. The time factor is of great importance when turnover of the capital is considered, as it would seem far more desirable to achieve the same return in a short period.

A high rate of return implies that stocks should be kept at a low level and replenished at short interval times by small batches. If the batch size that yields maximum rate of return is called the *economic batch size*, it would appear that the economic batch is far smaller than the minimum-cost batch.

The advantages of the economic batch size were first pointed out by F. E. Raymond (1930) and later by P. T. Norton (1933), who recognized the fact that production of economic batches will involve higher production costs per piece

[3] This section may be omitted at first reading.

than when Q_m is produced; however, they suggested that economic batches are more profitable because a higher rate of return on the capital invested is achieved.

If the rate of return obtained when Q_m is produced is considered an acceptable one, it is possible to define a smaller batch that will yield the same rate of return. The smaller batch will entail higher production costs per piece, but this will require far less capital. F. E. Raymond proceeded to find this batch size by using his formula for the minimum-cost batch size, modified by introducing fI instead of I, where the factor f "provides for the proper conservation of capital," as he put it. The value of f as suggested by Raymond is

$$f = 1 + 2\frac{r}{i} + \frac{(r/i)^2}{1 + \dfrac{[2(bv/h)][1 - (1/N)]}{I + \dfrac{2k\gamma}{A\{1 - \gamma[1 - (1/N)]\}}}}$$

where i is the rate of interest paid on the capital, and r is the rate of return normally expected on the invested capital, in excess of the interest rate. Raymond considered the range between this batch and Q_m as the "economic range," in which any selected batch size will yield at least the rate of return related to Q_m, the maximum rate of return being located somewhere at the middle of the range. This batch can be computed by the same method, except that the factor f is

$$f = 1 + 2\frac{r}{i}$$

A similar method was advocated by P. T. Norton, who considered for I the total interest paid, as well as taxes, insurance, etc., and the desired rate of return on the capital.

$$Q_e = \sqrt{\frac{2a_c sN}{(r + i)c + 2B(1 - \gamma)}}$$

where N = number of working days per year.
r = desired rate of return.
i = taxes, insurance, etc.
B = storage costs per piece per year.

In this method the interest is related only to the constant production costs and not to the total production costs. It is also based on the assumption that producers are free to select a desirable value for the rate of return with no relation to the sales price. In practice, the sales price is often determined by conditions prevailing in the market, those due to competition or to the preparedness on behalf of the customer to pay what seems a reasonable price for the article. The sales price will have a direct effect on the batch size and will determine the rate of return that may be expected.

Another method would be to find the rate of return in terms of sales price, production costs, and batch size, and then to maximize this expression. The

rate of return is defined as the profit realized for every monetary unit invested per unit time. The time during which the profit is obtained is the consumption period T_c. Hence the rate of return R is

$$R = \frac{\eta}{T_c}$$

or, by Eq. 10–28,

$$R = \frac{1}{T_c}\left(\frac{Y'}{Y} - 1\right) \tag{10–31}$$

In fact the rate of return may be slightly higher because not all the investment is made before the run and some is paid during the production period in the form of wages and even procurement of material. However, for any plant these factors remain fairly constant and do not affect the issue as far as batch sizes are concerned.

By substituting Eq. 10–1, $T_c = Q/a_c$, the maximum rate of return is obtained when

$$\frac{dR}{dQ} = 0$$

or

$$\frac{d}{dQ}\left(\frac{Y'}{YQ} - \frac{1}{Q}\right) a_c = 0$$

Hence

$$Y' \frac{d(YQ)}{dQ} = Y^2 \tag{10–32}$$

If Y is substituted by Eq. 10–8 and

$$\frac{d(YQ)}{dQ} = c + 2KQ$$

the equation obtained for the economic batch size is

$$K^2Q_e^4 - 2K(Y' - c)Q_e^3 + (c^2 + 2sK - Y'c)Q_e^2 + 2csQ_e + s^2 = 0 \tag{10–33}$$

While the batch size relating to maximum absolute profit has been shown to be larger than the minimum cost batch size Q_m, the economic batch is smaller than Q_m. This can be shown by substituting

$$\frac{d(YQ)}{dQ} = c + 2KQ = Y - \left(\frac{s}{Q} - KQ\right)$$

in Eq. 10–32; hence

$$Y'\left[1 - \frac{(s/Q_e) - KQ_e}{Y}\right] = Y$$

Since $Y' > Y$, it follows that

$$\frac{s}{Q_e} > KQ_e$$

As already shown, at the point of minimum costs

$$\frac{s}{Q_m} = KQ_m$$

and from Fig. 10–3 it is clear that

$$\frac{s}{Q} < KQ \qquad \text{for } Q > Q_m$$

and

$$\frac{s}{Q} > KQ \qquad \text{for } Q < Q_m$$

hence the economic batch size must be smaller than the minimum-cost batch.

Equation 10–33 for the economic batch size is of the fourth order and may be expressed in a simpler form by the use of the definitions 10–15 and 10–17:

$$p' = \frac{Y' - c}{Y_m - c}$$

or

$$Y' - c = p'(Y_m - c) = p'2KQ_m$$

and

$$q = \frac{Q_e}{Q_m}$$

Hence

$$q^4 - 4p'q^3 + 2(1 - up')q^2 + 2uq + 1 = 0 \tag{10–34}$$

or

$$p' = \frac{2}{2q + u}\left[\frac{1}{4}\left(q + \frac{1}{q}\right)^2 + \frac{u}{2q}\right]$$

By use of Eq. 10–18 this expression can be reduced to

$$2p'q^2 + (p'u - 2p^2)\, q - u = 0 \tag{10–35}$$

This form is a quadratic equation, which is simpler to solve than Eq. 10–34, when all the factors are known. Admittedly, p is a function of q, but as p varies slowly with q (especially when q is high), the equation can be solved by stages: First, it can be assumed that $p = 1$, and then a correction is introduced for p by use of Eq. 10–18.

From Eq. 10–35 very simple approximations can be obtained for the evaluation of q. The solution depends on the value of u, which in practice is finite. If an extreme case is taken where $u \longrightarrow \infty$, Eq. 10–35 is reduced to

$$p'q - 1 = 0$$

or

$$q = \frac{1}{p'} \tag{10–36}$$

At the other extreme, when $u \longrightarrow 0$, Eq. 10–35 becomes

$$2p'q^2 - 2p^2q = 0$$

or

$$q = \frac{p^2}{p'} \tag{10–37}$$

and the solution for q should in fact lie between the two extreme values obtained by Eqs. 10–36 and 10–37. When q is high, the solution obtained by these two

approximations do not greatly differ from each other. A satisfactory method to compute Q_e would be to find q from Eq. 10–36, then find a corrected value by successive use of Eq. 10–37 (the calculation of p being based on Eq. 10–18). Q_m has to be found, whereupon $Q_e = qQ_m$.

Example 8

The minimum-cost batch size is known to be 2,000 pieces. Find the economic batch size when the sales price yields a factor $p' = 1.40$.

Solution

From Eq. 10–36,

$$q = \frac{1}{p'} = \frac{1}{1.40} = 0.71$$

The factor p, by Eq. 10–18 is

$$p = \frac{1}{2}\left(q + \frac{1}{q}\right) = \frac{1}{2}(0.71 + 1.40) = 1.06$$

The corrected value for q, by Eq. 10–37, is

$$q = \frac{p^2}{p'} = \frac{(1.06)^2}{1.40} = 0.80$$

This value results in a modified value for p:

$$p = \frac{1}{2}\left(0.80 + \frac{1}{0.80}\right) = 1.025$$

Hence

$$q = \frac{p^2}{p'} = \frac{(1.025)^2}{1.40} = 0.75$$

A third step will bring us nearer to the desired value:

$$p = \frac{1}{2}\left(0.75 + \frac{1}{0.75}\right) 1.04$$

$$\therefore \qquad q = \frac{(1.04)^2}{1.40} = 0.77$$

A further check will show that this is the solution of Eq. 10–37. The real value of q lies between 0.71 and 0.77, depending on the magnitude of u. If u is high, a value for q nearer to 0.71 should be selected, while for a small u, the solution for q will be nearer 0.77. When $u = 2$, the value of q is about midway between the solutions obtained from Eqs. 10–36 and 10–37; in our case, 0.74. Suppose we have $u = 10$. We should select a value for q between 0.71 and 0.74. Take for instance, $q = 0.73$. Therefore

$$Q_e = qQ_m = 0.73 \times 2{,}000 = 1{,}460 \text{ pieces}$$

If the full equation, Eq. 10–34, is solved, it is found that $q = 0.72$, leading to the conclusion that the approximation method yields a very satisfactory result.

Another method for computing the economic batch size is given by Fig. 10–12, where the factor p' is plotted against q with u as a parameter. The curves are

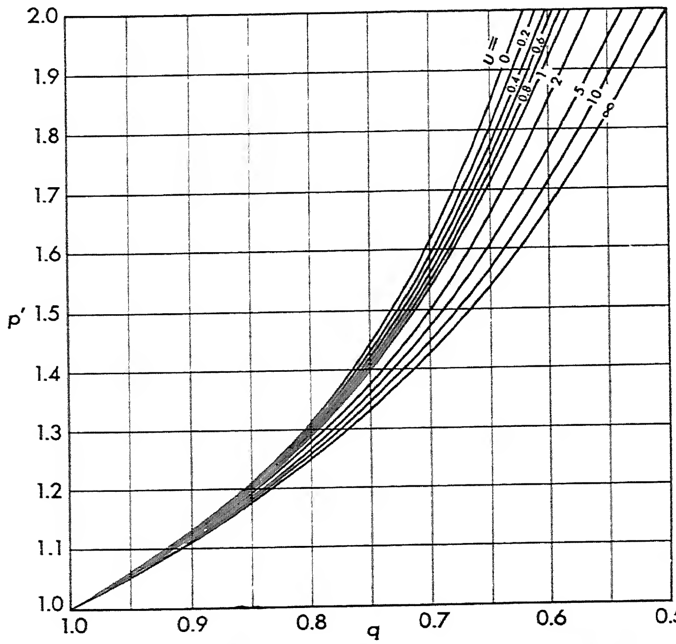


Figure 10–12. ***Non-dimensional curves for computing the economic batch.***

based on Eq. 10–34 and provide a simple and convenient means for finding q when p' and u are known. The curves lie between two limits: one where $u = 0$ (i.e., when Eq. 10–37 applies) and the other where $u = \infty$ (i.e., when Eq. 10–36 applies). All start at the same point ($q = 1.0$; $p' = 1.0$), thereby showing how manufacturers are compelled to produce minimum-cost quantities when competition is severe and selling prices are comparatively low. The role of u is insignificant in this region, but its effect becomes more and more noticeable as the selling prices increase.

Figure 10–13 has a similar series of curves, except that the ratio Y'/Y_m is plotted against q. The relation between Y'/Y_m and p' is readily obtained from Eqs. 10–22 and 10–23:

$$\frac{Y'}{Y_m} = \frac{u + 2p'}{u + 2}$$

If the ratios Y'/Y_m and u are known, the value of q is immediately obtained

without necessarily having to calculate p'. It is clear from Figs. 10–12 and 10–13 that reduction of setup costs will reduce the economic batch size, both because q becomes smaller (owing to reduction in u) and because Q_m is smaller (see Eq. 10–9).

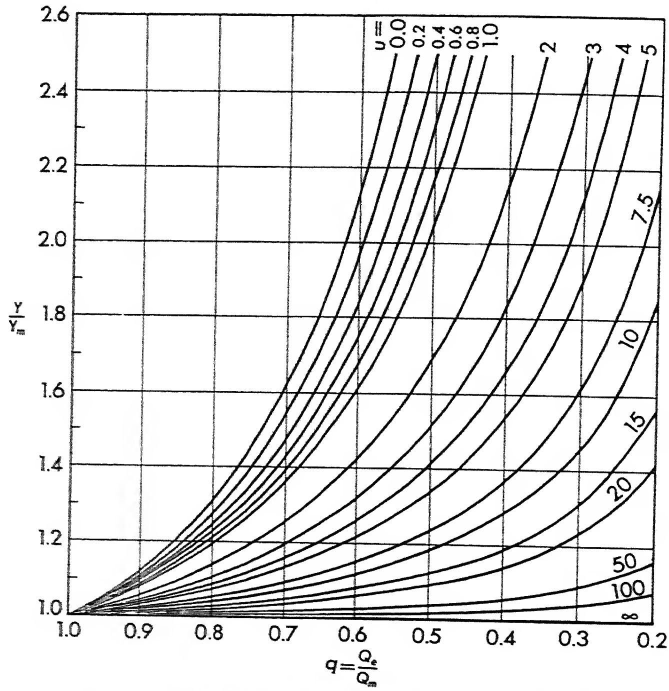


Figure 10–13. ***Non-dimensional curves for computing the economic batch.***

It is interesting to study the variations in rate of return when a batch size is selected. A convenient basis for comparison would be Q_m. The rate of return has been defined as

$$R = \frac{\eta}{T_c} = \frac{\eta}{Q} a_c$$

When Q_m is produced,

$$R_m = \frac{\eta_m}{Q_m} a_c$$

Hence

$$\frac{R}{R_m} = \frac{\eta}{\eta_m} \frac{Q_m}{Q} = \frac{\eta}{\eta_m} \frac{1}{q} \tag{10–38}$$

As $(\eta/\eta_m) < 1$, it is clear that producing batches larger than Q_m will adversely affect the rate of return.

The adoption of the economic batch size, leading to the production of a smaller batch than Q_m, has some noteworthy features when compared with Q_m:

1. As in the case of the maximum-profit batch size, higher than Y_m production costs per piece are incurred. However, now we are below Q_m, where the costs function is steep (Fig. 10–3) and far more sensitive to the batch size than it is beyond the point Q_m.
2. Less capital is tied to a particular production run.
3. Production time is shorter; hence production techniques may not get a chance to become fully developed as in long runs, when production rates could be greatly improved. Also, short runs increase the relative length of setup time and nonproduction time of the equipment.

Comparison of the Various Criteria[4]

Some of the implications resulting from the selection of batch sizes were discussed above, but it is useful to illustrate some of these effects by an example: Suppose the parameter $u = 5.0$ (i.e., the variable costs constitute 28.6 per cent of the total production costs) and $p' = 1.40$ (the sales price in this case being 11.4 per cent above Y_m). The effect of batch size on the variable costs factor p, production costs per piece, costs per batch, profit, profit/cost of production, and rate of return is given in Table 10–1, and some of these functions are shown in Fig. 10–14, the results for Q_m being taken as 100 per cent.

Table 10–1

Effect of Selecting a Lot Size

($p' = 1.40$; $u = 5.0$)

$q = \frac{Q}{Q_m}$	*(Variable costs):(Min. var. costs)* p	*Costs per piece,* Y	*Cost of Production*	*Profit*	*Profit / Cost of Prod.*	*Rate of Return*
0.50	1.250	107.1	53.6	18.8	35.1	70.2
0.60	1.133	103.8	62.2	40.0	64.2	107.0
0.70	1.064	101.8	71.3	58.8	82.3	117.6
0.73	1.050	101.4	74.1	63.9	86.2	118.1
0.80	1.025	100.7	80.6	75.0	93.1	116.4
0.90	1.005	100.1	90.1	88.8	98.6	109.6
1.00	1.000	100.0	100.0	100.0	100.0	100.0
1.10	1.005	100.1	110.1	108.6	98.6	89.6
1.20	1.017	100.5	120.6	114.9	95.3	79.4
1.30	1.034	101.0	131.3	119.0	90.6	69.7
1.37	1.050	101.4	138.9	119.9	86.3	63.0
1.40	1.057	101.6	142.2	120.1	84.5	60.4
1.50	1.083	102.3	153.5	118.9	77.4	51.6
1.60	1.113	103.2	165.1	114.8	69.6	43.5
1.70	1.144	104.1	177.0	108.8	61.5	36.2
1.80	1.178	105.1	189.2	99.9	52.8	29.3
1.90	1.213	106.1	201.6	88.8	44.1	23.2
2.00	1.250	107.1	214.2	75.0	35.0	17.5

[4] This section may be omitted at first reading.

An increase of 25 per cent in variable costs causes in our case an increase of 7.1 per cent in total production costs. The cost of production obviously increases with the batch size, but owing to the increase in costs per piece, the cost of production is more than doubled when the minimum-cost batch size is doubled ($q = 2.0$). The profit function is maximum at $q = 1.4$ and is 20 per cent higher than the profit at Q_m, but cost of production tied to the production run is 42 per cent higher, the ratio of profit to cost of production is down 15 per cent and rate of return is 40 per cent lower and only about a half of its maximum. Maximum rate of return is obtained at $q = 0.73$, and these two batch sizes are approximately the limits of the production range for $p = 1.05$. Ratio of profit to cost of production for the economic batch size is about the same as for the other end of the range, but the cost of production for the batch is almost halved.

The advantages of the economic batch size are self-evident from Fig. 10–14,

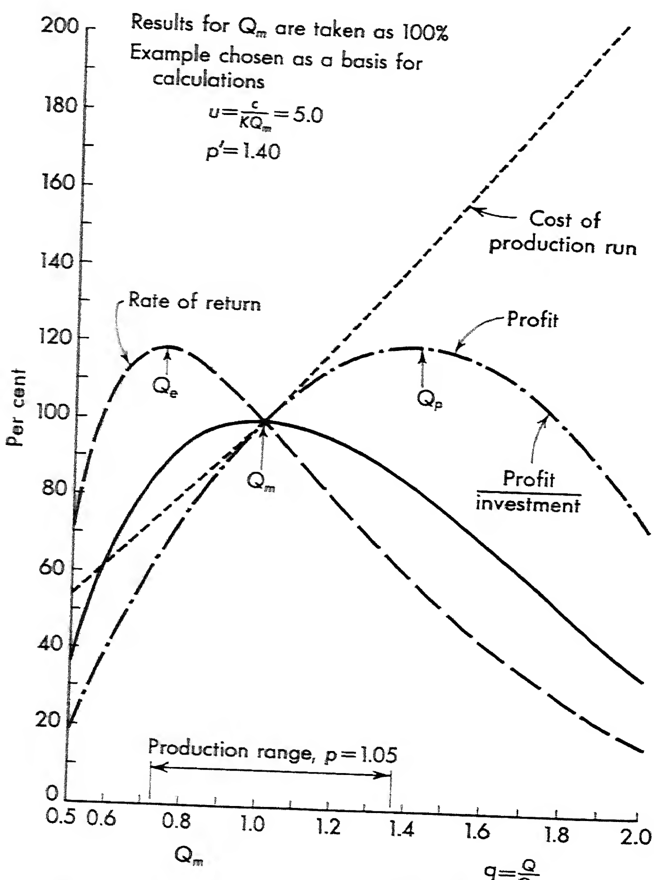


Figure 10–14. ***Some effects of selecting batch sizes.***

and unless special benefits are to be gained from a larger batch (such as better production methods, or envisaged changes in labor and materials costs), it is preferable to the other criteria for the choice of batch sizes. The function of rate of return is, however, very sensitive to the batch size, and this fact leaves little room for flexibility in the form of production ranges for scheduling purposes. Moreover, below the point of the economic batch, both the return and rate of return deteriorate very quickly. A change in pricing policy may adversely affect both the return and rate of return.

It would appear from the above that in most cases the best production range should probably lie between Q_e and Q_m, provided the production costs at Q_e are not too high. This range, however, may prove to be too narrow and may have to be expanded when scheduling difficulties arise. The range Q_e to Q_m may be termed *the economic production range.*

Example 9

The sales price of a household article is \$11.50. It has been established that minimum production costs of \$9.75 can be attained when a batch of 10,000 is scheduled. State the economic production range, if management's policy clearly states that production costs should not exceed \$10.00 per piece. It has also been established that $u = 7.5$.

Solution

The given data are as follows:

$$\begin{aligned} Y' &= \$11.50 \\ Y_m &= \$9.75 \\ Y &\leqslant \$10.00 \\ Q_m &= 10{,}000 \text{ pieces} \\ u &= 7.5 \end{aligned}$$

The economic quantity is obtained from Fig. 10–13:

$$\frac{Y'}{Y_m} = \frac{11.50}{9.75} = 1.18$$

This value corresponds to $q = 0.56$ (when $u = 7.5$), or

$$Q_e = q \times Q_m = 0.56 \times 10{,}000 = 5{,}600 \text{ pieces}$$

Had there been no restrictions on the production costs, the economic range would have been defined as between 5,600 to 10,000 pieces. It is necessary to check whether this range complies with the additional restriction.

If Q_e is produced, the p factor would be (by Eq. 10–18)

$$p = \frac{1}{2}\left(q + \frac{1}{q}\right) = \frac{1}{2}\left(0.56 + \frac{1}{0.56}\right) = 1.17$$

From Eqs. 10–22 and 10–23, we know that

$$\frac{Y}{Y_m} = \frac{u + 2p}{u + 2}$$

Hence, when Q_e is produced, we shall have

$$\frac{Y}{Y_m} = \frac{7.5 + 2.34}{7.5 + 2.0} = 1.035$$

The restriction in our case states that

$$\frac{Y}{Y_m} \leqslant \frac{1000}{9.75} = 1.025$$

Hence the economic batch cannot be produced without causing too high costs. To find the lowest acceptable batch, first find p from

$$\frac{Y}{Y_m} = \frac{u + 2p}{u + 2}$$

or

$$1.025 = \frac{7.5 + 2p}{9.5}$$

$\therefore$

$$p = 1.125$$

From Fig. 10–8 or by Eq. 10–19,

$$q = p - \sqrt{p^2 - 1} = 1.125 - \sqrt{(1.125)^2 - 1} = 0.61$$

or

$$Q = 6{,}100 \text{ pieces}$$

The economic range is therefore modified to 6,100 to 10,000 pieces.

Summary

In batch production a stock control is required to ensure that stock levels are adequate to meet demand but not too high as to cause excessive storage costs and low capital turnover. It is therefore desirable to include optimal batch sizes in the production schedule. Three optimal batch-sizes can be defined.

(i) A minimum-cost batch size, which ensures that the production costs per piece are minimum; this batch also yields a maximum ratio of profit to cost of production. The minimum-cost batch should be preferred in cases of keen competition or likely changes in pricing policies.

(ii) A maximum-profit batch size, which yields a maximum profit per batch and is larger than the minimum-cost batch; this batch involves, however, a high investment and results in a low capital turnover and low profit per annum.

(iii) An economic batch size, which yields a maximum rate of return and is smaller than the minimum-cost batch; this batch should be preferred, unless special benefits are to be gained from longer production runs.

The computation of optimal batch sizes by itself is not enough, since a certain amount of flexibility has to be allowed for scheduling purposes. This flexibility is achieved in the form of permissible deviations from the optimal values by defining production ranges within which any batch size may be safely selected. There are two such ranges.

(i) A production range, which is defined by the permissible increase in total production costs above the minimum costs; this range extends from a value below the minimum-cost batch size to a value well above it.

(ii) An economic production range, which is defined by the smaller of the two following ranges:

(a) A production range determined as in (i) but having the minimum-cost batch as its upper limit.

(b) A range having the economic batch and the minimum-cost batch as its two extreme limits.

The economic range ensures that the final selection of the batch size results in a better rate of return; however, if the limitations by which the range is defined are too tight, little room for flexibility is left.

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Problems

1. The production of an article involved machine setup and preparation costs amounting to \$12,000. The constant costs for each article were

Materials costs	15 cents
Labor costs	4 cents
Overhead	6 cents

The interest paid on the capital was 12 per cent. The time taken on the first operation was 8 min., while the total time required for all operations was 100 min. The volume of the article is given as 0.01 cu. ft., and the rate of consumption was uniform at 20 per day. It was decided to divide the produced batch into three lots, each of which is finished before the subsequent batch is started. Assume that the setup costs do not increase appreciably by the division of the batches into lots.

Compare the minimum-cost batch sizes obtained by the use of formula 10–9, and Camp and Raymond formulae, when the rate of production is given as (i) 500 per day, (ii) 200 per day, and (iii) 80 per day, and when for each of these cases the storage charge is: (a) 0.1 cent per cu. ft. per day; (b) 0.2 cent per cu. ft. per day; (c) 0.3 cent per cu. ft. per day.

Demonstrate the effect of these variables on the results by plotting the batch size against γ when $b = 0.1$ cent per cu. ft. per day and against b when $a_p = 200$ per day.

Find the total costs per piece according to the three formulae for the case $b = 0.1$ cent per cu. ft. per day when $a_p = 200$ per day.

Note: Assume there are 300 working days in a year.

2. In connection with batch production, the following data is known:

Preparation costs: \$1,500.
Rate of production: 2,000 per day.
Rate of consumption (linear): 500 per day.
Storage charges: \$3.00 per piece per annum.
Materials, labor, and overhead costs: \$2.00.
Rate of interest: 9% per annum.

(i) Find the effect of splitting the batch into lots (up to 10 in number) by using Raymond's formula. Plot batch size against number of lots and compare these results with formula 10–9. Assume:

$$k = 0.6$$
$$A = 1 + 0.05(N\text{–}1)$$

The setup costs will increase with the number of lots introduced, being 1,400 + $100N$ dollars.

(ii) Find the total costs per piece when Q_m is produced.
(iii) Find the rate of return (for $N = 1$).
(iv) Calculate the cost of the production run.

3. In Problem 2 the sales price is 20 per cent above the minimum costs per piece.
(i) Find the economic batch size (for $N = 1$).
(ii) Calculate the production costs per piece.
(iii) Find the rate of return.
(iv) Find the cost of the production run? Compare these results with those obtained in Problem 2.

4. The constant costs per piece are known to be \$4.00, the carrying charges factor $K = 0.5 \times 10^{-3}$ dollars per unit per day, and the setup costs per batch are \$10,000.

Find the production range if the allowable increase in total costs per piece is 2 per cent above the minimum costs per piece.

5. In Problem 4 define the economic production range, if the sales price is fixed at 16 per cent above the minimum costs per piece.

6. In Problem 4: (i) define the economic production range, if the sales price can be fixed at 10 per cent above the production costs; (ii) find whether the maximum-profit batch size lies in the production range when the sales price is fixed at 10 per cent above the production costs.

7. (i) Show that when a batch Q is produced, the ratio of profit to that which might result if Q_m were produced is

$$\frac{Z}{Z_m} = q\,\frac{p' - p}{p' - 1}$$

(ii) Find when this ratio becomes a maximum.

(iii) Given $p' = 1.4$, plot the change of Z/Z_m with q.

8. The economic batch and the maximum-profit batch involve approximately the same production costs per piece. Discuss this statement.

9. An analysis of the production costs of a shaft produced in batches revealed the following data:

Constant costs per piece: \$2.20.
Minimum total costs per piece: \$5.20.
Setup and preparation costs per batch: \$4,000.

Due to difficulties in scheduling, it was decided that the total costs per piece should be allowed to increase by a maximum 6 per cent.

(i) Find the economic production range of batch sizes that could be scheduled.

(ii) If the setup costs could be halved, how would this range be affected?

(iii) If the sales price is \$8.00 per unit, find the quantity that yields maximum profit. Should this batch size be adopted? Why?

10. A batch Q is produced during the period T_p at the rate of a_p units per unit time. The consumption takes the form of $y = A + Bx^n$, where y represents the number of articles in stock and x is the time. Assume an initial consumption of $-ma_c$, where a_c represents the theoretical uniform consumption and $m > 0$.

(i) Find the general formula of the consumption curve.

(ii) If T_c is the consumption period of the stock Q for the case of a uniform consumption (i.e., $n = 1$ in the formula and $x = T_p/T_c$), show that for any value of n:

$$T'_c = T_p\left[\frac{n}{m\gamma} + 1\right]^{1/n} - T_p$$

Assume:

$a_p = 200$ articles per day
$a_c = 20$ articles per day
$Q = 6{,}000$
$m = 1$
$n_1 = \frac{1}{2},\ n_2 = 1,\ n_3 = 2$

Compare the consumption periods for the three cases.

(iii) For $m = 1$, show how the minimum-cost batch size can be found.

Assume:

$a_p = 200$ articles per day
$a_c = 20$ articles per day
$n_1 = \frac{1}{2},\ n_2 = 1,\ n_3 = 2$
Setup costs, $s = \$600$
Interest rate, $i = 10\%$ p.a.
Cost per piece, $c = \$0.50$

Find Q for these three cases.

(iv) For the data given in (iii) find graphically the value of the time interval (between two successive production periods) and the value of the minimum stock Q_0.

11. A product, for which it is known that $u = 4.0$, is produced in a quantity which results in an increase of 6 per cent of the variable costs above the minimum. What increase of total costs above Y_m should be expected?

12. Plot curves to show how the ratios η/η_m and R/R_m change with q.

13. A stock Q is produced at a rate of a_p units per day for a period T_p. It is then necessary to leave the batch in stock for a period T_d, during which sorting, inspection, and painting are carried out. A quantity Q_1 is then supplied to the assembly line at the rate of a units per day for a period T_c. The supply to the assembly line is intermittent, so that after a supply period T_c, there is an interval T_0 before supply is resumed for another period T_c, and so on.

(i) Draw the change in stock level for the four cases:

$$T_0 > T_p + T_d$$
$$T_p + T_d > T_0 > T_d$$
$$T_d > T_0 > 0$$
$$T_0 = 0$$

(ii) If it is desirable to have minimum costs per unit, what is the optimal batch Q that should be aimed at in each case?

(iii) What is the level of minimum stock in each case?

14. In the previous problems, T_d was held constant. Suppose that it actually varies linearly with the batch size Q (due to the fact that when a larger quantity is produced, more time is required to complete the various necessary operations in the store); how would you determine the optimal batch size?

15. A survey of the stock control policy in a plant producing furniture by mass production methods yielded the following data:

A batch of components Q is produced at a rate of a_p units per day for a period T_p and then the batch has to be delayed in the store for a delay period T_d for marking, polishing, and inspection before it can be issued to the assembly line. If the supply to the assembly line is continuous (at the rate of a units per day), and if it is undesirable for the stock level to come down below a safety level Q_0, find the optimal batch size for minimum costs per unit. How would you present your solution to the production department? Detail your suggestions by adding an appropriate drawing of stock-level changes and by explaining how your solution could be applied.

For one component the rate of production is 1,000 per week; rate of supply, 120 per week; the delay period, $T_d = 2\frac{1}{2}$ weeks. If $s = \$720$, $I = \$0.082$ unit per day, $B = \$0.0014$ unit per day, and $c = \$4.08$ per unit,

(i) Find the optimal batch size.

(ii) What is the minimum safety stock Q_0?

(iii) Plot the function of unit costs.

(iv) What are the limits of the production range if the costs per unit should not exceed 0.25 per cent above the minimum costs?

11[1]

BATCH-SIZE DETERMINATION UNDER BOUNDARY CONDITIONS

Considerations of the optimal batch size in the preceding chapter were based on the assumptions that

(i) The constant cost term, c, remains constant, irrespective of the quantity produced.

(ii) The carrying costs factor, K, is unaffected by the batch size (the carrying cost term KQ continuously increasing with Q).

(iii) The cost of preparation, s, for production of a batch is the same, whatever the size of the batch.

In this chapter we shall examine these assumptions in more detail; it would particularly be useful to study the effect of abrupt changes in any of these three factors on the total production costs function, on the optimal batch size, and on the production range. Since the optimal batch sizes judged by various criteria are all related to the minimum-cost batch size Q_m, we shall confine our remarks in this chapter to Q_m.

Abrupt Changes in the Constant Cost Term, c

The constant cost term, c, was defined as the cost of materials, labor, and certain overhead, and it was assumed that the cost of these items increases linearly with the quantity produced, so that the cost per unit remains constant.

In practice, however, we find that c is likely to be affected by the quantity Q. As the quantity planned for production increases, there is more scope for improving work methods, investing in better aids to production, and planning the work-place layout and the flow of materials and products. These improvements affect the preparation costs per batch s, but they may often lower the labor costs and hence the term c. Similarly, the materials cost content may be lowered if discounts are allowed when larger quantities are purchased. The reduction in c may take two forms (Fig. 11–1):

1. Cost factor, c, may slowly decline as the quantity Q increases; say, as a linear function (which may often be considered a reasonable approximation if

[1] This chapter may be omitted at first reading.

the range of Q that we are interested in is not too wide). In this case c may be expressed as

$$c = c_0 - \beta Q$$

where β describes the rate of decline in c. The total production costs per unit become

$$Y = c_0 - \beta Q + \frac{s}{Q} + KQ$$
$$= c_0 + \frac{s}{Q} + (K - \beta)Q$$

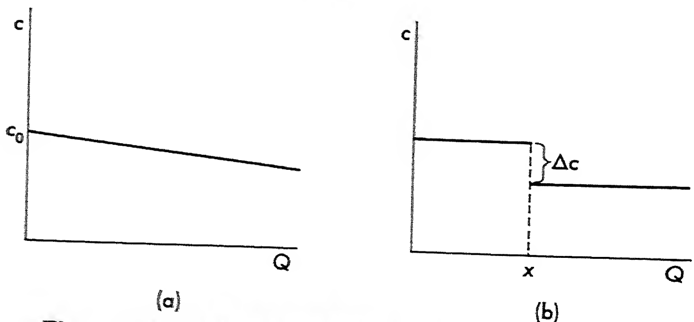

Figure 11–1. ***Changes in the cost term, c:***

(a) Gradual (linear) change, $c = c_0 - \beta Q$
(b) Abrupt change

The analysis of the minimum-cost batch size is not fundamentally affected, since the cost function is still a continuous one and all we have to do is to substitute c_0 for c and $K - \beta$ for K in the results obtained in the preceding chapter. Hence the minimum-cost batch size would be

$$Q_m = \sqrt{\frac{s}{K - \beta}}$$

2. Cost c may abruptly change at a certain quantity x and decline from c_1 to c_2. This often occurs because of discounts allowed in the purchase of materials, the cost per unit being constant until x is reached but subsequently reducing to a lower level. There may, in fact, be more than one "break"[2] in the cost term c, depending on the discount terms. If the cost of preparation s and the carrying cost factor K are unaffected, the total cost function (Fig. 11–2) is

For $Q < x$: $$Y_1 = c_1 + \frac{s}{Q} + KQ$$

For $Q \geqslant x$: $$Y_2 = c_2 + \frac{s}{Q} + KQ$$

[2] The term *price break* was used in "Introduction to Operations Research," Chapter 9, by C. W. Churchman, R. L. Ackoff, and E. L. Arnoff (John Wiley & Sons, Inc., 1957).

Evidently, the point at which the total costs become minimum does not change, as $Q_m = \sqrt{s/K}$ for both cases.

The total cost function follows curve 1 up to $Q = x$ and then follows curve 2. The questions that we have to study are: When is the cost function at x lower than the minimum cost Y_m, and what effect does the price break have on the production range?

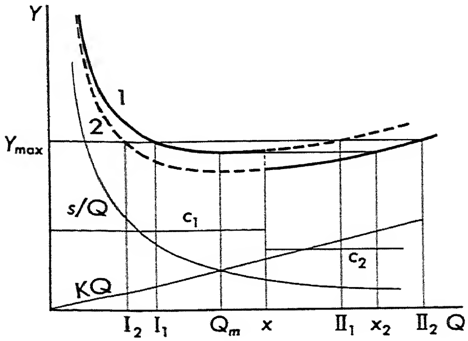

Figure 11–2. ***Total cost function with an abrupt break in c.***

Clearly (Fig. 11–2), the mathematical optimum (derived from $dY/dQ = 0$), which is located at Q_m, is on curve 2 when $Q_m > x$ and on curve 1 when $Q_m < x$. In any event the cost function at x is higher than the costs at the optimal point of the second curve ($Y_x > Y_{2m}$). Hence, in the first case (namely, $Q_m > x$), the minimum costs would occur at Q_m. In the second case ($Q_m < x$), Y_x would be lower than the costs at Q_m, provided the reduction in c is large enough, as

$$Y_x = c_2 + 2p_2KQ_m$$

and

$$Y_{1m} = c_1 + 2KQ_m$$

where the suffixes 1 and 2 relate to curves 1 and 2, respectively.

The relation $Y_x \leqslant Y_{1m}$ applies when

$$c_2 + 2p_2KQ_m \leqslant c_1 + 2KQ_m$$

or

$$c_1 - c_2 = \Delta c \geqslant 2KQ_m(p_2 - 1) \tag{11-1}$$

If this condition is satisfied, we can proceed to find that quantity x_2 corresponds to Y_{1m} (see Fig. 11–2), so that by specifying any quantity between x and x_2, we shall have total production costs per unit lower than those obtained at Q_m. Similarly to Eq. 11–1, $Y_{x_2} = Y_{1m}$ yields

$$\Delta c = 2KQ_m(p_2 - 1)$$

$$\therefore \qquad p_2 = 1 + \frac{\Delta c}{2KQ_m}$$

but
$$\frac{\Delta c}{KQ_m} = \frac{c_1 - c_2}{KQ_m} = u_1 - u_2 = \Delta u$$

$\therefore$
$$p_2 = 1 + \tfrac{1}{2}\Delta u$$

and
$$q = p_2 + \sqrt{p_2^2 - 1}$$

where
$$q = \frac{x_2}{Q_m}$$

or
$$q = 1 + \tfrac{1}{2}\Delta u + \sqrt{\Delta u + (\Delta u)^2} \tag{11-2}$$

as shown graphically in Fig. 11-3.

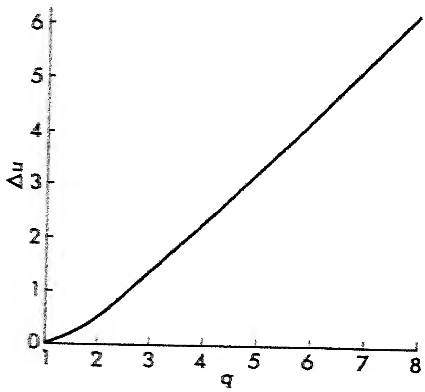

Figure 11-3. ***Quantity q yielding minimum costs Y_m when a reduction Δu occurs at the boundary.***

If a maximum level $Y_{\max}$ is not to be exceeded, the production range would have been Q_{I_1} to Q_{II_1}, had there been no break in c (curve 1), or Q_{I_2} to Q_{II_2}, had curve 2 been the only one to portray the change in total costs per unit, where (from Chapter 10)

$$Q_{I_1} = Q_m[p_1 - \sqrt{p_1^2 - 1}]$$
$$Q_{II_1} = Q_m[p_1 + \sqrt{p_1^2 - 1}]$$
$$Q_{I_2} = Q_m[p_2 - \sqrt{p_2^2 - 1}]$$
$$Q_{II_2} = Q_m[p_2 + \sqrt{p_2^2 - 1}]$$

and the relation between p_1 and p_2 is obtained from

$$p_1 = \frac{Y - c_1}{2KQ_m}$$

$$p_2 = \frac{Y - c_2}{2KQ_m}$$

or
$$p_2 = p_1 + \tfrac{1}{2}\Delta u \tag{11-3}$$

The limits of the production range would therefore be:

When $x < Q_{I2}$:

$$\text{Lower limit, } Q_{I} = Q_{I2}$$
$$\text{Upper limit, } Q_{II} = Q_{II2}$$

When $Q_{I2} < x < Q_{I1}$:

$$\text{Lower limit, } Q_{1} = x$$
$$\text{Upper limit, } Q_{II} = Q_{II2}$$

When $Q_{I1} < x < Q_{II1}$ (see Fig. 11–4):

$$\text{Lower limit, } Q_{I} = Q_{I1}$$
$$\text{Upper limit, } Q_{II} = Q_{II2}$$

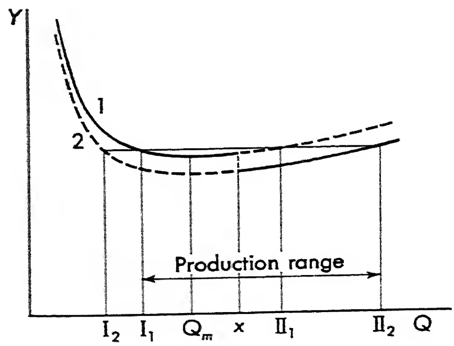

Figure 11–4. ***The production range in case of a break in c when $Q_{I1} < x < Q_{II1}$***

When, however, $Q_{II1} < x < Q_{II2}$, there would be two production ranges (see Fig. 11–5):

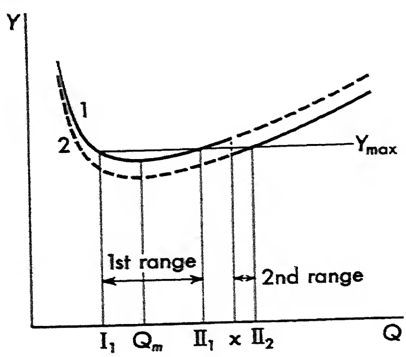

Figure 11–5. ***The production ranges in case of a break in c when $Q_{II1} < x < Q_{II2}$.***

First range: Lower limit, Q_{I1}
Upper limit, Q_{II1}

Second range: Lower limit, x
Upper limit, Q_{II2}

Finally, when $Q_{II_2} < x$:

$$\text{Lower limit, } Q_I = Q_{I_1}$$
$$\text{Upper limit, } Q_{II} = Q_{II_1}$$

These five situations are summarized in Fig. 11–6, which gives the upper and lower limits for each case.

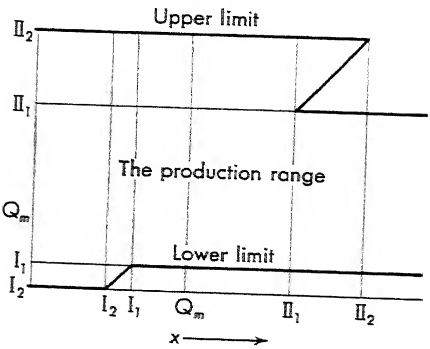

Figure 11–6. ***The production range when a break in c occurs at x.***

Breaks in the Carrying Costs Factor, K

When a certain quantity x is reached, we may find that a reduction in the varying cost factor becomes possible, owing to one or a combination of the following reasons:

- Better credit terms, leading to reduction in the interest charges per unit
- Reduction in the constant cost c; hence lower interest charges per unit
- With increased production we may offer better terms and expect an increase in the consumption rate
- At certain quantities it may pay to improve storage methods so that storage charges per unit may become lower.

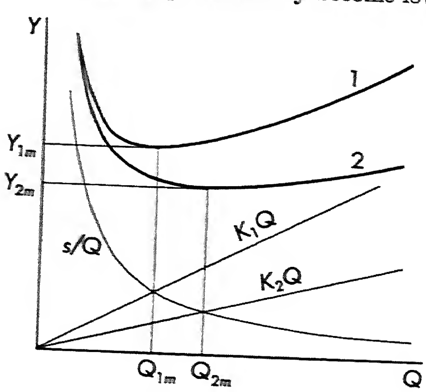

Figure 11–7. ***Production costs curves for the values of K.***

If the carrying costs factor K reduces at $Q = x$ from K_1 to K_2, we get a new total costs curve, as shown in Fig. 11–7. Q_{1m} is the minimum point on the first curve;

Q_{2m}, on the second. If we assume that c and s remain unchanged when we move from curve 1 to curve 2 because of a break in K_1, then

$$\frac{Q_{1m}}{Q_{2m}} = \frac{(s/K_1)^{\frac{1}{2}}}{(s/K_2)^{\frac{1}{2}}} = \left(\frac{K_2}{K_1}\right)^{\frac{1}{2}} = \kappa^{\frac{1}{2}} \tag{11-4}$$

where $\kappa = K_2/K_1$. As $K_2 < K_1$, it follows that $Q_{2m} > Q_{1m}$. It can also be shown that

$$Y_{2m} < Y_{1m}$$

The relation between the p factors is obtained from

(curve 1) $$p_1 = \frac{Y - c}{2K_1Q_{1m}}$$

(curve 2) $$p_2 = \frac{Y - c}{2K_2Q_{2m}}$$

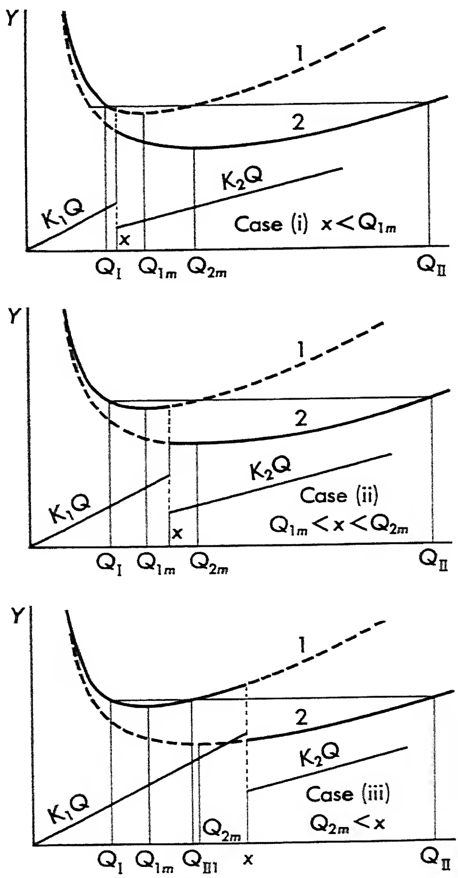

Figure 11–8. ***Three cases of a break in K.***

Hence
$$\frac{p_1}{p_2} = \frac{K_2 Q_{2m}}{K_1 Q_{1m}} = \left(\frac{K_2}{K_1}\right)^{\frac{1}{2}}$$

or
$$p_1 = p_2 \kappa^{\frac{1}{2}} \tag{11-5}$$

The total costs curve is a combination of curves 1 and 2, following curve 1 up to the break at $Q = x$ and then proceeding along curve 2. Three situations may arise (see Fig. 11–8).

Case (i)

$$x < Q_{1m}$$

Since any point on the portion left of curve 1 is above Y_{1m}, and as Y_{2m} is even lower, the optimal point is obviously at Q_{2m}. From the point of view of minimum costs per unit, it is therefore advantageous to operate beyond the breaking point.

Case (ii)

$$Q_{1m} < x < Q_{2m}$$

The combined total costs curve has now two optimal points, of which Q_{2m} would correspond to lower costs; so, again it is advantageous to operate beyond the breaking point in order to reduce production costs per unit. The interval on curve 2 below Y_{1m} is x to x_2, similar to the one shown in Fig. 11–2. The costs at x_2 are $Y_{x2} = Y_{1m}$. From Eq. 11–5,

$$p_1 = p_2 \kappa^{\frac{1}{2}}$$

but here
$$p_1 = 1$$

$\therefore$
$$p_2 = \kappa^{-\frac{1}{2}}$$

Since
$$q = \frac{x_2}{Q_{2m}} = p_2 + \sqrt{p_2^{\,2} - 1}$$

$\therefore$
$$\frac{x_2}{Q_{2m}} = \kappa^{-\frac{1}{2}} + \sqrt{\kappa^{-1} - 1}$$
$$= \kappa^{-\frac{1}{2}}(1 + \sqrt{1 - \kappa})$$

or, by Eq. 11–4,

$$x_2 = \frac{Q_{1m}}{\kappa}(1 + \sqrt{1 - \kappa}) \tag{11-6}$$

Case (iii)

$$Q_{2m} < x$$

This is somewhat similar to the case of Fig. 11–2. The total costs curve has only one mathematical minimum point (at Q_{1m}), but costs at x may be lower; again, if they are, the interval x to x_2 is better than Q_{1m} (x_2 being obtained by Eq. 11–6).

As to the production range, the results obtained for a break in c apply here as well, and essentially Fig. 11–6 represents these results graphically (but

instead of one optimal point Q_m, two should be marked when a break in K occurs). When the break is at x below I_2, the production range is between I_2 and II_2. The upper limit remains at II_2 until x reaches the point II_2, and then the upper limit falls to II_1. The lower limit is x between I_2 and I_1, and then remains at I_1.[3] Between II_1 and II_2 there are two ranges, but the second one narrows as x increases.

Breaks in the Preparation Costs, *s*

Investing in better production planning or production aids (tools, jigs, handling systems, loading and unloading devices, inspection methods, etc.) often means that there is an abrupt increase in the preparation costs s, and such additional expenditure may be worth while undertaking when a certain quantity $Q = x$ is reached. Suppose that c and K remain unchanged, but that the preparation costs increase abruptly from s_1 to s_2 (as, for example, in Fig. 11–9).

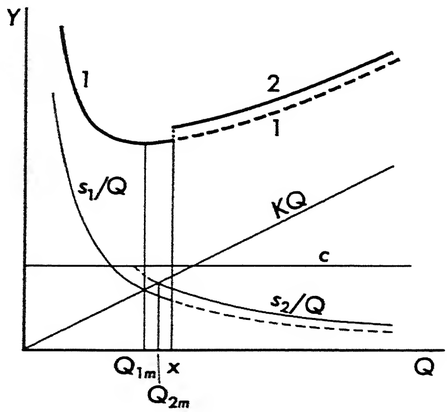

Figure 11–9. ***A break in s.***

Again, there are two mathematical optimal points, Q_{1m} on curve 1 (which involves s_1) and Q_{2m} on curve 2 (which involves s_2). It is simple to show that

$$\frac{Q_{2m}}{Q_{1m}} = \frac{p_1}{p_2} = \left(\frac{s_2}{s_1}\right)^{\frac{1}{2}} = \nu^{\frac{1}{2}} \tag{11–7}$$

where $s_2/s_1 = \nu$. The break in s may occur at three points:

Case (i)

$$x < Q_{1m}$$

The mathematical optimum would be at Q_{2m}, but the costs at x may be lower, provided

$$p_1 \leqslant \nu^{\frac{1}{2}} \tag{11–8}$$

where
$$p_1 = \frac{Y_x - c}{2KQ_{1m}}$$

[3] Note that Q_{II_1} and Q_{III_1} can be computed, provided $p_1 > 1.0$; otherwise $Y_{\max} < Y_{1m}$, which means that the whole of curve 1 is above the maximum allowable production costs.

If Eq. 11–8 is satisfied, an interval x_2 to x would involve lower costs than Y_{1m}; x_2 is derived in a similar way to expression 11–6, except that now

$$x_2 = Q_{2m}\left(1 - \sqrt{\frac{\nu - 1}{\nu}}\right) \tag{11–9}$$

Case (ii)

$$Q_{1m} < x < Q_{2m}$$

There are two optimal points at Q_{1m} and Q_{2m}, the second being the lower of the two. The interval x to x_2 below Y_{1m} is obtained as in (i).

Case (iii)

$$Q_{2m} < x$$

There is only one optimal point at Q_{1m}. As the portion of the cost curve beyond Q_{2m} involves higher costs than Y_{2m}, and as Y_{1m} is even lower, it follows that operating before the breaking point is advantageous.

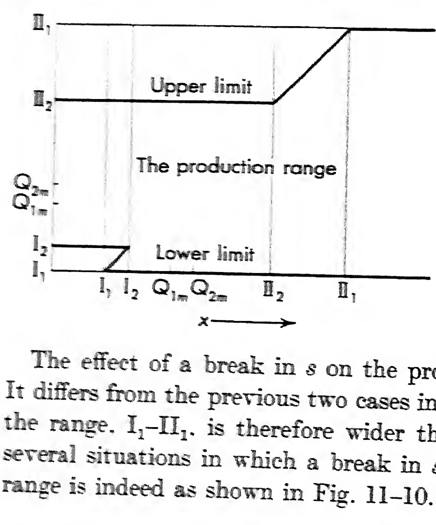

Figure 11–10. ***The production range for a break in s at x.***

The effect of a break in s on the production range is shown in Fig. 11–10. It differs from the previous two cases in that curve 2 is now above curve 1 and the range, I_1–II_1, is therefore wider than I_2–II_2. The reader can try to plot several situations in which a break in s occurs, to verify that the production range is indeed as shown in Fig. 11–10.

Combined Breaks

It was assumed above that when a break occurs, it applies to one parameter at a time, while the others remain constant. However, breaks in two or even three cost terms may often occur simultaneously. In fact a break in one parameter may often lead to a break in another; for instance, a reduction in cost of materials often implies a reduction in interest charges per unit, and similarly, an increase in preparations costs may reduce labor costs and thereby the interest charges.

It is useful, therefore, to study the effect of a simultaneous multibreaks situation where, at $Q = x$: c_1 reduces to c_2, K_1 reduces to K_2, and s_1 increases to s_2. The cases discussed above for single breaks may in point of fact be considered

special cases of this general problem, case 1 (a break in c) being derived when we put $K_1 = K_2$; $s_1 = s_2$; etc.

Curve 1 (prior to the break) is described by

$$Y_1 = c_1 + \frac{s_1}{Q} + K_1 Q$$

and curve 2 (after the break) by

$$Y_2 = c_2 + \frac{s_2}{Q} + K_2 Q$$

The relation between the two optimal points, Q_{1m} and Q_{2m}, is

$$\frac{Q_{1m}}{Q_{2m}} = \left(\frac{s_1/K_1}{s_2/K_2}\right)^{\frac{1}{2}} = \nu^{-\frac{1}{2}}\kappa^{\frac{1}{2}} \tag{11–10}$$

and as $\kappa < 1$ and $\nu > 1$, it follows that

$$Q_{2m} > Q_{1m}$$

If a certain $Y_{\max}$ is not to be exceeded, then

$$Y_{\max} = c_1 + 2p_1 K_1 Q_{1m} \quad \text{(on curve 1)}$$

$$Y_{\max} = c_2 + 2p_2 K_2 Q_{2m} \quad \text{(on curve 2)}$$

$$\therefore \qquad c_1 + 2p_1 K_1 Q_{1m} = c_2 + 2p_2 K_2 Q_{2m}$$

or

$$(u_1 + 2p_1) K_1 Q_{1m} = (u_2 + 2p_2) K_2 Q_{2m}$$

Substituting Eq. 11–10,

$$\frac{u_1 + 2p_1}{u_2 + 2p_2} = (\nu\kappa)^{\frac{1}{2}} \tag{11–11}$$

Special cases

1. A break in c only: $\nu\kappa = 1$; hence Eq. 11–3.
2. A break in K only: $u_1 = u_2\kappa^{\frac{1}{2}}$, $\nu = 1$; hence Eqs. 11–4 and 11–5.
3. A break in s only: $u_1 = u_2\nu^{\frac{1}{2}}$, $\kappa = 1$; hence Eq. 11–7.

Summary

Price breaks may occur at certain production or procurement quantities owing to improved production methods, to heavier expenditure in better production aids, and to discounts in the cost of materials (if bought in large enough quantities). The total costs curve is obtained as a combination of costs curves prior to and after the break, and if this break is abrupt, the final costs curve becomes discontinuous at the breaking point. Effects of price breaks on the optimal point and on the production range are worthy of study so that we may determine when and how to take advantage of such breaks. The treatment in this chapter is limited to those situations where abrupt breaks occur in the cost of materials, in carrying costs, in preparation costs, or in combinations of all these three factors.

Problems

1. In the case of a break in c, when condition 1 is satisfied, the interval x to x_2 is below Y_{1m} (Fig. 11–2). Show that if Δu is comparatively large

$$q = \frac{3}{2}\left(\Delta u + 1\right)$$

where

$$q = \frac{x_2}{Q_m}.$$

Find the error incurred by this approximation when $\Delta u = 3;\ 5;\ 10;\ 20$.

2. In the case of a discontinuity in the carrying costs factor, when K changes abruptly from K_1 to K_2 at a certain quantity $Q_{1m} < x_1 < Q_{2m}$ (while s and c remain unchanged), show that if $\kappa = K_2/K_1$,

(i)
$$\frac{u_1}{u_2} = \kappa^{\frac{1}{2}}$$

(ii)
$$\frac{Y_{1m}}{Y_{2m}} = \frac{u_2 + 2\kappa^{-\frac{1}{2}}}{u_2 + 2}$$

(iii) Prove that $Y_{2m} < Y_{1m}$ [not by (ii)].

3. For a break in s (Fig. 11–9), (i) prove Eqs. 11–8 and 11–9 in the text; (ii) show that $Q_{1m} > Q_{2m}$; (iii) show that $Y_{1m} < Y_{2m}$.

4. For the case of a break in c, K, and s, all occurring at $Q = x$ as described in "Combined Breaks," page 276;
   (i) Draw the total production costs function when the break occurs before Q_{1m}; between Q_{1m} and Q_{2m}; after Q_{2m}.
   (ii) If Y_1 is the total cost described by curve 1 (before the break) and Y_2 is the total cost described by curve 2 (after the break), state under what conditions $Y_2 < Y_1$ for any Q. Is it possible for the two curves to intersect, so that $Y_1 = Y_2$?
   (iii) Under what conditions is $Y_{2m} < Y_{1m}$?
   (iv) Suppose the condition for (iii) is satisfied; find the interval x to x_2 that would involve lower costs than Y_{2m}.
   (v) What effect would the break have on the production range if $Y_{2m} < Y_{1m}$ and if a certain $Y_{\max}$ should not be exceeded?

5. For a given product it is known that:

$$c = \$5.00$$
$$s = \$1{,}500$$
$$K = \$1.5 \times 10^{-3}$$

When $Q = 2{,}000$ units, there is a drop in c to $4.00; in K to 1.2×10^{-3}; and a rise in s to $2,000. For $Q = 4{,}000$ units, c reduces further to $3.80; K to 1.0×10^{-3}; and s rises to $2,500.
   (i) Plot the total production costs function.
   (ii) Find the production range when $p_1 = 1.05$; $p_1 = 1.10$, where $p_1 = (Y_{\max} - c_1)/(2K_1Q_{1m})$.
   (iii) Find the economic production range if the sàles price is given by $p'_1 = 1.20$.

6. A store orders a product for which the following data are given:

$$c = \$10.00$$
$$s = \$2{,}500$$
$$a_c = 20 \text{ per day}$$
$$\text{Interest, } i = 0.25 \times 10^{-3} \text{ per day}$$

Storage costs are negligible. The sales price is fixed at 20 per cent above the minimum point of total costs. Suppose that the rate of consumption increases linearly with reduction in the sales price, and that if the sales price were to reduce by 10 per cent, demand should increase by 20 per cent.

If a quantity of 12,000 were to be ordered, a break in c would be expected from \$10.00 to \$8.00.

(i) Plot the total costs function.

(ii) Comment on the effect of the break on the point of minimum costs and suggest a range in which p_1 will not exceed 1.06.

12

MACHINE CAPACITY

Once the production quantities have been ascertained (either by definite orders or through batch-size calculations in the case of batch production) and the operation sheets have been worked out, we are in possession of the basic data required for machine loading and scheduling. We know now:

1. The breakdown of operations and the sequence
2. How long each operation should take and therefore the time required for these operations for the whole order or batch
3. The type of machine or process capable of performing the required tasks.

In addition to these data we must have information about the dates on which

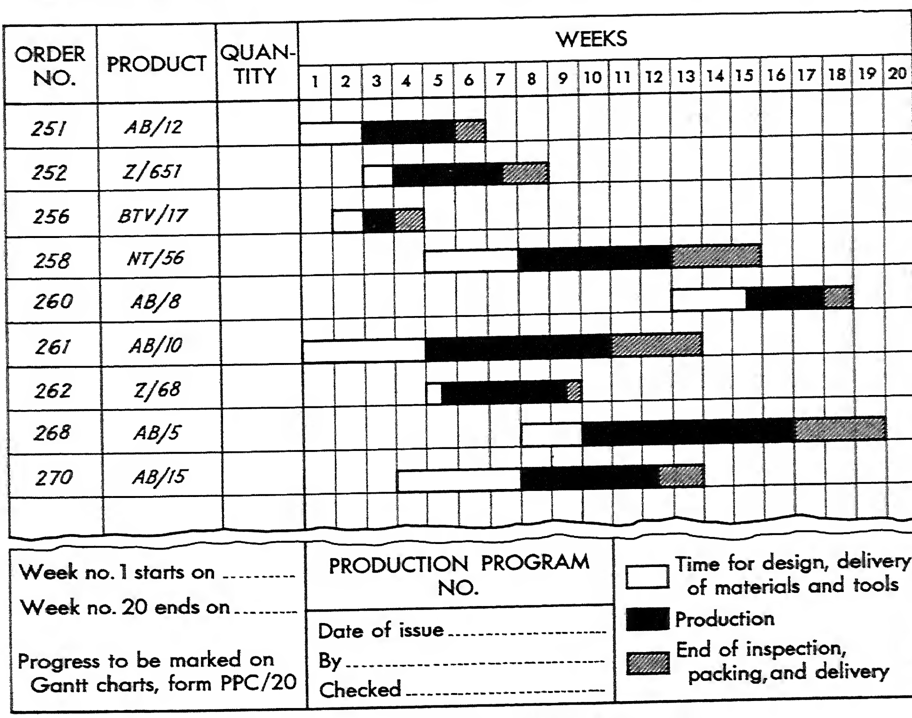

Figure 12–1. ***A master production program.***

the job should be finished. This information may be presented in a graphical form as a production program (see for example, Fig. 12–1). Machine loading is an attempt to match all these requirements with the available machine capacity, and a machine loading card (Fig. 12–2) is very useful for this purpose. The card —designed, say, for four weeks—contains information about the basic capabilities and specifications of the machine, data about its performance in the past

PERFORMANCE DATA
AVAILABLE WEEKLY HOURS: REGULAR ___ OVERTIME ___
AV. WEEKLY BREAKDOWNS __________ HRS.
AV. WEEKLY MAINTENANCE __________ HRS.
AV. PER CENT UTILIZATION __________ HRS.
(ALL AVERAGES BASED ON LAST FOUR WEEKS)

MACHINE LOADING CHART
SYMBOLS:
* PART OF THE JOB
→ TRANSFER TO NEXT WEEK
← TRANSFERRED FROM LAST WEEK

DESCRIPTION NO.

CAPACITY

WEEK ENDING *Jan. 20*				WEEK ENDING *Jan. 27*				WEEK ENDING				WEEK ENDING			
ORDER NO.	PART NO.	TIME HRS	BALANCE AVAILABLE	ORDER NO.	PART NO.	TIME HRS	BALANCE AVAILABLE	ORDER NO.	PART NO.	TIME HRS	BALANCE AVAILABLE	ORDER NO.	PART NO.	TIME HRS	BALANCE AVAILABLE
361	A/622	12	33	376	B/32	←13	32								
364	A/625	14	19	381	J/21	2	30								
372	B/29	7	12												
376	B/32	12*	0												
"	"	13→													

Figure 12–2. ***A machine loading card (calculations are based on the assumption that 45 machine-hours per week are available).***

(breakdowns, maintenance, etc.) to enable us to make appropriate allowances, and details about commitments already assigned to the machine. In determining machine or process capacity, we have to study such problems as: What is the relation between productive and nonproductive times? What is the effect of one process on another, when processing times differ? What restricts the machine output and what is the relation between machine capacity and plant capacity? These, in brief, are the problems discussed in this chapter.

Machine Output

Machine output is inversely proportional to the cycle time. If the cycle time is T minutes, the theoretical output per machine (Fig. 12–3) would be

$$Q_{th} = \frac{60}{T} \text{ units per hour} \tag{12–1}$$

Maximum output is therefore obtained when the cycle time is absolutely at its minimum value. Take for example the operation described in Fig. 12–4a, where an operator is supervising one machine. Both the operator and the machine are

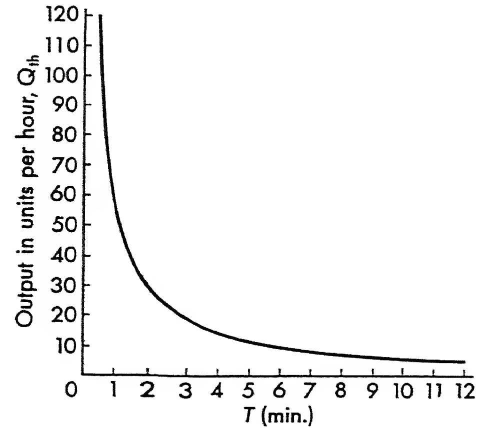

Figure 12–3. ***Theoretical machine output*** $Q_{th} = \frac{60}{T}$ ***(Units/hour).***

idle part of the time. The length of the cycle is determined by the total activity time, provided activities that are carried out concurrently are accounted for only once. In our case (Fig. 12–4a)

$$T = 0.2 \text{ (unloading)} + 0.4 \text{ (inspection)} + 0.4 \text{ (loading)} + 0.8 \text{ (machine running)}$$

$$= 1.8 \text{ min.}$$

and the output would be

$$Q_{th} = \frac{60}{1.8} = 33.3 \text{ units per hour}$$

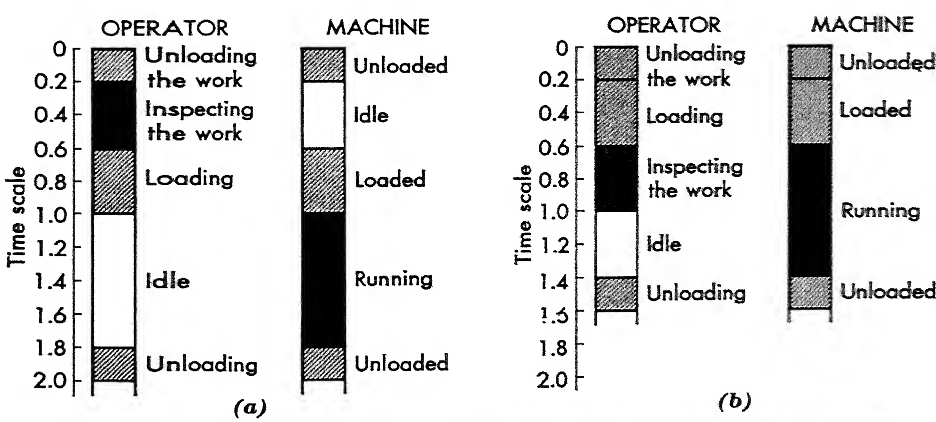

Figure 12–4. ***Reduction in cycle time by changing the sequence of operations.***
(a) Original work cycle $T = 1.8$ min.
(b) Improved work cycle $T = 1.4$ min.

The cycle can obviously be shortened by transferring the task of inspection so that it is carried out while the machine is running (assuming the machine requires only a limited amount of supervision while running), as shown in Fig. 12–4b. The cycle is now only $T = 1.4$ min. long, and the output is

$$Q_{\text{th}} = \frac{60}{1.4} = 42.8 \text{ units per hour}$$

We have not completely eliminated the idle time in the cycle (the operator is 0.2 min. idle), but the machine is fully busy, and the cycle time cannot be reduced any further without shortening the loading and unloading operations or increasing the speed at which the machine is running. The length of the cycle is therefore determined by the busy "partner" in the multiactivity chart, and that partner (the machine in Fig. 12–4b) causes the "bottleneck," so to speak, that restricts an increase in output.

Three periods of time in activity charts can be classified as follows:

Independent activity, which is performed by one of the partners without need for aid or services of another partner. Denote the independent activity per cycle of the machine by t.

Concurrent activity, which must be undertaken by several partners, each contributing his time or work to ensure that the task will be successfully completed. Denote the total length of the concurrent activity by a.

Idle time, which signifies that one partner is waiting for the other to complete his task. Denote the idle time per cycle by i_0 for the operator and i_m for the machine.

The cycle time would be

$$T = a + t + i_m \tag{12–2}$$

When the machine is the partner of longest independent activity and has no idle time, the cycle becomes

$$T = a + t \tag{12–2a}$$

which is the case in Fig. 12–4b.

How can we reduce the cycle time to a minimum? Some common methods are listed below:

1. Try to ensure that no idle time[1] is involved for the partner of longest activity by:

(i) Changing the sequence of tasks (as in Fig. 12–4).
(ii) Eliminating delays at end of cycles.

[1] The idle time of the other partners does not affect the output as long as the composition of partners in the multiactivity team does not change. However, this idle time is worth investigating, since output per machine or per man may increase by adding or deleting partners to the team.

2. Reduce the independent activity time by:
   (i) Increasing the running speeds, feeds, etc.
   (ii) Selecting a better machine for the job.
   (iii) Transferring a part of the operations to another machine or another operator (see example in Fig. 12–5, where the cycle is shortened by relieving the operator from the responsibility for inspection).

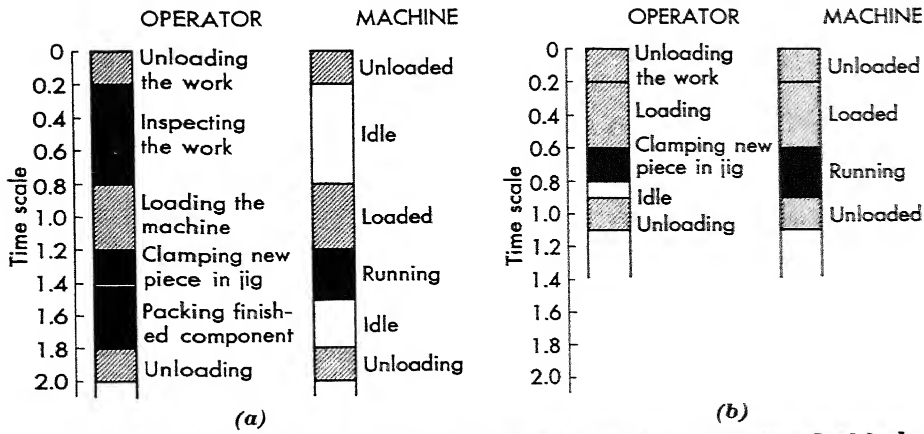

Figure 12–5. ***Reduction in cycle time by relieving operator of independent tasks.***
(a) Original work cycle $T = 1.8$ min.
(b) Improved work cycle $T = 0.9$ min.

3. Reduce the concurrent time a by:
   (i) Using better methods and jigs for feeding the work to the machine, for loading and setting.
   (ii) Setting the work and carrying out most of the preparatory tasks while the machine is running.
   (iii) Ejecting automatically the finished work from the machine.

The actual output Q tends to fall below the theoretical figure Q_m because of time lost in delays between cycles, adjustments, repairs, breakdowns, etc., so

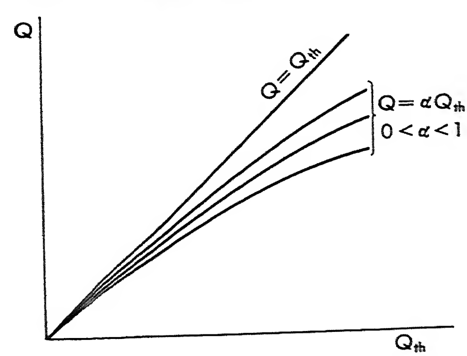

Figure 12–6. ***Actual output Q in relation to the theoretical output Q_{th}.***

that $Q = \alpha Q_{th}$, where $0 < \alpha < 1$ is a coefficient that describes quantitatively the discrepancy between the theoretical and actual output figures. Very often we find that the factor α is dependent on the cycle time (see Fig. 12–6), so that when the cycle is, say, halved, the output does not quite double.

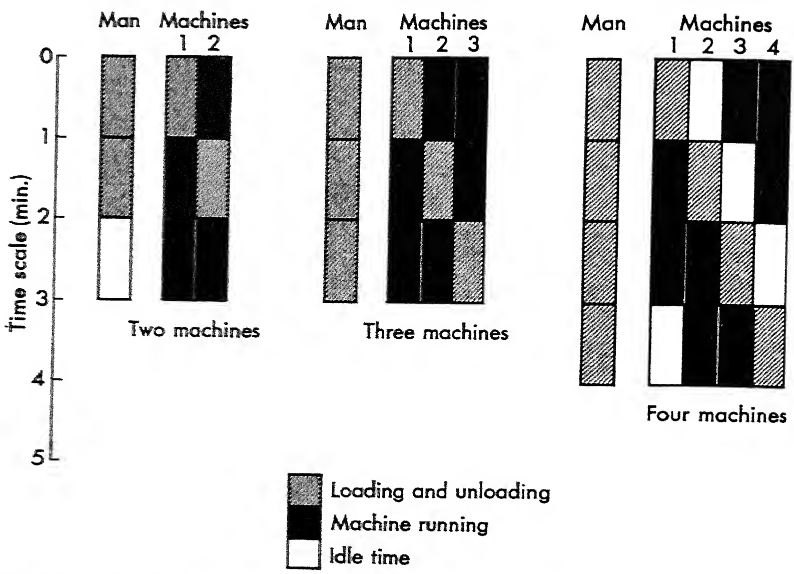

Figure 12–7. ***Assigning two, three, or four machines to one operator.***

Multimachine Supervision by One Operator

When a machine performs an operation automatically, so that only a limited amount of attention on the part of the operator is required (such as loading the machine, unloading, inspection, setting, and adjustments), it is sometimes possible to put several machines under the care of one operator. The operator and the machines form a production center, and the tasks of the operator are carried out in accordance with a certain sequence that ensures maximum utilization of the production cycle.

Consider the following example: An operator is busy loading a machine for 0.5 min. and unloading for 0.5 min. Between these two operations, the machine is running for 2.0 min. How many machines could this operator supervise? The total direct preparation time of the operator is 1.0 min., which is one-third of the total cycle time. If we assign two machines to the operator, he will be idle a third of the time; if we assign four machines, each machine will be idle a quarter of the time; with three machines we get perfect matching, no idle time being incurred either on the part of the operator or the machines (Fig. 12–7). Perfect matching, however, is not always possible; as, for example, when the

ratio of direct preparation time to running time is 1.0:2.5. In this case, operator idle time is incurred when the number of machines is two or three, while with four machines, the operator is fully occupied, but we get machine idle time.

In general terms, when the machine time is t and preparation time is a, the total cycle time is $T = a + t$, and the number of machines that should be allocated to one operator (assuming all the machines require the same preparation time and have the same running time) is

$$n' = \frac{a + t}{a} \tag{12-3}$$

If we select n' machines, there would be neither man nor machine idle time, but when n' is not a whole number and we have to select either n or $n + 1$ machines, where

$$n < n' < n + 1$$

perfect matching is not possible. The choice between n and $n + 1$ machines would be governed by cost analysis of the operation.

Let c_o be the cost of labor per operator hour and c_m the cost per machine hour. Suppose we choose n machines: The total cost per hour is $c_o + nc_m$. Since the output per machine is $60/(a + t)$ units per hour, the cost per unit is

$$Y_n = \frac{c_o + nc_m}{60/(a + t)n} = \frac{c_m}{60n} (\epsilon + n)T \tag{12-4}$$

Where Y_n is the cost per unit when n machines are chosen, and $\epsilon = c_o/c_m$ is the ratio between labor and machine costs.

When $n + 1$ machines are chosen, the total cost of the operation is $c_o + (n + 1)c_m$. The cycle time is now $(n + 1)a$, since the operator is fully occupied and the machines have to wait their turn until his services become available. The output per machine is $60/[(n + 1)a]$ units per hour, and the cost per unit is therefore

$$Y_{n+1} = \frac{c_o + (n + 1)c_m}{[60/(n + 1)a](n + 1)} = \frac{c_m}{60} (\epsilon + n + 1)a \tag{12-5}$$

Hence
$$\frac{Y_n}{Y_{n+1}} = \frac{\epsilon + n}{\epsilon + n + 1} \frac{T}{a} \frac{1}{n}$$

But
$$\frac{T}{a} = n'$$

$\therefore$
$$\frac{Y_n}{Y_{n+1}} = \frac{\epsilon + n}{\epsilon + n + 1} \frac{n'}{n} \tag{12-6}$$

When $\dfrac{Y_n}{Y_{n+1}} > 1$, choose $n + 1$ machines

When $\dfrac{Y_n}{Y_{n+1}} < 1$, choose n machines

The relation $Y_n/Y_{n+1} = 1$ is shown in Fig. 12–8, where n' is plotted against ϵ with n as a parameter, so that for known values of n' and ϵ, the appropriate number of machines to be allotted to one operator is indicated. If, for example, $a = 1.2$ min., $T = 6.9$ min. (hence $n' = 5.75$), the number of machines to be allotted would be five when $\epsilon = 0.8$, but six when $\epsilon = 5.5$. The curves in Fig. 12–8 are rather flat, which means that the solution is quite insensitive to changes in ϵ. Since the allocation of several machines to one operator affects the organization and layout of the production centers, it is important to know that when changes in the cost of labor or cost of operating machines occur, they are not likely to affect the number of machines allocated.

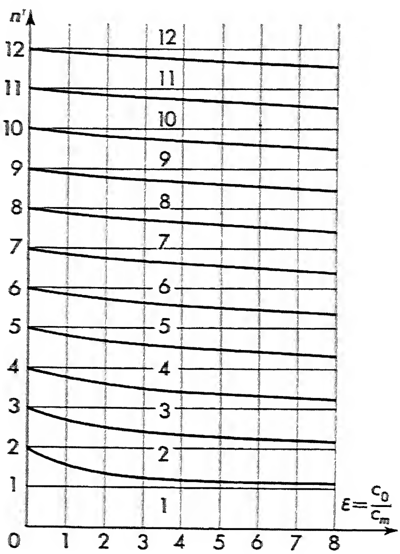

Figure 12–8. ***Allocation of a number of machines to one operator (under deterministic conditions).***

The amount of relative idle time resulting from selection of n or $n + 1$ machines can easily be calculated. When n machines are chosen,

$$\text{Operator idle time} = \text{cycle time} - \text{operator time} = T - na$$

and in relation to the cycle it is

$$\frac{i_o}{T} = \frac{T - na}{T} = 1 - n\frac{a}{T} = 1 - \frac{n}{n'}$$

When $n + 1$ machines are chosen, the operator is fully busy and each machine is idle $(n + 1)a - T$ each cycle, the relative idle time i_m for each machine is

$$\frac{i_m}{T} = \frac{(n + 1)a - T}{T} = (n + 1)\frac{a}{T} - 1 = -\left(1 - \frac{n + 1}{n'}\right)$$

The two expressions for i_o and i_m are similar and could be combined into

$$\frac{i}{T} = 1 - \frac{n}{n'} \tag{12-7}$$

where n now signifies the number of machines selected. If $n \leqslant n'$ machines are taken, $i \geqslant o$ is the operator idle time; if $n \geqslant n'$ are taken, $i \leqslant o$ is the machine idle time. Figure 12–9 shows i/T.

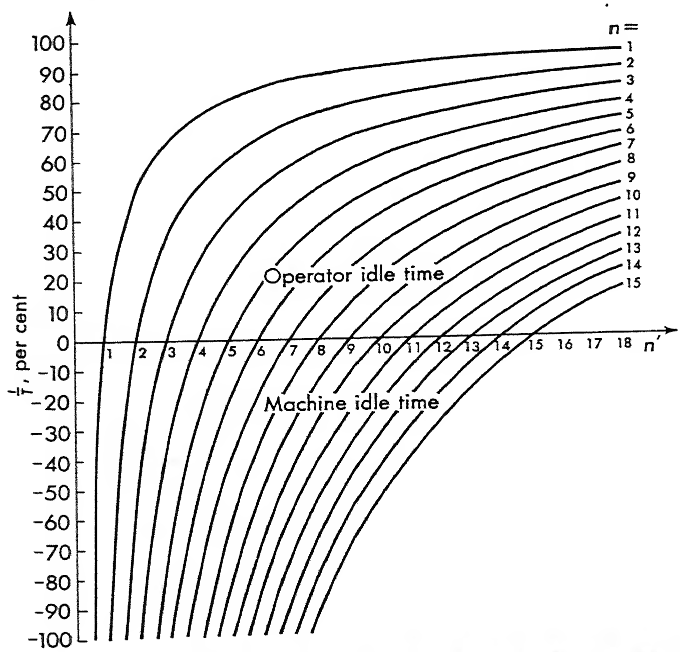

Figure 12–9. ***Relative idle time when assigning n machines to one operator.***

This simplified analysis does not make allowances for several noteworthy factors.

1. Operator walking time from one machine to another.
2. Operator additional independent duties, such as:
   - (i) Inspection of machines
   - (ii) Inspection of the product (the time required depending on the output and the inspection method)
   - (iii) Subsequent treatment to the work (removing machining marks, polishing, greasing, matching or assembling several components)
   - (iv) Packing
   - (v) Transportation of work to and from the production center
   - (vi) Personal allowances

3. Machine interference: When the elements of the cycle are not deterministic but liable to random changes, perfect matching is virtually impossible, and deviations of operational times from the rigid cycle timetable cause the machines to "interfere" with each other.

These factors may increase the independent activity time very appreciably, and in as much as the cycle time thereby increases, the output is adversely affected.

An allowance for an operator's independent activities (such as walking between machines, additional duties, or even relaxation time) can be accounted for without great difficulty. Suppose these activities amount to b minutes per machine, in addition to the concurrent activity a. The machine work cycle is $a + t$, but the operator's total activity per machine is $a + b$; hence the number of machines for perfect matching is

$$n' = \frac{a + t}{a + b} \tag{12–8}$$

When n' is not a whole number, its two neighboring whole numbers may be considered, namely,

$$n < n' < n + 1$$

By following the same procedure adopted above, we find that the cost per unit when n machines are selected is again

$$Y_n = \frac{c_m}{60n}(\epsilon + n)T$$

and in the case of $n + 1$ machines,

$$Y_{n+1} = \frac{c_m}{60}(\epsilon + n + 1)(a + b)$$

Hence, again

$$\frac{Y_n}{Y_{n+1}} = \frac{\epsilon + n}{\epsilon + n + 1}\,\frac{n'}{n}$$

as in Eq. 12–6, except that now the new definition for n' given by Eq. 12–8 applies.

An operator may be asked to supervise nonidentical machines, i.e., machines that differ in the amount of time required for preparation, loading and unloading, and in the running time. An example for perfect matching of operator time and machine time, when the operator is supervising three nonidentical machines, is shown in Fig. 12–10. It is evident from this multiactivity chart that:

1. In order to avoid machine idle time, the cycle times for the machines must be the same, and they must be longer than the operator's work cycle. These two conditions may be expressed as

$$T_1 = T_2 = T_3 (= a_1 + t_1 = a_2 + t_2 = a_3 + t_3) = T$$
$$T \geqslant a_1 + a_2 + a_3$$

2. In order to avoid operator idle time,

$$a_1 + a_2 + a_3 \geqslant T$$

Hence, for perfect matching of n machines and one operator we must have

$$T_1 = T_2 = \cdots = T_n = T = \sum_1^n a \tag{12-9}$$

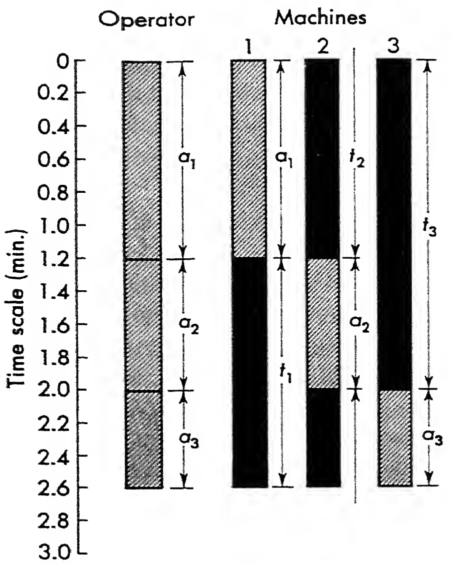

Figure 12–10. ***Assigning three non-identical machines to one operator for perfect matching between operator and machines' work cycles.***

This, incidentally, is similar to the condition we have to satisfy for perfect matching of a multiproduct production schedule (Chapter 14). If perfect matching cannot be achieved, and we have to choose between assignment of n machines or $n + 1$ machines to one operator, the matter has to be considered from the point of view of cost, since in the case of n machines the operator is idle part of the time (his idle time being $T - \sum_1^n a$ minutes per cycle), while in the case of $n + 1$ machines, the operator is fully occupied, but machine idle time is incurred (each machine being $\sum_1^{n+1} a - T$ minutes idle per cycle).

Machine Interference[2]

When the machine cycle time is deterministic, and provided the number of identical machines, n per operator, is such that $T \geqslant na$, the output of the production center will increase linearly with the number of machines. Automatic machines usually have a fairly constant running time t, since the speed and the automatic stop at the end of the operation control the running time within very

[2] This section may be omitted at first reading.

close limits. The operator's time, a, however, is subject to some variations, depending on the pace at which the operator chooses to work, so that the total cycle time $T = a + t$ will be affected accordingly. This may mean that the cycle time differs for different machines; also that it may differ for the same machine in different cycles. If the number of machines is selected for perfect matching (or nearly perfect matching) based on the average cycle time T, we may very often get a situation where a machine has finished its task and has to wait for the operator, who has not quite finished his job on another machine. The first machine is therefore idle. It has to wait merely because of the variations in the cycle time. The machines are, so to speak, "interfering" with each other during parts of the cycle, and this interference adversely affects the output of the center. The larger the variations in the cycle time and the larger the number of machines, the more interference can be expected.

A similar situation occurs in the case of semiautomatic machines, which can run and perform their task without requiring the constant attendance of the operator. But every now and then the machine stops and demands attention. Looms are a common example for machines of this type, where the machine may stop, owing to several reasons (such as a break in the thread or end of a cop). The time spent by the operator at the machine depends on the kind of repair or service he has to give to the machine. Here, again, some machines may be waiting for the operator's services, who is then engaged, while at other times the operator may be idle, waiting for one of the machines to require his attention. This leads to a host of problems, such as: how many machines to allocate to one operator, how to determine the average waiting time of a machine, how to calculate the cycle time and thereby the expected output, and how to determine the optimal procedure for repairs (of several waiting machines, should short repairs be tackled first?). It is not difficult to see that these problems are common to other situations where customers require the attention of servers or service stations. For example:

Operators go to the store to obtain from the servers (= storemen) tools or materials.

Various departments obtain machine components from a centralized machine shop.

Customers buying stamps or sending parcels at a post office.

Customers buying tickets at box offices (= servers).

Ships come to a harbor having a limited number of berths (= servers).

In each of these situations there is random arrival of customers, involving varying service times, which depend on the kind of attention these customers require. Interference occurs when a customer cannot get immediate attention and has to wait for a server to become free, and if more than one customer demands this service, a queue is formed. The problem of scheduling orders which arrive at random is further discussed in the next chapter.

In the case of assignment of n semiautomatic machines to one operator, the

interference increases not only with the number of machines but also with the proportion of attention time received per cycle. Thus, when a/t increases, interference increases and causes the output to fall. The operator's attention time a may depend on two factors:

1. The type of repair or service
2. The operator, his skill, training, and incentive to perform the job

The Ashcroft tables

The research of H. Ashcroft on the productivity of several machines under the care of one operator (1950) has greatly contributed to our knowledge in this field, and his tables facilitate quick calculations of the efficiency of the machines and their output, when the ratio $\frac{a}{t}\left(=\frac{\text{attention time}}{\text{machine time}}\right)$ is known.

Ashcroft made the following assumptions in connection with his tables:

1. Machines stop and demand attention at random. The probability of a machine stopping in the future is independent of the time that elapsed since the last time the machine received attention.

2. The service time of the operator is constant.

Needless to emphasize, these assumptions are seldom justified in practice. If a machine stops and the cause for the breakdown is removed, one would expect the machine to operate satisfactorily for some time before stopping again. In other words, there should be some sort of dependency of the frequency of stoppages on the current running time of the machine. If, on the other hand, many causes are responsible for stoppages and indicate the suggested independency, the second assumption could hardly be justified, since with different causes one would expect an inherent difference between the times required for servicing (apart from the variability that occurs for any one particular cause, due to inconsistencies in operators' performance).

Thus, it could be argued that in most situations the assumptions suggested by Ashcroft are not valid, and that, moreover, there is a certain degree of incompatibility between the two assumptions. It has often been found, however, that his tables are very valuable in providing good approximations in cases where reality is not appreciably different from his suggested model.

If machines stop at random, and if future stoppages are independent of the last attention they received, then the probability that a machine will continue to run for a period t is e^{-kt}, where k is a coefficient depending on the number of machines. In the case of two machines where one machine will be running while the other is being attended to by the operator, the probability is e^{-p}, where $p = a/t$ is the ratio of attention time to machine running time (not to be confused with the ratio p defined in Chapter 10). The probability that the second machine stops is $1 - e^{-p}$ (since the second machine must be either running or stopping, the probability of the two events occurring has to be 1).

Table 12-1

Ashcroft Numbers, Part I

$n = 1$ to 10, $p = 0.01$ to 1.0. Average number of machine hours per hour, A, for an operator having the care of n semiautomatic machines. The cycle ratio p is the ratio a/t.

p	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 7$	$n = 8$	$n = 9$	$n = 10$	p
0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	0.00
0.01	0.99	1.98	2.97	3.96	4.95	5.94	6.93	7.92	8.91	9.90	0.01
0.02	0.98	1.96	2.94	3.92	4.90	5.88	6.85	7.83	8.81	9.78	0.02
0.03	0.97	1.94	2.91	3.88	4.84	5.81	6.77	7.74	8.70	9.66	0.03
0.04	0.96	1.92	2.88	3.84	4.79	5.74	6.69	7.64	8.58	9.52	0.04
0.05	0.95	1.90	2.85	3.79	4.74	5.67	6.61	7.53	8.45	9.37	0.05
0.06	0.94	1.88	2.82	3.75	4.68	5.60	6.51	7.42	8.31	9.19	0.06
0.07	0.93	1.86	2.79	3.71	4.62	5.52	6.42	7.29	8.15	8.99	0.07
0.08	0.93	1.85	2.76	3.67	4.56	5.44	6.31	7.16	7.98	8.76	0.08
0.09	0.92	1.83	2.73	3.62	4.50	5.36	6.20	7.01	7.78	8.50	0.09
0.10	0.91	1.81	2.70	3.58	4.44	5.28	6.08	6.85	7.57	8.21	0.10
0.11	0.90	1.79	2.67	3.53	4.38	5.19	5.96	6.68	7.33	7.89	0.11
0.12	0.89	1.77	2.64	3.49	4.31	5.10	5.83	6.50	7.08	7.55	0.12
0.13	0.88	1.76	2.61	3.44	4.24	5.00	5.69	6.31	6.81	7.19	0.13
0.14	0.88	1.74	2.58	3.40	4.18	4.90	5.55	6.10	6.53	6.83	0.14
0.15	0.87	1.72	2.55	3.35	4.11	4.80	5.40	5.90	6.25	6.48	0.15
0.16	0.86	1.71	2.52	3.31	4.04	4.70	5.25	5.68	5.97	6.14	0.16
0.17	0.85	1.69	2.50	3.26	3.97	4.59	5.10	5.47	5.70	5.82	0.17
0.18	0.85	1.67	2.48	3.22	3.90	4.48	4.94	5.26	5.44	5.52	0.18
0.19	0.84	1.66	2.44	3.17	3.83	4.37	4.79	5.05	5.19	5.24	0.19
0.20	0.83	1.64	2.41	3.12	3.75	4.26	4.63	4.85	4.95	4.99	0.20
0.21	0.83	1.62	2.38	3.08	3.68	4.15	4.48	4.66	4.73	4.75	0.21
0.22	0.82	1.61	2.35	3.03	3.61	4.04	4.33	4.47	4.53	4.54	0.22
0.23	0.81	1.59	2.33	2.98	3.53	3.94	4.18	4.30	4.34	4.34	0.23
0.24	0.81	1.58	2.30	2.94	3.46	3.83	4.04	4.13	4.16	4.16	0.24

0.25	0.80	1.56	2.27	2.89	3.39	3.72	3.90	3.98	4.00	4.00	0.25
0.26	0.79	1.55	2.24	2.85	3.31	3.62	3.77	3.83	3.84	3.84	0.26
0.27	0.79	1.53	2.22	2.80	3.24	3.52	3.65	3.69	3.70	3.70	0.27
0.28	0.78	1.52	2.19	2.75	3.17	3.42	3.53	3.56	3.57	3.57	0.28
0.29	0.77	1.51	2.16	2.71	3.10	3.33	3.42	3.44	3.45	3.45	0.29
0.30	0.77	1.49	2.14	2.67	3.03	3.23	3.31	3.33	3.33	3.33	0.30
0.31	0.76	1.48	2.11	2.62	2.97	3.14	3.21	3.22	3.22	3.22	0.31
0.32	0.76	1.46	2.09	2.58	2.90	3.06	3.11	3.12	3.12	3.12	0.32
0.33	0.75	1.45	2.06	2.53	2.84	2.98	3.02	3.03	3.03	3.03	0.33
0.34	0.75	1.44	2.03	2.49	2.77	2.90	2.93	2.94	2.94	2.94	0.34
0.35	0.74	1.42	2.01	2.45	2.71	2.82	2.85	2.86	2.86	2.86	0.35
0.40	0.71	1.36	1.89	2.25	2.43	2.49	2.50	2.50	2.50	2.50	0.40
0.45	0.69	1.30	1.78	2.07	2.19	2.22	2.22	2.22	2.22	2.22	0.45
0.50	0.67	1.24	1.67	1.90	1.98	2.00	2.00	2.00	2.00	2.00	0.50
0.55	0.64	1.19	1.57	1.76	1.81	1.82					
0.60	0.62	1.14	1.48	1.63	1.66	1.67					
0.65	0.61	1.10	1.40	1.51	1.54	1.54					
0.70	0.59	1.05	1.32	1.41	1.43	1.43					
0.75	0.57	1.01	1.25	1.32	1.33	1.33					
0.80	0.55	0.97	1.19	1.24	1.25	1.25					
0.85	0.54	0.94	1.13	1.17	1.17	1.18					
0.90	0.53	0.91	1.07	1.11	1.11	1.11					
0.95	0.51	0.87	1.02	1.05	1.05	1.05					
1.00	0.50	0.84	0.98	1.00	1.00	1.00					

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Table 12–2

Ashcroft Numbers, Part II

$n = 11$ to 20, $p = 0.005$ to 0.27. For $n = 17$ to $n = 20$ and values of p between 0.145 and 0.27 may be taken as for $n = 16$.

p	$n = 11$	$n = 12$	$n = 13$	$n = 14$	$n = 15$	$n = 16$	$n = 17$	$n = 18$	$n = 19$	$n = 20$	p
0.000	11.00	12.00	13.00	14.00	15.00	16.00	17.00	18.00	19.00	20.00	0.000
0.005	10.94	11.94	12.93	13.93	14.92	15.92	16.91	17.91	18.90	19.89	0.005
0.010	10.88	11.87	12.86	13.85	14.84	15.83	16.82	17.80	18.79	19.78	0.010
0.015	10.82	11.80	12.79	13.77	14.75	15.73	16.71	17.69	18.69	19.65	0.015
0.020	10.76	11.73	12.71	13.68	14.65	15.62	16.59	17.56	18.53	19.50	0.020
0.025	10.69	11.66	12.62	13.58	14.54	15.50	16.46	17.41	18.37	19.32	0.025
0.030	10.62	11.57	12.53	13.48	14.42	15.37	16.31	17.24	18.17	19.10	0.030
0.035	10.54	11.48	12.42	13.36	14.29	15.21	16.13	17.04	17.94	18.82	0.035
0.040	10.46	11.39	12.31	13.23	14.13	15.03	15.92	16.79	17.64	18.48	0.040
0.045	10.37	11.28	12.18	13.08	13.95	14.82	15.66	16.48	17.27	18.03	0.045
0.050	10.27	11.16	12.04	12.91	13.75	14.57	15.35	16.10	16.81	17.45	0.050
0.055	10.17	11.04	11.89	12.71	13.51	14.27	14.98	15.64	16.23	16.75	0.055
0.060	10.05	10.90	11.71	12.49	13.23	13.92	14.54	15.09	15.56	15.93	0.060
0.065	9.93	10.74	11.51	12.24	12.91	13.52	14.04	14.47	14.80	15.04	0.065
0.070	9.80	10.57	11.29	11.96	12.55	13.06	13.47	13.78	14.00	14.14	0.070
0.075	9.65	10.38	11.05	11.65	12.15	12.56	12.87	13.08	13.20	13.28	0.075
0.080	9.50	10.18	10.79	11.30	11.72	12.03	12.25	12.38	12.45	12.48	0.080
0.085	9.33	9.96	10.50	10.94	11.27	11.49	11.63	11.71	11.74	11.76	0.085
0.090	9.15	9.72	10.19	10.55	10.80	10.96	11.05	11.09	11.10	11.11	0.090
0.095	8.96	9.47	9.87	10.16	10.34	10.45	10.49	10.52	10.52	10.52	0.095
0.100	8.76	9.21	9.54	9.76	9.89	9.96	9.98	9.99	10.00	10.00	0.100
0.105	8.55	8.94	9.21	9.38	9.46	9.50	9.52	9.52	9.52	9.52	0.105
0.110	8.34	8.67	8.88	9.00	9.06	9.08	9.09	9.09	9.09	9.09	0.110
0.115	8.12	8.39	8.56	8.64	8.68	8.69	8.69	8.69	8.69	8.69	0.115
0.120	7.89	8.12	8.24	8.30	8.32	8.33	8.33	8.33	8.33	8.33	0.120
0.125	7.67	7.85	7.94	7.98	7.99	8.00	8.00	8.00	8.00	8.00	0.125
0.130	7.44	7.59	7.65	7.68	7.69	7.69	7.69	7.69	7.69	7.69	0.130
0.135	7.22	7.34	7.38	7.40	7.41	7.41	7.41	7.41	7.41	7.41	0.135

0.140	7.01	7.09	7.13	7.14	7.14	7.14	7.14	7.14	7.14	7.14	0.140
0.145	6.80	6.86	6.89	6.89	6.90	6.90	6.90	6.90	6.90	6.90	0.145
0.150	6.59	6.64	6.66	6.66	6.67	6.67					
0.160	6.21	6.24	6.25	6.25	6.25	6.25					
0.17	5.86	5.88	5.88	5.88	5.88	5.88					
0.18	5.55	5.55	5.55	5.55	5.55	5.55					
0.19	5.26	5.26	5.26	5.26	5.26	5.26					
0.20	5.00	5.00	5.00	5.00	5.00	5.00					
0.21	4.76	4.76	4.76	4.76	4.76	4.76					
0.22	4.54	4.54	4.54	4.54	4.54	4.54					
0.23	4.35	4.35	4.35	4.35	4.35	4.35					
0.24	4.17	4.17	4.17	4.17	4.17	4.17					
0.25	4.00	4.00	4.00	4.00	4.00	4.00					
0.26	3.85	3.85	3.85	3.85	3.85	3.85					
0.27	3.70	3.70	3.70	3.70	3.70	3.70					

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Average number of consecutive servicing tasks

What is the average number of consecutive servicing tasks or repairs, X_2, that the operator has to perform when supervising two machines? After he attends to one machine, there may be two situations:

1. The other machine may still be running, in which case the number of consecutive tasks is 1 (being the service that he has just completed).
2. The other machine may have stopped in the meantime, so that the operator is faced with a second job waiting for him. But as the occurrence of machine stoppages is a random one, the operator is now placed in precisely the same position as when he was engaged on the first job, i.e., when he finishes this second job, the first machine may still be running (in which case the chain of consecutive tasks is terminated), or it may have stopped while the second task was performed (in which case the operator will have a third task to tackle), and so on. In other words, when the chain of operator tasks is not terminated after the first task, the average number of consecutive tasks would be X_2. But as the probability of second machine stopping while the first one is attended to is $1 - e^{-p}$, the average number of tasks for this case would be $X_2(1 - e^{-p})$.

Situation 1 always occurs, since whenever there is a chain of tasks for the operator, there must necessarily be at least one task in this chain (otherwise, there is no chain). The amount $X_2(1 - e^{-p})$ for situation 2 denotes the additional average number of tasks. The total average number of consecutive tasks is therefore

$$X_2 = 1 + X_2(1 - e^{-p})$$

or

$$X_2 = e^{p} \tag{12–10}$$

Let us now examine the case of an operator supervising three machines, the average number of operator consecutive servicing tasks being X_3. Suppose the operator is called upon to service one machine. When his task is completed, he may be faced with the following situations:

1. The other two machines are still running, in which case his chain of tasks would consist of one task only.
2. Machine 2 has stopped and demands attention. The operator is faced precisely with same situation as before the first machine stopped, namely, the average number of tasks he has to expect is X_3. But how often would this situation occur? The probability of the second machine stopping while the third is still running is $(1 - e^{-p})e^{-p}$; therefore the average number of tasks to be expected in this case is $X_3(1 - e^{-p})e^{-p}$.
3. Machine 3 has stopped and demands attention. The number of tasks on the average awaiting the operator is the same as in situation 2, namely, $X_3(1 - e^{-p})e^{-p}$.
4. Both machines 2 and 3 have stopped. As the operator can handle only one machine at a time, and as all machines are assumed to be identical, he may

disregard the third machine. He may proceed to attend to the second machine, and should the first one stop in the meantime, he would return to the first one. In short, he would act precisely as though he were in charge of only two machines, the number of tasks on the average that he has to expect being X_2. When eventually he gets the first two machines running, he can turn his attention to the third machine. But now the situation is reduced to one machine demanding attention while two are running; in other words this is the original situation of a three-machine system where the average number of consecutive tasks expected is X_3. The total number of tasks for situation 4 is therefore $X_2 + X_3$. The probability of a machine stopping is $1 - e^{-p}$; hence the probability of two machines stopping is $(1 - e^{-p})^2$, so that the average number of consecutive operator tasks for situation 4 is $(X_2 + X_3)(1 - e^{-p})^2$.

Situation 1 always occurs. The additional average number of tasks is described by situations 2, 3, and 4, and if we substitute $X_2 = e^p$, we get

$$X_3 = 1 + 2X_3(1 - e^{-p})e^{-p} + (e^p + X_3)(1 - e^{-p})^2$$

or

$$\begin{aligned} X_3 &= 1 + X_3(1 - e^{-p})(2e^{-p} + 1 - e^{-p}) + e^p(1 - e^{-p})^2 \\ &= 1 + X_3(1 - e^{-2p}) + e^p - 2 + e^{-p} \end{aligned}$$

$$\therefore \qquad X_3 = e^{3p} - e^{2p} + e^p \tag{12–11}$$

Ashcroft has shown that this expression can be written in the following form:

$$X_3 - 2X_2 + 1 = (e^p - 1)(e^{2p} - 1) \tag{12–12}$$

Similarly, for four machines,

$$X_4 - 3X_3 + 3X_2 - 1 = (e^p - 1)(e^{2p} - 1)(e^{3p} - 1) \tag{12–13}$$

and so on.

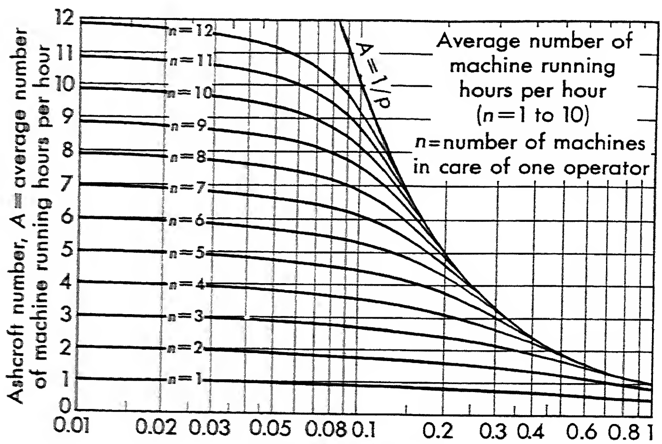
The coefficients of the X's on the left-hand side of these equations reminds one of Pascal's triangle. The values for X can be progressively calculated so that when all the values up to X_n are known, we can proceed to calculate X_{n+1}, and so on.

The Ashcroft number[3]

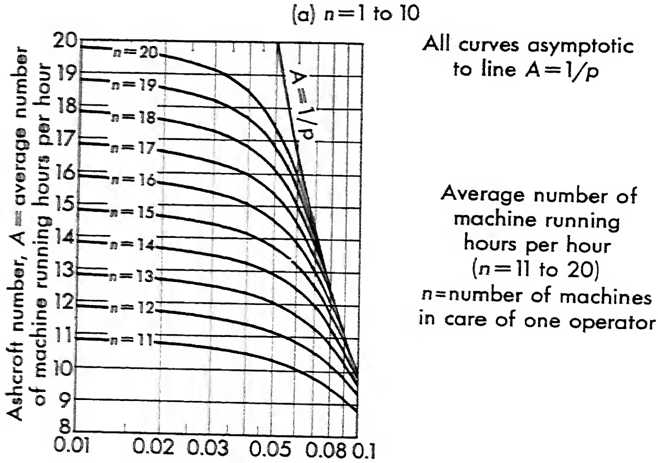
The Ashcroft number A_n is defined as the average number of effective (= running) machine hours per hour, when n machines are assigned to the supervision of one operator; in other words, the Ashcroft number is a measure of the expected output of the n machines. The number A_n can be expressed in terms of X_n as follows:

[3] This section may be omitted at first reading.

The average number of consecutive operator tasks is X_n. If each task lasts λ hours, the operator is busy on the average $X_n\lambda$ hours at a time, during which some machines are nonproductive. Between these "chains" of tasks, the operator waits for a customer to demand service, and as long as he waits, all the machines are running. Suppose the number of times a machine stops (on the average) is m per machine running hour. This would mean that the average length of a running period (between machine stoppages) would be $1/m$ hours. As there are n machines (all assumed to be identical), each one stopping m times per machine



(a) $n=1$ to 10



(b) $n=11$ to 20

Figure 12–11. ***The Ashcroft number.*** (Reproduced by permission from *Product and Probability*, by T. F. O'Connor, Manchester, England, Emmott & Co. Ltd., 1952)

hour, the average length of the interval in which all the machines are running would be $(1/m) \div n$, or $1/mn$ hours.

The operator, therefore, has an average service interval of $X_n\lambda$ hours followed by an average idle interval of $1/mn$ hours, so that the "cycle" is $X_n\lambda + 1/mn$ long, of which $1/mn$ is the effective machine running time.

The ratio of service time to effective machine time was defined as p. But as a machine stops m times per machine running hour, and each stop would necessitate λ hours of immediate attention before it can run again, therefore

$$p = \lambda m \tag{12–14}$$

Since the average number of running machines is A_n, the proportion of service time per hour is A_np. Therefore

$$A_np = \frac{X_n\lambda}{X_n\lambda + (1/mn)}$$

or

$$A_n = \frac{1}{p + (1/X_n\lambda)(1/mn)p}$$

but since $p = m\lambda$,

$$A_n = \frac{1}{p + (1/nX_n)} \tag{12–15}$$

and this is the formula on which the Ashcroft tables are based, since X_n can be found for any given n and p. Values for A_n are also shown in Fig. 12–11.

Example

Given $p = 0.18$; output per machine is 100 units per running hour. The ideal number of machines per operator is

$$n' = \frac{a + t}{t} = \frac{1 + p}{p} = \frac{1.18}{0.18} = 6.56$$

From the tables we find the corresponding values of A_n as follows:

n	A_n	*Output (units per hour)*	*Efficiency, A_n/n 100*
1	0.85	85	85
2	1.67	167	83.5
3	2.48	248	82.7
4	3.22	322	80.5
5	3.90	390	78.0
6	4.48	448	74.7

These results are shown in Fig. 12–12.

The last column in the accompanying table gives the efficiency of the production center. The maximum output is attained when all n machines are

always running, and since the machines are assumed to be identical, the maximum output is n times the machine output, whereas the actual output is only A_n times the machine output. The productive efficiency is therefore A_n/n.

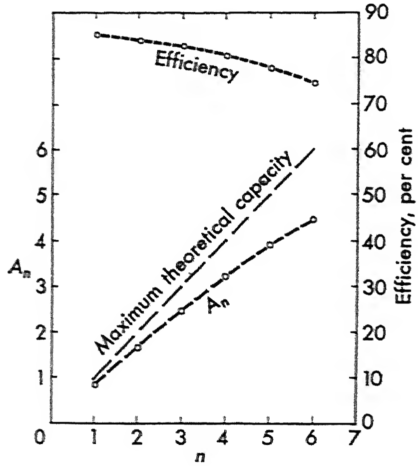


Figure 12–12. ***Examples showing the effect of machine interference on output and efficiency.***

Balancing

If a product has to undergo a sequence of operations through several machines, each stage involving a certain amount of concurrent activity a and machine running time t, the output would be the same for all stages when the total cycle time, $a + t$, per stage is the same throughout. In other words we get perfect matching of the stages in the sequence. In practice, however, the operation times vary in length, so that some machines manage to cope more quickly than others with the job on the line. If one machine has a higher output than the one succeeding it in the sequence, components will obviously accumulate at the input to the second machine and form a queue, and if the first machine goes on producing, the queue will constantly increase in length. If, on the other hand, the first machine is slower than the second, there will be no queue in front of the second machine, which will, in fact, have to be idle part of the time and run intermittently whenever a component is available to be fed into the machine. Through necessity, therefore, the second machine adjusts its cycle time to that of the first machine, unless of course we allow the first machine to run for some time and build up a stock of components at its output end before the second machine starts operating. If the output of one stage is directly fed into the next one on the line, or if the pile of components at the input end is to be kept at a constant level, the "pace" or output of the line is determined by the slowest machine. The problem of machine balancing is that of equalizing the idle times at the different stages on the line and matching, or balancing, the outputs.

Example

Take the example shown in Table 12–3, in which a product has to undergo 20 stages in the production sequence. At each stage the operation is carried out on one machine, involving a certain operation time $T = a + t$, where a is the preparation time (which is a concurrent activity for the operator and the machine) and t is the machine time. The operation times differ at the various stages, ranging from 0.6 minutes to 10.0 minutes, and the "pace" or the cycle time for

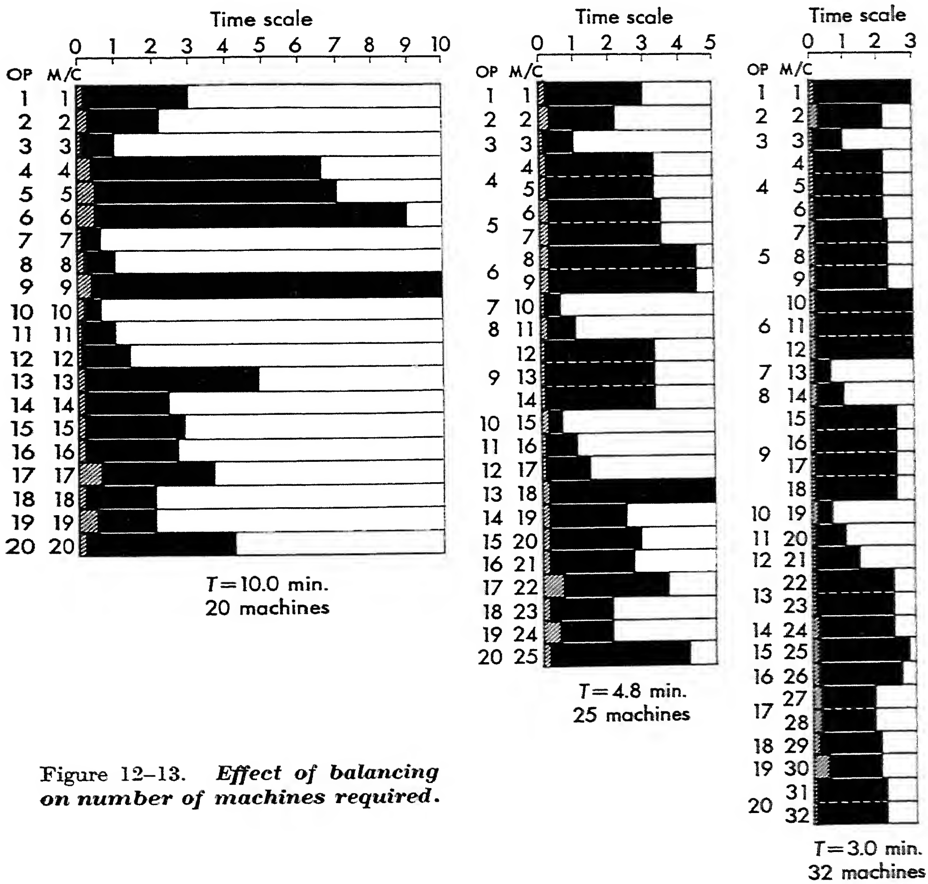


Figure 12–13. ***Effect of balancing on number of machines required.***

this line is the longest time (namely, 10.0 minutes), if only one machine is allocated to each stage. This is shown in Fig. 12–13. The machine at stage 9, which requires 10.0 minutes, is fully occupied, but there is idle time at all the other stages. Stage 9 is therefore the "bottleneck," and if we want to speed up production, we have to assign another machine to this stage. With two machines

at stage 9 we can produce two units per 10.0 minutes; thus the time per unit is reduced to 5.0 minutes. The next "bottleneck" is stage 6, then stage 5, then stage 4, etc.

Table 12 3

Operation	*Machine Time (t min.)*	*Preparation Time (a min.)*	*Total Op. Time ($T' = a + t$)*	$n' = \dfrac{T'}{a}$
1	2.8	0.2	3.0	15
2	1.9	0.3	2.2	7.3
3	0.9	0.1	1.0	10
4	6.2	0.4	6.6	16.5
5	6.5	0.5	7.0	14
6	8.5	0.5	9.0	18
7	0.5	0.1	0.6	6
8	0.8	0.2	1.0	5
9	9.6	0.4	10.0	25
10	0.4	0.2	0.6	3
11	0.9	0.1	1.0	10
12	1.3	0.1	1.4	14
13	4.6	0.2	4.8	24
14	2.2	0.2	2.4	12
15	2.6	0.2	2.8	14
16	2.4	0.2	2.6	13
17	3.0	0.6	3.6	6
18	1.8	0.2	2.0	10
19	1.5	0.5	2.0	4
20	4.0	0.2	4.2	21
Total	62.4	5.4	67.8	

Suppose we want the cycle time to be 4.8 minutes (corresponding to the time required by stage 13). The number of machines required for the first three stages remains at 1 machine per stage, but at stage 4 we need 2 machines; 2 at stage 5; 2 at stage 6; 3 at stage 9; etc. The total number of machines now required is $n = 25$, compared with $n = 20$ originally planned, but the total amount of idle time is considerably reduced. For cycle time $T = 10.0$, the total machine time available per cycle is $nT = 20 \times 10 = 200$ min. The total operation time is $\Sigma(a + t) = 67.8$ min.; thus the efficiency of the cycle is[4]

$$\eta = \frac{\Sigma(a + t)}{nT} 100 = \frac{67.8}{200} 100 = 33.9\%$$

and the output of the line is $60/10.0 = 6$ units per hour. But when the cycle time is reduced to 4.8 minutes, the efficiency of the cycle is

$$\eta = \frac{67.8}{25 \times 4.8} = 56.5\%$$

[4] For the purpose of the present discussion efficiency is defined as the ratio of machine active time (including concurrent activity) to total cycle time. In some applications efficiency is defined as the ratio of machine running time to total cycle time.

and the output of the line is $60/4.8 = 12.5$ units per hour. If we reduce the cycle time even further to $T = 3.0$ min. (see Fig. 12–13), the number of machines required increases to 32, the efficiency to 70.6%, and the output to 20 units per hour. The number of machines required at each stage as the balancing process continues is shown in Table 12-4 for some selected values of cycle time. The change in number of machines and in efficiency is shown in Fig. 12–14, which is

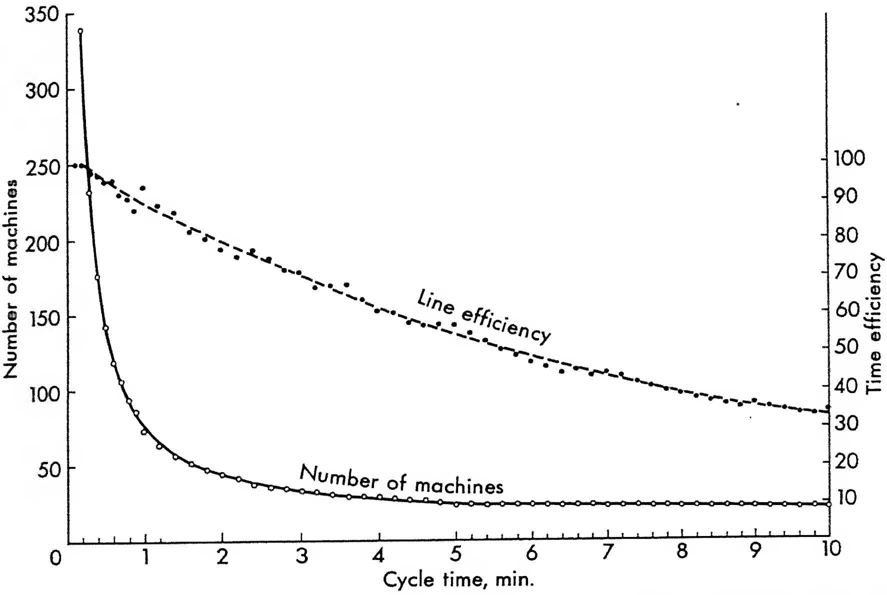


Figure 12–14. ***Effect of balancing on the number of required machines and on the time efficiency of the line. (Calculated at intervals of 0.2 min. for $T > 1.0$ min. and intervals of 0.1 min. for $T < 1.0$ min.)***

based on calculations at intervals of 0.2 minutes in the cycle time. The number of machines naturally increases very rapidly at the lower end of the cycle time range, and so does the theoretical line output. When the cycle time coincides with the highest common factor of all the operation times (which in our case is 0.2 minute), all the machines should theoretically be busy all the time. In practice, obviously, this ideal situation is not so easily attainable, since operation times are liable to variations, especially when part of the cycle is dependent on the behavior and effort of the operator.

Variation of time efficiency

It is important to point out that the increase in time efficiency is a discontinuous function, since it depends on the number of machines allotted to each stage, and these increase abruptly when the cycle time reduces below certain

Table 12-4

Number of Machines Required

Op.	*Op. time (min.)*	*T* = 10.0 *min.*	9.0	8.0	7.0	6.0	5.0	4.0	3.0	2.8	2.6	2.4	2.2	2.0	1.8	1.6	1.4	1.2	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
1	3.0	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	3	3	3	4*	4	5	5	6	8	10	15	30
2	2.2	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	3	3	3	4	4	5	6	8	11	22
3	1.0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	3	4	5	10
4	6.6	1	1	1	1	2	2	2	3	3	3	3	3	4	4	5	5	6	7	8	9	10	11	14	17	22	33	66
5	7.0	1	1	1	1	2	2	2	3	3	3	3	4	4	4	5	5	6	7	8	9	10	12	14	18	24	35	70
6	9.0	1	1	2	2	2	2	3	3	4	4	4	5	5	5	6	7	8	9	10	12	13	15	18	23	30	45	90
7	0.6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	3	6
8	1.0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	3	4	5	10
9	10.0	1	2	2	2	2	2	3	4	4	4	5	5	5	6	7	8	9	10	12	13	15	17	20	25	34	50	100
10	0.6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	3	6
11	1.0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	3	4	5	10
12	1.4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	3	3	4	5	7	14
13	4.8	1	1	1	1	1	1	2	2	2	2	2	3	3	3	3	4	4	5	6	6	7	8	10	12	16	24	48
14	2.4	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	3	3	3	4	4	5	6	8	12	24
15	2.8	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	3	3	4	4	4	5	6	7	10	14	28
16	2.6	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	3	3	3	4	4	5	6	7	9	13	26
17	3.6	1	1	1	1	1	1	1	2	2	2	2	2	2	2	3	3	3	4	4	5	6	6	8	9	12	18	36
18	2.0	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	3	3	4	4	4	5	7	10	20
19	2.0	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	3	3	4	4	4	5	7	10	20
20	4.2	1	1	1	1	1	1	2	2	2	2	2	2	3	3	3	3	4	5	5	6	6	7	9	11	14	21	42
Total *n*	67.8	20	21	22	22	24	24	28	32	34	35	37	41	44	47	52	56	64	73	86	94	106	118	142	176	232	339	678
nT (min.)		200.0	180.0	176.0	154.0	144.0	120.0	112.0	96.0	95.2	91.0	88.8	90.2	88.0	84.6	83.2	78.4	76.8	73.0	77.4	75.2	74.2	70.8	71.0	70.4	69.6	67.8	67.8
Efficiency % $= \frac{\Sigma(a+t)}{nT} 100$		33.9	35.9	38.5	44.0	47.1	56.5	60.5	70.6	71.2	74.5	76.4	75.2	77.0	80.1	81.5	86.5	88.3	92.9	87.6	90.2	91.4	95.8	95.5	96.3	97.4	100.0	100.0
Output per hour		6.0	6.7	7.5	8.6	10.0	12.0	15.0	20.0	21.4	23.1	25.0	27.2	30.0	33.3	37.5	42.8	50.0	60.0	66.7	75.0	85.7	100	120	150	200	300	600

limits. Figure 12–15 shows how this efficiency changes when the cycle time is reduced from 10.00 to 9.80 and from 0.30 to 0.10 minute (the calculations were carried out in steps of 0.01 minute), and we see that below 0.2 minute the efficiency decreases and is again restored to 100 per cent at 0.10 minute. The highest common factor of all the operation times is the first point at which maximum efficiency is reached as we reduce the cycle time, but this is attained in fact when the cycle time coincides with any common factor (0.1, 0.05, 0.04 minute, etc.).

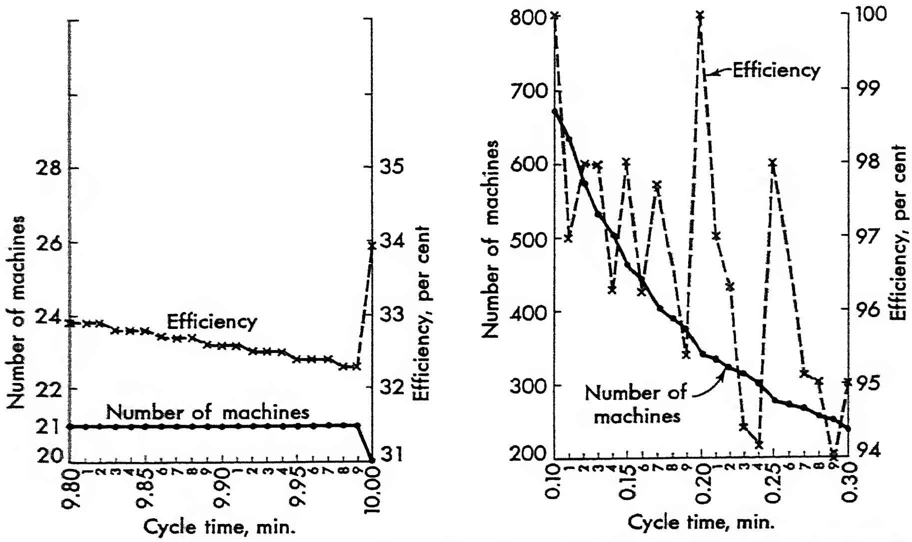


Figure 12–15. ***Two examples showing the effect of the cycle time on the regularity of the number of machines and efficiency functions (based on calculations at intervals of 0.01 min.).***

Broadly speaking, therefore, the time efficiency increases as the cycle time decreases, so that the product should become more and more economical to produce. Furthermore, as more machines per stage are added, the operator's time can be better utilized by multimachine supervision. However, coupled with the increase in the number of machines, the layout problem becomes more and more complicated, and capital expenditure tends to increase, owing to handling and flow control systems, and these factors have to be carefully considered in each case.

Balancing to meet demand

One of the limiting factors in this balancing process is the demand for the product, and this figure will greatly determine the selected cycle time. Suppose that the product referred to in Table 12–3 is required in quantities of 800 pieces per week. The "pace" or cycle time can now be calculated on the basis of the

number of normal working hours per week. Suppose there are 40 normal hours, the required output per hour is 800/40 = 20; therefore the cycle time that we should aim at is 60/20 = 3.0 min. The allocation of machines would be as shown in Fig. 12–13, the time efficiency being 70.6 per cent. We could, of course, increase this efficiency by selecting a shorter cycle time, but then we would be producing at a higher rate than required and would have to abandon flow production and resort to batch production. Overtime can also be used to adjust the output level. Suppose the demand for our product increases from 800 to 1,000 pieces per week. Instead of trying to shorten the effective cycle time by adding machines, the new demand figure can be met by employing overtime.

To summarize, in tackling problems of machine balancing, the following considerations should be borne in mind:

1. The "pace" of production is determined by the slowest operation in the sequence.
2. Total machine idle time increases with the effective cycle time and can theoretically be removed when the cycle time is a common factor of the individual operation times. As the effective cycle time is reduced, there is better scope for placing identical machines under the supervision of one operator.
3. The "bottleneck," or slowest stage in the line, should have the most expensive machine, so that the idle time of the expensive equipment will be kept to a minimum level.
4. The effective cycle time should be selected in relation to the required output, but some adjustments can be made by use of overtime, double shifts, etc.

Analysis of Process Capacities in a Multiproduct System

Estimating capacity

When several products have to be produced on the same machines, the problem of matching the capacities of the various stages in the production sequence becomes rather involved. Take, for example, the following two products, which have to be processed through four stages.

Stage	*Product A*	*Product B*
1	0.2 min./unit	0.4
2	0.3	0.3
3	0.48	—
4	0.4	0.24
5	—	0.2

Assuming there are 480 minutes available per day, the maximum output of the stages when no overtime is used is as shown in the following table:

Stage	*Product A*	*Product B*
1	2,400 units/day	1,200
2	1,600	1,600
3	1,000	—
4	1,200	2,000
5	—	2,400

The capacity of stage 1 is shown graphically in Fig. 12–16. When only product *A* is produced, the maximum output is 2,400 units (marked on the ordinate); when only product *B* is produced, the maximum output is 1,200 units (marked on the abscissa). Any combination of the two products, if stage 1 is employed

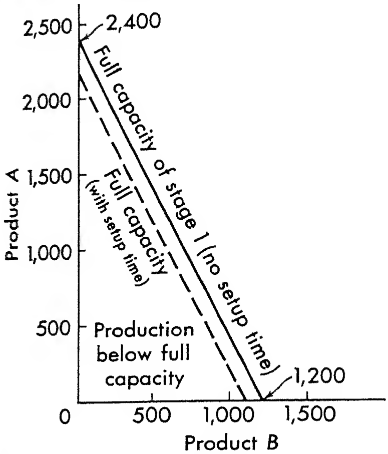


Figure 12–16. ***Process capacity in the case of two products.***

at full capacity, is given by a point on the straight line indicated. This assumes that no setting time is required when changing over from one product to another. If, say, 20 minutes have to be taken into account for that purpose, the maximum output figures when either product *A* or product *B* are solely produced remain unchanged at 2,400 and 1,200 units, respectively, but combination of the two lie on a somewhat lower line, as indicated by the broken line in Fig. 12–16. Points above the line for full capacity refer to output figures that cannot be

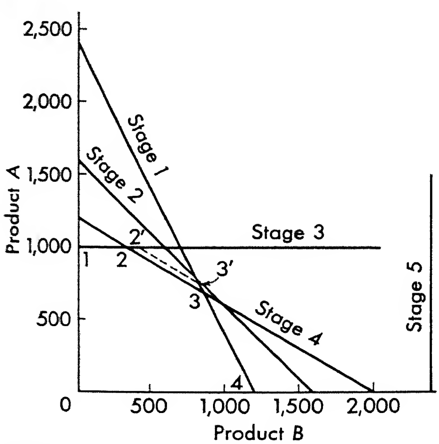


Figure 12–17. ***Capacity of five processes for the products.***

complied with by stage 1; points below the line refer to output below full-stage capacity. The reader will note that a similar analysis for plant capacity was presented in Fig. 5–17.

The capacities of the five processes (assuming no setup time is required) are shown in Fig. 12–17. Since stage 3 is not employed for the manufacture of product B, it is represented by a horizontal line, while stage 5, which is not required for product A, is shown by a vertical line. This graphical presentation tells us immediately which processes limit the output. If, for example, we want to produce only product A, the limiting factor is the process at stage 3 (as it involves the longest operation time per unit) and the maximum output is 1,000 pieces per day, while all the other stages will have to be idle part of the time (stage 5 will, in fact, be idle throughout). We can now increase our total output by using some of the available machine time for producing product B, and for a while this will not affect the output of A, since process 3 is not required for B at all. The maximum output for B that can be achieved while A's output is maintained at 1,000 units is given by point 2, which is the point of intersection of the line representing stage 3 with that of stage 4.

The line for stage 4 is given by

$$\frac{x_A}{1{,}200} + \frac{x_B}{2{,}000} = 1$$

where x_A = the amount produced of product A

x_B = the amount produced of product B.

$$\therefore \qquad x_A + 0.6x_B = 1{,}200$$

The line for stage 3 is given by

$$x_A = 1{,}000$$

Hence the value of x_B at point 2 is $(x_B)_2 = 333$ units.

If we wish to produce a larger quantity of B, the limiting process is given by stage 4 along the line 2–3. As the amount for product B increases, that for A decreases, and the amounts at point 3 are given by the intersection of the lines representing stages 1 and 4, namely, by solution of

$$\text{Equation for line 4:} \qquad x_A + 0.6x_B = 1{,}200$$

$$\text{Equation for line 1:} \qquad \frac{x_A}{2{,}400} + \frac{x_B}{1{,}200} = 1$$

$$\therefore \qquad (x_A)_3 = 684 \text{ units}$$

$$(x_B)_3 = 858 \text{ units}$$

Further increase in product B is now restricted by stage 1 along the line 3–4, as if the other processes did not exist (as, in fact, shown in Fig. 12–16). Thus the condition of maximum capacity is given by the broken line 1–2–3–4, different processes being the limiting factor, depending on the "mix," or balance, between the amounts for the two products. Any point inside the figure 0–1–2–3–4 indicates that the plant will not be operating at maximum capacity. It is

interesting to note that stages 2 and 5 impose no restrictions on the maximum output because at no point on the line 1–2–3–4 are these two stages fully employed. This is a very important piece of information, since it would be futile to spend money and increase the capacity of these two stages when in fact the restriction lies in the other processes.

Should we wish to increase the maximum capacity, Fig. 12–17 would tell us the capacity of which process should be raised for this purpose. Suppose we require 950 units of product A and 500 units of product B. This point lies outside the figure 0–1–2–3–4, but only the line for stage 4 runs below this point. Should this line be raised so that it passes through the point (the new line being parallel to the old one), the required figures would be compatible with the new maximum capacity (presented by the broken line 1–2′–3′–4 in Fig. 12–17). We have found, therefore, that stage 4 needs expansion, and we can easily calculate the required capacity for this stage:

$$\text{Line 4 is given by } x_A + 0.6x_B = 1{,}200$$

If we need $x_B = 600$, then the possible output of stage 4 at present is

$$x_A + 0.6 \times 600 = 1{,}200$$

or

$$x_A = 840$$

But since we need $x_A = 900$, we have to increase the capacity of stage 4 by 60 units per day, so that the maximum output when only product A is produced would be 1,260 as against 1,200 specified so far. If we know how much it costs to increase this capacity, we can proceed to analyze whether this is worth while.

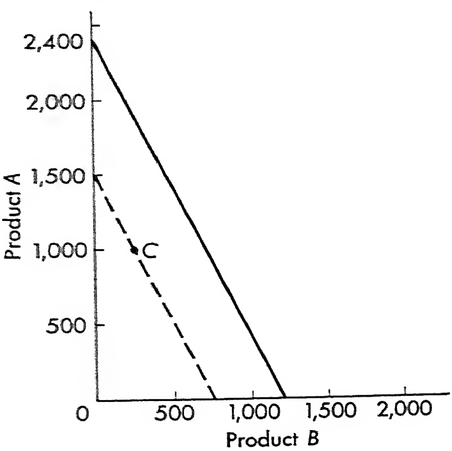


Figure 12–18. ***Machine-time efficiency at point*** $C = \frac{1{,}500}{2{,}400} = 0.625$.

Machine time efficiency

It may also be of interest to study the machine time efficiency. If the maximum capacity of a machine or a process is given by Fig. 12–18, and the machine is operating at point C, the time utilization is denoted by the intercept of a parallel

line through point C (if this time is measured by output of product A). The efficiency is therefore

$$\frac{1{,}500}{2{,}400}\,100 = 62.5\%$$

As we proceed along the line 1–2 in Fig. 12–17 and increase the output for product B, we can draw for each point lines parallel to the appropriate maximum capacity lines and find the efficiency for each stage. Up to point 2 stage 3 would be operating at 100 per cent (i.e., it is fully busy); at point 2, which is an intersection of two lines, two stages would operate at 100 per cent (stages 3, 4); and along the line 2–3 stage 4 would be at 100 per cent, and so on. The change of efficiency with the amount required for products A and B is shown in Fig. 12–19.

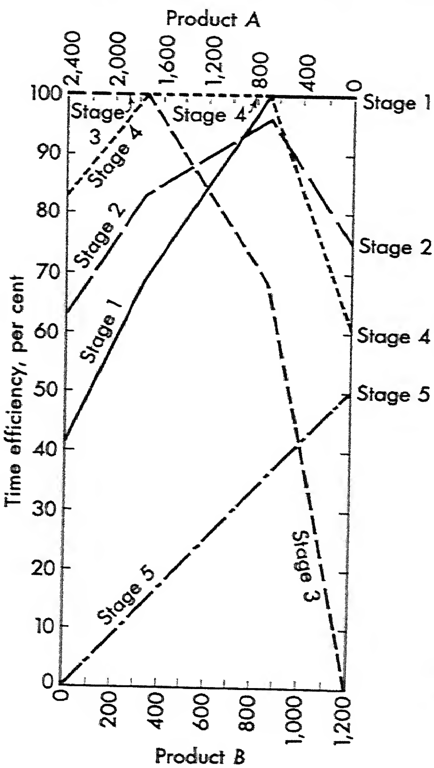


Figure 12–19. ***Machine time efficiencies of the five stages.***

Let us now return to the problem of machine balancing and examine it in the case of a multiproduct schedule. Take, for example, two products A and B, each produced on two machines as follows:

	Product A		Product B	
Machine	*Cycle time (min.)*	*Output/day*	*Cycle time (min.)*	*Output/day*
1	6.0	80.0	4.0	120
2	9.0	53.3	3.2	150

If we consider product A alone, the output is determined by machine 2 at 53.3 pieces per day, while machine 1 is partially idle. Through machine balancing, we can match the capacities of process 1 and process 2 by taking two machines for process 1 and three machines for process 2. At this point the cycle time is 3.0 minutes (= the highest common factor of 6.0 and 9.0) corresponding to an output of 160 units (= the lowest common multiple of 80 and 53.3). Graphically, this increased capacity can be represented by a displacement of the line for full capacity, as in Fig. 12–20, so that when perfect matching is attained, the lines for full capacity meet at the point of 160 units. We see, however, that at their new capacity levels, the processes are balanced only for product A, but they are far from being balanced for product B.

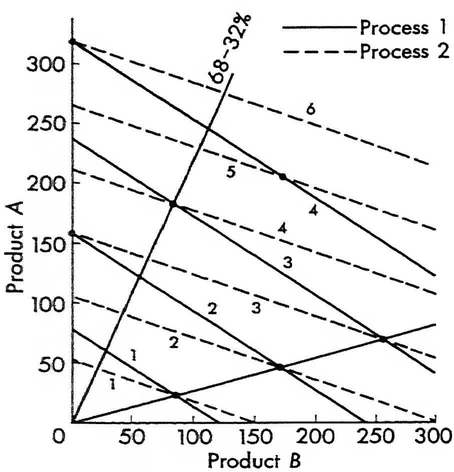


Figure 12–20. ***Balancing process capacities for two products. (There are seven points of balance in this chart.)***

Perfect matching is possible for any given ratio of quantities of products A and B. Take, for example, a ratio of 68:32 per cent, which is represented by a straight line from the origin (Fig. 12–20), namely, by

$$\frac{x_A}{x_B} = \frac{68}{32}$$

or

$$x_A = 2.12x_B$$

The full-capacity lines are

$$\text{Process 1:} \qquad \frac{x_A}{80} + \frac{x_B}{120} = 1 \quad \text{or} \quad 3x_A + 2x_B = 240$$

$$\text{Process 2:} \qquad \frac{x_A}{53.3} + \frac{x_B}{150} = 1 \quad \text{or} \quad 45x_A + 16x_B = 2{,}400$$

Both processes will be employed at full capacity when n_1 machines are assigned for process 1 and n_2 machines for process 2, the lines for full capacity being

$$\text{Process 1:} \qquad 3x_A + 2x_B = 240n_1$$

$$\text{Process 2:} \qquad 45x_A + 16x_B = 2{,}400n_2$$

By substituting $x_A = 2.12x_B$, we get

$$x_B = 28.6n_1$$

$$x_B = 21.5n_2$$

As n_1, n_2 are whole numbers, the lowest amount of x_B for full capacity would simply be the lowest common multiple of 28.6 and 21.5, which is 86. Thus, perfect balance is obtained when $x_A = 183$, $x_B = 86$ (see Fig. 12–20). In general, when the lines for full capacity for production of two products A and B by m processes are given by

$$\text{Process 1:} \qquad \frac{x_A}{A_1} + \frac{x_B}{B_1} = n_1 \quad \text{(using } n_1 \text{ machines)}$$

$$\text{Process 2:} \qquad \frac{x_A}{A_2} + \frac{x_B}{B_2} = n_2 \quad \text{(using } n_2 \text{ machines)}$$

$$\vdots$$

$$\text{Process } m\text{:} \qquad \frac{x_A}{A_m} + \frac{x_B}{B_m} = n_m \quad \text{(using } n_m \text{ machines)}$$

and when the required ratio between the products is $x_B = \alpha x_A$, full capacity is obtained when all these lines intersect at one point. This can be derived by solution of the preceding simultaneous equations.

In Fig. 12–20, the first two lines of the processes 1 and 2 intersect at $x_A = 23$ and $x_B = 86$ when each process employs one machine. This means that whenever we assign the same number of machines n to each process, we shall get a point of balance, which will occur at $x_A = 23n$ and $x_B = 86n$, and all these points of balance lie on a straight line (as in Fig. 12–20).

Multiproduct analysis

The problem of balancing in the case of one product is a one-dimensional one, the condition for full capacity of the processes being indicated by a series of points on a straight line. In the case of two products, we have a two-dimensional problem, the condition for full capacity being represented by straight lines. For any given ratio between the amounts required for the two products, a point of full balance can be found, but unless the lines of full capacity are parallel to each other, perfect matching when the ratio varies is not possible.

A three-product problem can be analyzed by a three-dimensional model, the conditions for full capacity being represented by planes. Take, for example, a three-product system, in which there are two processes as given in the accompanying table.

Product No. and Max. Output	*1*	*2*	*3*
Process I (per hour)	80	50	20
Process II (per hour)	50	70	50

The maximum capacity of process I is expressed by

$$\frac{x_1}{80} + \frac{x_2}{50} + \frac{x_3}{20} = 1$$

and that of process II by

$$\frac{x_1}{50} + \frac{x_2}{70} + \frac{x_3}{50} = 1$$

(where x_1 is the amount of product 1; x_2, of product 2; and x_3, of product 3), and these are two planes, as shown in Fig. 12–21, the intercepts of these planes at the axis being the values denoted in the above table. Maximum capacity for

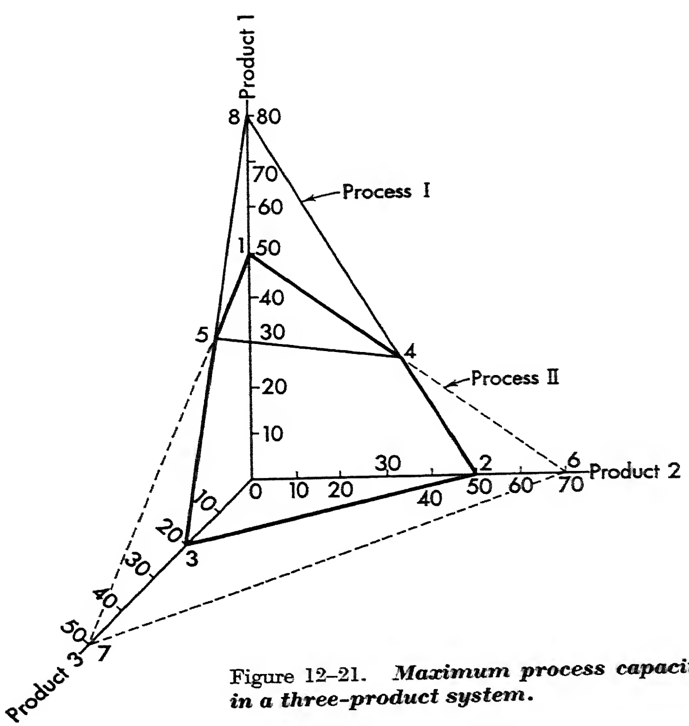


Figure 12–21. ***Maximum process capacity in a three-product system.***

process I is described by the plane 2–3–8 and that for process II by 1–6–7. The maximum capacity of the two processes combined would therefore be given by whichever plane is "lower," i.e., by the figure 1–4–2–3–5 (the line 4–5 being at the intersection of the two planes), and any point below this surface (within the space 0–1–4–2–3–5) would correspond to operation below full capacity.

Similarly, in the case of n products, the problem has to be expressed by an n dimensional system. The analysis is basically the same as that for a two- or three-product system, though naturally it is difficult to visualize or to present in a graphical form. The capacity restrictions are expressed by linear equations, being of the form

$$\begin{aligned} a_1x_1 + a_2x_2 + \cdots + a_nx_n &\leqslant A \\ b_1x_1 + b_2x_2 + \cdots + b_nx_n &\leqslant B \\ c_1x_1 + c_2x_2 + \cdots + c_nx_n &\leqslant C \end{aligned} \tag{12—16}$$

and so on, where x_1 is the amount of product 1, x_2 the amount of product 2, etc., and A being the capacity of process 1, B of process 2, etc. The symbol $\leqslant$ indicates that capacities cannot exceed the maximum values on the right. If the capacity of process 1 can be expanded, the restriction would read

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n \leqslant n_1A$$

where n_1 is the ratio between the expanded capacity to the original capacity (if this expression is obtained through addition of identical machines, n_1 would be a whole number, unless some allowance has to be made for the fact that the output of a process or a stage does not rise linearly with the number of machines allocated). The restriction for the other processes would be likewise amended. As we are dealing only with positive and real values for the various quantities, our considerations are restricted to the first quadrant in our n dimensional system of coordinates, namely:

$$\begin{aligned} x_i &\geqslant 0 \\ A;B;C &\geqslant 0 \\ n_i &\geqslant 0 \end{aligned} \tag{12–17}$$

Profit Maximization

We saw that the maximum capacity of a plant is determined by one or more restrictions, depending on process capacities. Of all the points at which the plant can operate, which is the best?

Graphical presentation

Let us examine the example for two products A and B, shown in Fig. 12–17. Maximum capacity is indicated by the line 1–2–3–4, but we have already seen

that operating at point 2 is preferable to that at point 1, since in both cases the quantity produced for product A is the same, whereas at point 2 we produce 333 units of product B against none at point 1. Assuming the profit per unit of product B is positive, it is obviously worth while to move from point 1 to point 2.

Likewise, one can argue that operating on the line of maximum capacity (namely, on 1–2–3–4) is better than operating at partial capacity (i.e., at a point within the figure 0–1–2–3–4). Suppose we take a point relating to 500 units of A and 500 units of B, which is obviously below the line of maximum capacity. We can draw a horizontal line through this point until it meets the line 1–2–3–4. The point of intersection corresponds to 500 units of A (being the same as for the original point) and 950 units of B. Again, if the profit per unit of B is positive, it would be far better to produce 500 of A and 950 of B, rather than 500 of A and only 500 of B.

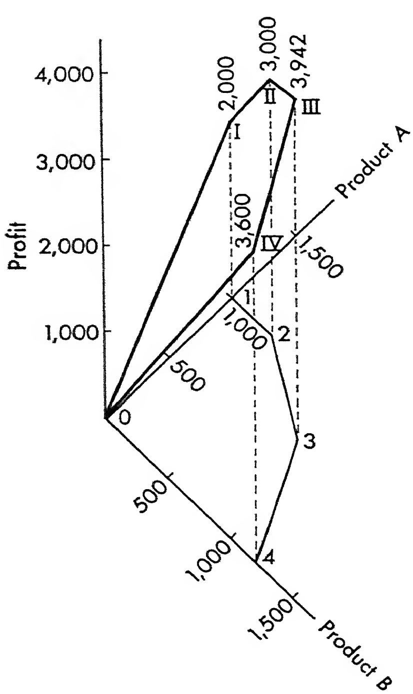


Figure 12–22. ***The profit function for a two-product system.***

Analysis by linear programing

Let us now examine numerically the profit function for our two-product system. Suppose the profit for product A is $z_A = \$2.00$ per unit and that for product

B is $z_B = \$3.00$ per unit. The profit (in dollars) corresponding to the corners of the figure 0–1–2–3–4 would be as shown in the accompanying table.

Point	x_A	x_B	*Profit for A* z_Ax_A	*Profit for B* z_Bx_B	*Total Profit Z for A and B*
0	0	0	0	0	0
1	1,000	0	2,000	0	2,000
2	1,000	333	2,000	1,000	3,000
3	684	858	1,368	2,574	3,942
4	0	1,200	0	3,600	3,600

The profit function along any of the sides of the figure 0–1–2–3–4 is linear, since the total profit is

$$Z = z_Ax_A + z_Bx_B$$

and the relation between x_A and x_B as given by the restrictions is linear. The profit function can therefore be described as a plane, shown by the figure 0–I–II–III–IV in Fig. 12–22. Maximum profit is attained at point 3, where 684 units of A and 858 units of B are produced, and we see that

1. The maximum is obtained at a corner point; this conclusion obeys a theorem in linear programing, which states that the point of optimum must lie at a corner point.

2. The maximum point lies on the line of maximum capacity 1–2–3–4, and it consists of a series of linear functions (see Fig. 12–23).

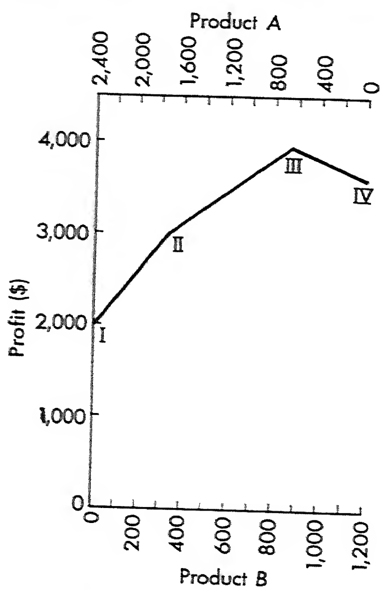


Figure 12–23. ***The profit function along the line of maximum capacity (a two-product system).***

In the preceding example we selected a two-product system again because it is simple to illustrate graphically what happens to the profit functions as we change the ratio between the amounts produced. If we have n products on our program,

the problem can be similarly stated: The best point for which production should be planned lies on the maximum capacity space and is obtained by maximizing the profit function:

$$Z = z_1x_1 + z_2x_2 + \cdots + z_nx_n \tag{12–18}$$

where x_1 is the quantity for product 1, x_2 for product 2, etc., and z_1 is the profit per unit of product 1, z_2 the profit per unit of product 2, etc. This system is, however, subject to certain process output restrictions, namely,

$$\begin{aligned} a_1x_1 + a_2x_2 + \cdots + a_nx_n &\leqslant n_1A \\ b_1x_1 + b_2x_2 + \cdots + b_nx_n &\leqslant n_2B \end{aligned} \tag{12–19}$$

and so on, and where

$$\begin{aligned} x_i &\geqslant 0 \\ A;B;C \cdots &\geqslant 0 \\ n_i &\geqslant 0 \end{aligned} \tag{12–20}$$

This is a typical linear programing problem; *linear* because all the relations between variable may be expressed by linear functions; *programing* because we are trying to define the best program from a large number of possible alternatives. The optimal solution can be sought by use of conventional linear programing techniques, which are well described in several textbooks on the subject.

If we accept the maxim that the production planning and control department is responsible for working out the best ways and methods with which the production facilities should be deployed, then it follows that it should be very much concerned with the optimal solution to the problem stated above for the several reasons discussed in the subsequent paragraphs.

Correlation of production and sales volumes

The optimal solution indicates the best quantities of production that the plant should try to operate at, so that the sales department can take appropriate action in endeavoring to adjust the sales volumes in that direction.

Isoprofit spaces

It is possible to work out what potential profit is lost when deviations from the optimum have to be undertaken. In the case of Fig. 12–23, for instance, we see that deviations from the optimum in favor of product B are preferable to deviations in favor of A. It is often useful to construct what may be termed *isoprofit spaces*. An isoprofit space is the locus of points of plant outputs that yield the same profit. Take the example of the two products presented in Fig. 12–17. The profit is given by

$$Z = z_Ax_A + z_Bx_B$$

If Z is kept constant, the isoprofit space in this case is a straight line, intercepting the x_A axis at $x_A = Z/z_A$ (i.e., all the profit Z is obtained by marketing only product A) and the x_B axis at $x_B = Z/z_B$ (i.e., all the profit Z is obtained by marketing only B). The isoprofit lines are parallel to each other, as only the

value Z changes for each line, as shown in Fig. 12–24. Take, for instance, the isoprofit line $Z = \$3,000$. The intercepts are $3,000/2.00 = 1,500$ units on the product A axis and $3,000/3.00 = 1,000$ units on the product B axis. As all the points on this line yield the same profit of \$3,000, we see that we need not work

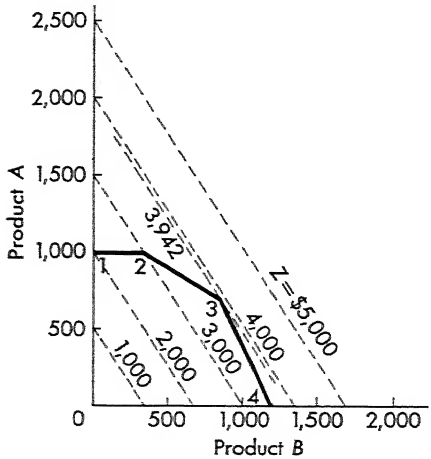


Figure 12–24. ***Isoprofit lines in a two-product system.***

at maximum capacity (point 2) in order to achieve this profit, as any other combination of quantities on the isoprofit line and below the maximum capacity line (i.e., when $x_A < 1,000$) is just as effective. As we increase the profit, the

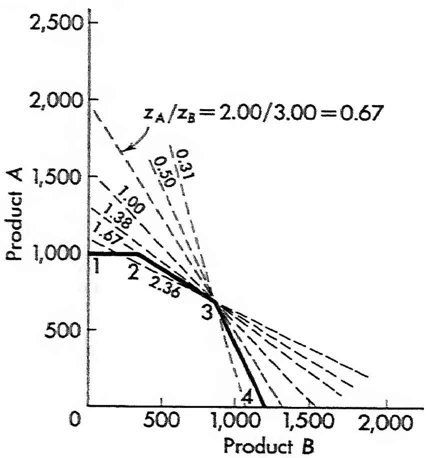


Figure 12–25. ***Effect of the ratio $z_A : z_B$ on the location of the optimal point.***

portion of the isoprofit line below the maximum capacity line becomes shorter and shorter, until eventually for $Z = \$3,942$, the isoprofit line touches the maximum capacity line at one point only (point 3), and this is the optimal point. A higher profit cannot be attained, since all isoprofit lines for $Z > \$3\ 942$ are above the line 1–2–3–4 (see, for example, the line $Z = \$4,000$).

It is clear from this concept of isoprofit lines that the optimal point must be a corner point on the line 1–2–3–4, and the location of this point depends very much on the ratio of z_A/z_B, since this ratio determines the slope of the isoprofit lines. For our case, $z_A/z_B = 2.00/3.00 = 0.67$, and the optimum is at 3. What happens when this ratio changes? If it increases, say, to 1.00 or 1.38 (see Fig. 12–25), the isoprofit line passing through 3 is still the maximum one. But for a ratio of 1.67 the isoprofit line coincides with the portion 2–3 on the maximum capacity line, so that now any combination on 2–3 will yield the same profit (this is rather important, since this situation provides useful and advantageous flexibility in production programing). But as soon as we go below the ratio 1.67, the isoprofit line lies partially below the maximum capacity line, and now point 2 becomes the "extreme" one that would yield maximum profit. Similarly, when we reduce the ratio z_A/z_B below 0.67, we get eventually an isoprofit line that coincides with the portion 3–4 of the maximum capacity line (when the ratio is 0.50), and when the ratio is below 0.50, point 4 becomes the optimal one.

In the case of a three-product system, the isoprofit space is a plane, given by

$$Z = z_1x_1 + z_2x_2 + z_3x_3$$

and the intercepts of the plane at the axes would be

$$x_1 = \frac{Z}{z_1}; \quad x_2 = \frac{Z}{z_2}; \quad x_3 = \frac{Z}{z_3}$$

Any point on this plane within the positive quadrant (i.e., we are dealing with positive values of x_1, x_2, x_3) will yield the same profit. The isoprofit planes in this three-dimensional system would be parallel to each other, since only Z changes from one plane to another (see Fig. 12–26).

Through these isoprofit spaces we gain some information about many points of partial plant capacity that yield the same profit as points relating to maximum capacity. This may give us some clues as to the best policy we ought to adopt in order to adjust our production program and attain optimum performance.

Limiting factors to production increase

The analysis provides us with data about the utilization of each process, and if we manage to set our plant to work at the optimal solution, we know precisely which processes are the factors limiting the increase of our production volumes. Take, for instance, Fig. 12–19. At the optimal point ($x_A = 684$; $x_B = 858$), processes 1 and 4 are working at full capacity and process 2 very nearly at full capacity (its time utilization being 96 per cent), while processes 3 and 5 are far from being highly utilized. This information allows us to draw some important conclusions relating to efficient operation of the plant, such as:

1. The machines at stages 1, 4, 2 should receive first priority in maintenance and repair schedules. Preventive maintenance should preferably not be carried

out during normal hours, in order not to reduce the time efficiency. The machines at stages 3 and 5 may be maintained during normal working hours.

2. If occasional orders are obtained for additional products, they may be accepted as long as they require the use of available time of processes 3 and 5, but deployment of the other processes that operate at full capacity may curb the output targets set for the basic products A and B on our program.

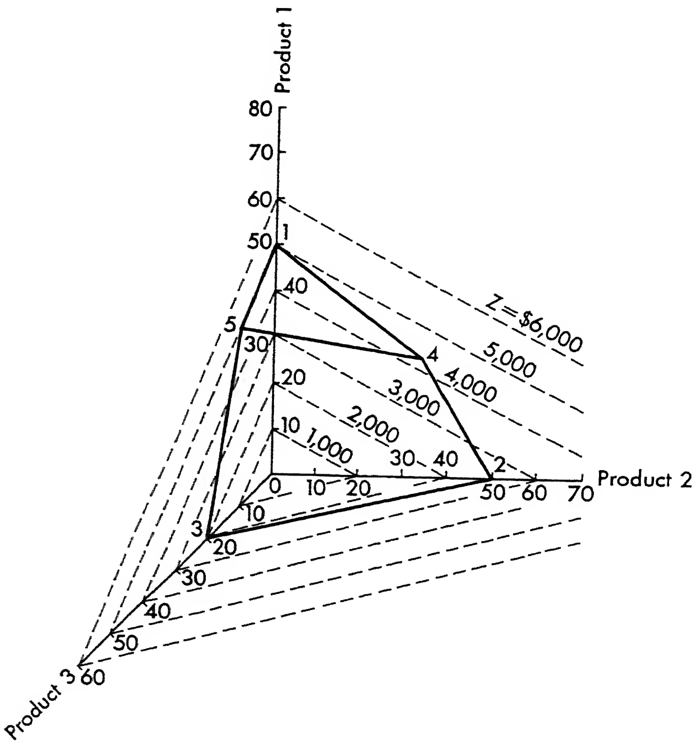


Figure 12–26. ***Isoprofit planes for the three-product program given in Figure 12-21 (1-4-2-3-5 is the maximum capacity surface; z_1 = \$100/unit, z_2 = \$50/unit, z_3 = \$100/unit).***

Choice of expandable process

An "opportunity analysis" may be carried out to indicate which process could expand most profitably. Suppose we have a certain amount of capital that may be invested in order to increase the capacity of one of the processes, and we know by how much we expect this capacity to increase as a result of this investment—which process should be selected for the purpose? In the case of Fig. 12–17, we know from Fig. 12–19 that there is little point in expanding stages 3 and 5. It is even futile to expand stage 2 on its own, since such an expansion would just displace the output restriction of this stage in Fig. 12–17 to the right,

and this would have little effect on the optimal point at 3. In short, if any of the processes 2, 3, or 5 is increased in capacity, this will not increase the maximum plant capacity, and when curves for time efficiency of the processes are drawn, we shall realize that the efficiencies of these processes merely decline. Evidently we have to expand either process 1 or process 4, since these limit the maximum plant capacity. But which one? Suppose that by investing our money, we may expect an increase of 25 per cent in capacity of either process 1 or 4. If we

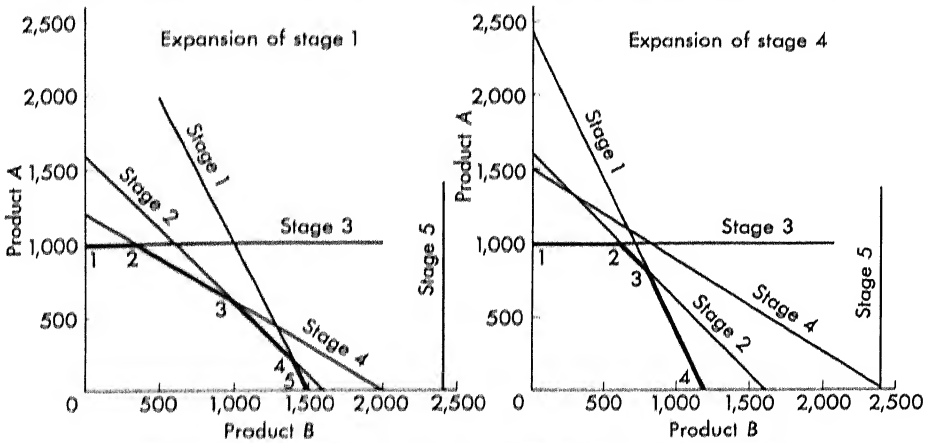


Figure 12–27. ***Examining alternative expansion plans.***

expand process 1, we get the new maximum-capacity line 1–2–3–4–5, as shown in Fig. 12–27; if we expand process 4, we get the line 1–2–3–4. The profit to be expected along the new maximum-capacity lines is given in the accompanying table.

Expansion of Stage 1				*Expansion of Stage 4*			
Point	x_A	x_B	*Total profit*	*Point*	x_A	x_B	*Total profit*
1*	1,000	0	2,000	1*	1,000	0	2,000
2*	1,000	333	3,000	2	1,000	600	3,800
3	600	1,000	4,200	3	800	800	4,000
4	200	1,400	4,600	4*	0	1,200	3,600
5	0	1,500	4,500				

* These points coincide with corner points on the maximum-capacity line in Fig. 12–17.

The profit function is plotted in Fig. 12–28, from which we see that expansion of stage 1 is definitely preferable. The study of opportunities when a larger number of products is involved is similar to that employed for a two-product system.

Reduction of variety: simplification

The analysis of plant capacity provides us with some valuable information for any simplification program we may have in mind. In the two-product system

considered above, the profit plane is inclined upward as we move from the origin, but the slope along the axis x_B is higher than the slope along x_A, since the profit per unit of B is higher. If we could increase the plant output in product B at the expense of product A, unit for unit, it would obviously be advantageous to eliminate A altogether. The capacity restrictions, however, prevent us from doing that, and this is why we sometimes get an optimal profit solution which corresponds to a certain combination of quantities for A and B.

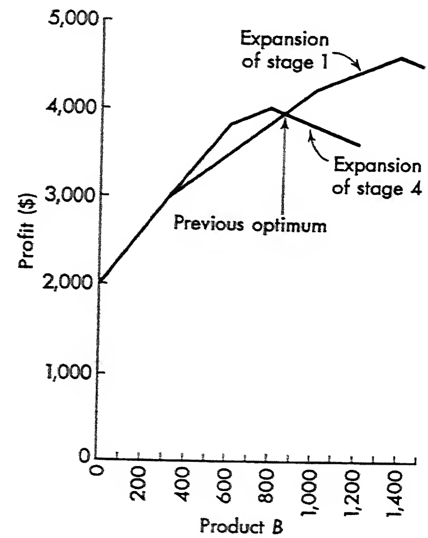


Figure 12–28. ***The profit function along the line of maximum capacity for two alternative expansion plans.***

But when expansion is undertaken, we soon discover whether a basic preference of one product is inherently indicated. In the case cited above for expansion of 25 per cent in process capacity, we see from Fig. 12–28 that the system is moving very rapidly toward the elimination of product A. In fact the profit at point 4 (see preceding table) is not appreciably higher than that at 5, where the program consists of product B alone. Furthermore, we could study the effect of changes in profit per unit on the optimal solution. In the case of a two-product system, such changes would affect the inclination of the profit plane, and it is very important to know to what extent the optimum solution is sensitive to them.

Changes of this kind may occur because of effects of competition in the market on the pricing policy of the firm, or because contemplated technical improvements in the plant may reduce the cost of production. In both cases we can use the capacity analysis to determine at what stage variety reduction is definitely indicated and whether our longer-term plans for expansion or specialization are reasonably stable and are not likely to be greatly affected by these changes. If

our long-term plans involve large sums in capital expenditure or fundamental orientation of the labor force (in training for special skills, for instance), the importance of such studies is self-evident.

Summary

Machine output increases as the cycle time decreases, and assignment of tasks to operators as well as analysis of nonproductive time should be carried out with this point in mind. The ratio of total cycle time to operator time gives an indication as to the number of automatic machines that one operator can look after under deterministic conditions. In practice, however, there are variations in operator times, which cause machine interference and idle time. In the case of automatic machines, which stop at random and demand the services of the operator (his attention time being constant for each stoppage), machine interference increases with the ratio of attention time to machine running time and with the number of machines assigned to one operator.

When the product emerges from the series of operations performed on several machines, the operation times usually differ, and the rate of production is then determined by the slowest process. This often results in excessive idle times for the quick machines, and to reduce it and increase the rate of production, we try to balance the output rates along the line by adding machines at the slow stages. Perfect matching is theoretically attained when the lowest common factor of the operation times is selected as the cycle time.

When n products are manufactured by use of m processes in the plant, the capacity restrictions may often be expressed by linear equations, such as (for process 1)

$$\sum_{1}^{n} a_i x_i \leqslant A$$

where x_i is the quantity produced of product i, a_i is a coefficient that describes the rate of production of product i, and A is the process capacity limitation. Of the many alternatives (i.e., different combinations of quantities of products) that are possible, we may want to maximize the profit, i.e., the function

$$Z = \sum_{1}^{n} z_i x_i$$

where z_i is the profit per unit of product i. Solutions to these problems are derived through linear programing techniques.

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Problems[5]

1. An article is processed on three machines, A, B, and C, as shown in the following table:

	M/c Operation Time (min.)			*Preparation before Production (min./day)*	*Cleaning after Production (min./day)*	*Operation Costs (including labor) ($/hour)*
	Loading and Unloading	*Processing*	*Total*			
A	2.0	2.5	4.5	15	10	10
B	3.0	10.0	13.0	30	10	14
C	2.0	3.0	5.0	15	10	25

A study revealed that if the jigs for machines B and C were to be redesigned, loading and unloading times could be reduced to 2.0 and 1.0 minutes, respectively.

(i) Find the number of pieces produced per day (8 hr.).
(ii) Costing has shown that unless production is increased by 20 per cent the installation of new jigs would not be worth while. Would you recommend redesigning the jigs?
(iii) If the number to be produced is large, suggest changes in the present arrangement and estimate the new production figure.

2. The problem of determining the number of identical machines to be assigned to one operator under deterministic conditions was tackled in this chapter.
A work cycle of a machine involves a minutes preparation time (loading and unloading) and t minutes running time; since the machine stops automatically at the end of its task and requires no supervision while running, the operator can supervise more than one machine. When there is complete matching of the operator cycle time and the machine cycle time, the number of machines that the operator can look after is $n' = (a + t)/a$. If n' is not a whole number, we have to select between n or $n + 1$ machines, where $n < n' < n + 1$. Show that these are the only two alternatives we need consider, i.e., that figures below n or above $n + 1$ may always be ruled out.

3. (i) Show that if an operator is required to supervise several machines with nonidentical cycle times, idle time inevitably occurs (express this idle time quantitatively). (ii) Two identical machines with a cycle time $T = a_1 + t_1$ and one machine with a cycle time $2T = a_2 + t_2$ are assigned to one operator. Show that no perfect matching is possible.

[5] In some of the problems linear programing techniques are required to arrive at final numerical solutions. Readers not familiar with such techniques are advised just to formulate the problems along the lines suggested in the text.

4. An order of 10,000 components have to be produced by a milling operation. Three identical milling machines are available in the shop. The operation times and costs are as follows:

Inserting piece in machine, 1.8 min.	Material cost, \$0.40/piece
Starting machine, 0.1 min.	Labor cost, \$4.00/hour
Machining time, 2.5 min.	Overhead cost, \$6.00 machine hour
Unloading, 0.6 min.	
Inspecting piece, 0.25 min.	
Walking between machines, 0.05 min.	

If a special milling fixture is used, the inserting and unloading times can be reduced to 1.0 minutes and 0.4 minute, respectively. The cost of one fixture is \$800.

(i) If only one operator is employed on this job, there are nine possible combinations of utilizing one, or two, or three machines with or without fixtures. Find the costs per piece incurred by each of these combinations, and hence the one involving the lowest costs per piece. Assume that if machines are not used for this job, they can be utilized for some other work.

(ii) Which combinations would you rule out from the practical point of view?

(iii) Draw a man and multimachine activity chart for the best arrangement.

(iv) If the plant is pressed for an early delivery date, it is possible to expedite production by adding another operator. How will the work be arranged and what will the costs per piece be?

(v) Suppose it is now known that the order will be repeated in the future. What arrangement would you recommend? What saving will be effected by this arrangement over the method recommended in (ii), when the order is repeated?

5. A machine shop has to produce 100,000 components for which the following times have been estimated:

Inserting piece into the machine, 0.30 min.
Starting the machine and engaging feed lever, 0.10 min.
Running time (automatic stop at end of operation), 3.25 min.
Unloading component, 0.20 min.
Inspecting component, 0.35 min.
Walking between machines, 0.10 min.

The following costs figures were provided by the accounting department:

Labor costs, \$4.00/hour
Machine costs, \$5.00/machine hour
Overhead and Materials, \$3.20/unit

(i) Find the cost per piece by using one operator with one, two, or three machines.

(ii) Draw a man and multimachine chart for the three cases.

(iii) Assume four machines are available, which, if not employed for this production, will remain idle; more operators (if required) can be engaged. Recommend and analyze a method where cost per piece will be lowest. How long will it take to produce the batch of 100,000 by your recommended method?

6. A number of operators are performing the same task in a production center, where the same machine is used by each in turn. The task includes:

Check and set the components	0.65 min.
Assemble	0.44 min.
Work with hand tools	0.32 min.
Walk to the machine	0.04 min.
Operate the machine	0.30 min.
Inspect before the next operation	0.10 min.
Operate the machine again	0.15 min.
Inspect the assembly	0.04 min.
Walk back to bench	0.04 min.
Further operation by hand tools	1.14 min.
Final inspection and packing	0.46 min.
Total cycle time	3.68 min.
Cost of machine	\$1.70/hour
Cost of labor	\$3.80/hour

(i) How many operators should be engaged in this production center?
(ii) Draw a multiactivity chart for the center. What maximum output would you expect to attain?
(iii) Find the production costs per piece for (i).
(iv) Would you expect operators' interference? Why?
(v) Would it be worth while allotting the final inspection and packing operations to one man? Compare the costs per unit for this case with the figure you obtained in (ii).
(vi) Similarly, would it be worth while allotting only the first task ("check and set") to one operator?
(vii) Would you recommend combining plans (v) and (vi)? What would the costs analysis be in this case?

7. (i) Show that when one machine is assigned to one operator, the column in the Ashcroft tables for $n = 1$ is obtained by

$$A_1 = \frac{1}{1 + p}$$

(ii) Show that when n fully automatic machines are supervised by one operator (assume no random stoppages for repairs),

$$A_n = \frac{n}{1 + p}$$

(iii) If the number of semiautomatic machines n is increased, show that the maximum value of A is given by

$$A_\infty \to \frac{1}{p}$$

(iv) For (a), $p = 0.20$, (b), $p = 0.30$, plot the change of A_n with n and the efficiency A_n/n with n. Compare your figures with those obtained for fully automatic machines [derived by (ii)].

(v) If $n = 8$ machines, plot the change of A_8 and the efficiency when p is increased from 0 to 0.5, and compare with values obtained for the case of fully automatic machines.

8. In a small plant engaged in the manufacture of biscuits, the process is composed of the following operations (see Fig. 12–29):

(1) The raw materials are brought from store A to temporary storage B; ·
(2) From B to mixers C, D;
(3) From mixers to rolling and forming machine E; then to tables F for arranging in forms and trays;
(4) From F to shelves G, waiting for oven;
(5) From G to oven H;
(6) After baking; to tables I for sorting and inspection;
(7) Then to packing section J;
(8) Finally to storage K, ready for delivery.

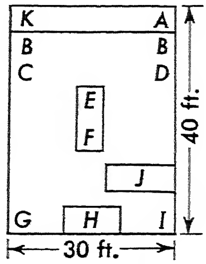


Figure 12–29.
A small plant for making biscuits

It was found convenient to measure the volume of production by the number of cartons, and the appropriate times are as follows:

Mixer C: Capacity 1 carton; preparation, 3 min.; mixing, 10 min.; pouring from mixer, $1\frac{1}{2}$ min.; cleaning at end of day, 6 min.

Rolling and Forming E (hand-operated): Operation, 15 min./carton; cleaning at end of day, 15 min.

Oven H (gas-operated): Capacity, 5 cartons, warming up before baking, 20 min.; baking time, 40 min.; loading, $2\frac{1}{2}$ min.; unloading, $2\frac{1}{2}$ min.; cleaning at end of day (including trays), 25 min.

Other operations:
Handling time: take an average figure of 0.2 min./carton/each transportation. Arranging in trays on tables F, 12 min./carton; inspection, 1 min./carton; packing, 8 min./carton; cleaning and sweeping of shop, 30 min./day.

Assume: 6-day week, 8 hr. a day on weekdays, 5 hr. on Saturdays. No dough can be left overnight.

Your Task:

(i) Draw an operation process chart.
(ii) Draw a flow diagram (as in Fig. 7–10 in Chapter 7) of present layout (scale $\frac{1}{2}$ inch to 1 foot).
(iii) Discuss the present layout. Suggest a new layout and draw a flow diagram. Suggest suitable place for running-water basins.
(iv) Draw activity machine charts for C, D, E, H.
(v) Calculate the maximum possible output with present equipment.
(vi) Calculate the total time per carton, per day, per week, spent on handling.
(vii) How many operators should be engaged to produce the production figure found in (v)?
(viii) Discuss means of increasing the maximum production figure found in (v).

9. In the shop described in Problem 8, it is required to produce four different products:

Product I: About 50 per cent of total volume; operation times as above. Product requires rolling in Rolling Machine E1.

Product II: About 20 per cent of total volume; requires rolling in E1.

Operations:

Premixing	8 min.
Second addition of materials to mixer	1 min.
Mixing	6 min.
Additional work in decoration	10 min./carton
Baking	60 min.

Product III: About 20 per cent of total volume; requires forming in E2.

Operations:

Mixing	As No. I
Additional work in decoration	8 min./carton
Baking	60 min.

Product IV: About 10 per cent of total volume; requires forming in E2.

Operations:

Mixing	10 min.
Baking	30 min.

Note: Baking temperature is appreciably higher here than that required for products I, II, III.

Additional Data: Mixer, rolling, and forming machines need cleaning (5 min.) when changing from one product to the other. Oven needs 10-min. temperature adjustment period when switching from product IV to the others, and vice versa.

Your Task:

(i) Determine the weekly production schedule. Draw weekly machine activity charts. Find the total weekly output and the percentage of total of each product.

(ii) The most costly equipment involved is the oven. Suggest means of utilizing this equipment to its maximum. Repeat (i) for this revised method.

(iii) For (ii) find the number of operators required and draw their activity charts.

10. A product is manufactured through ten operations in the following sequence:

Op. No.	*Machine*	*Operation Time (min.)*
1	1	3.0
2	2	4.2
3	3	6.0
4	4	1.2
5	5	0.6
6	6	3.6
7	7	6.0
8	8	2.4
9	9	3.0
10	10	1.8

If the product is manufactured on a flow basis, the demand by the store being 1,600 pieces per week, suggest machine balancing for minimum costs per piece, assuming:

(i) The cost per machine is virtually the same when it is running or idle.

(ii) An operator can supervise no more than four machines, which may belong either to one operation in the flow sequence or to two successive operations in the sequence (e.g., he is allowed to supervise two machines on operation 5 and two on operation 6 but not two on 5, one on 6, and one on 7, or two on 5 and two on 7).

(iii) Labor overtime costs 50 per cent more than regular time.

11. Two products are produced in a plant where five processes are mainly employed. The maximum output in units per day for each process when employed for one product only is given in the following table:

Process	*Product 1*	*Product 2*
I	300	100
II	200	—
III	120	—
IV	100	200
V	200	150

(i) Describe graphically the maximum plant capacity for any combination of quantities of products 1 and 2 in the schedule.

(ii) Find the efficiency for each process as the quantity produced of product 1 varies from 0 to maximum.

(iii) If the profit per unit is \$4.00 for product 1 and \$3.20 for product 2, find the point at which profit would be maximum. Show the profit function in a three-dimensional system.

(iv) A sum of money is available for expansion of the processes, and if spent only on one process at a time, the maximum output of process I could be increased by 30 per cent, process II or III by 20 per cent, process IV by 40 per cent, and process V by 10 per cent. Assuming that a proportion of this investment on one process would yield the same proportion of increased output (e.g., if half the money is spent on process I and the other half on process II, the maximum outputs would be increased by 15 per cent and 10 per cent respectively), how should this money be spent in order to increase the profit as much as possible?

12. Three products processed through three departments have the following maximum output figures (units per week):

	Product		
Department	*1*	*2*	*3*
I	5,000	8,000	10,000
II	7,500	4,000	3,000
III	4,500	4,000	4,000

(i) Show the maximum capacity surface in a three-dimensional model.

(ii) If the profits per unit are \$5.00 for product 1, \$6.00 for product 2, and \$6.50 for product 3, calculate the profit for the corner points of this surface and determine the optimal one.

(iii) It has been suggested that the capacity of department III should be doubled (by adding another production line). What are your views?

(iv) There is chance that department I will get a contract from another factory to process 2,000 pieces a week. Is this desirable? What effect would this have on your considerations above?

13. A factory produces n different kinds of skirts and m different blouses, which can be marketed either separately or in "sets," each set comprising one skirt and one blouse. The number of possible sets is nm. The sales department recommends that there should be at least 4 different skirts, 12 blouses, and

16 sets, and it is estimated that the total number of skirts required for the market during the period under consideration is between 4,000 and 6,000, and the total number of blouses required is between 8,000 and 12,000. It is further known that design costs for a skirt amount to \$200 and design costs for a blouse to \$300, but total design costs should not exceed \$5,000.

If the profit per blouse is \$4.00 and the profit per skirt is \$5.00, irrespective of whether they are marketed separately or as a part of a set,

(i) Express the above restrictions in algebraic form.

(ii) Find the optimal point for maximum profit. How many skirts and blouses (number of models and quantities) should be planned?

(iii) If the profit per "set" is \$10.00, how would your former conclusions be affected?

(iv) Due to reorganization of the plant, the production department estimates that maximum plant capacity will be 12,000 units (irrespective of whether these are blouses or skirts). Reconsider (i), (ii), and (iii) in the view of this new restriction.

14. (a) Referring to Fig. 12–25, plot the change in profit for points 2, 3, 4 when z_A/z_B is varied between 0.1 and 5.0 (the profit at point 3 when $z_A/z_B = 0.67$ is to be taken as 100). (b) For Fig. 12–26:

(i) Find the optimal profit.

(ii) Find the point of full plant capacity that would yield a profit of \$4,000.

(iii) Plot the change in profit along the line 4–5.

15. A plant is engaged on the production of five products, which are processed through three departments; the number of hours required to finish each is indicated in the table:

Department	*Product 1*	*2*	*3*	*4*	*5*	*Max. Hrs. Available per Week*
I	0.7	0.8	4.0	0.5	1.2	160
II	0.8	1.2	3.2	0.8	1.2	160
III	1.5	1.7	5.0	0.6	1.0	160

(i) If the profit for the products is \$6.00 for a unit of products 1, 2 but only \$4.00 for the others, what quantities per week should be planned to optimize profit?

(ii) If the cost per hour in department I is \$20; in department II, \$40; and in department III, \$50, what quantities should be planned to minimize the cost of production? Compare your results with those obtained for (i).

16. (i) A production engineer was faced with the problem of evaluating operational costs of a new machine, where the following data were given: The effective life of the machine is E days, after which major cleaning and repairs have to be undertaken. The effective time can be expressed as $E = i + \alpha t$, where i is the idle time, t the running time, and α a factor that allows for quicker wear of the machine when running. If the operation costs (including maintenance) of the machine is g dollars per day when it is running, and h dollars

per day when it is idle, find an expression for the average operational costs per day as a function of t.

(ii) The engineer was asked to select one of two machines, the initial cost of which was the same, and the effective life being 1,000 days for each; the other data were as follows:

	Machine A	*Machine B*
Operation costs when running, \$/day	$g = 20.00$	30.00
Operation costs when idle, \$/day	$h = \ \ 2.00$	1.00
Life factor	$\alpha = \ \ 2.50$	2.00

If the machine is known to run 20 per cent of the time, which one should the engineer select to have lower average operational costs per day?

(iii) At what percentage of running time are the two machines equal as far as operational costs per day are concerned? Which machine should then be selected?

13

SCHEDULING

Scheduling is the final stage in production planning, the stage at which all the production activities are coordinated and projected on a time scale. A production schedule is in fact a timetable that tells us what machine or department should be doing what and when.

It was pointed out in the preceding chapter that machine loading and scheduling is a procedure with which we try to match the requirements set out in the production order (quantities, dates of delivery) with the available facilities. There may be several ways in which the requirements can be met, several ways of routing the products through machines or processes, of determining the sequence of products and a priority scale, of determining when and how to expand facilities by use of overtime. That method which (after an appropriate analysis) is found to be the most satisfactory is formulated as a production schedule.

How is the best method selected? The very act of differentiating between better and worse methods implies that we have a measure of effectiveness, a criterion with which each solution can be quantitatively compared and evaluated. Usually this criterion is based on costs, and the solution that ensures the attainment of the objectives of the production order at the lowest possible costs is considered the best one.

Forms of Schedules

In what form should a production schedule be presented? This depends on the purpose of the schedule. Several forms can be found in this book, some of which were discussed in previous chapters.

1. *A production flow program* (see Figs. 9–7 and 9–8): If a number of components or assemblies have to be manufactured for the final assembly line and these components are to be made concurrently, the master flow program takes into account the sequence of operations and indicates when work on each component should start, in order to comply with the required date for completion of the product.

2. *A production master program for integrating work on large objects* (see Figs. 9–10 and 9–11): This program is particularly useful in cases of static layouts where the tools, materials, machines, and teams of operators flow from one

object to another, and it is extremely important to coordinate the activities of these facilities by appropriate phasing.

3. *A cumulative output progress chart* (as in Fig. 9–9): When a new job undertaken is likely to last for several months or years, the program must take into account the initial period required for preparation and the incremental change in the rate of production due to the learning curve. This rate is normally reduced toward the end of the job, to allow for gradual phasing out, and the effect of changes in the projected rate of production on the output is clearly indicated in the cumulative output progress chart.

4. *An outline master program* (as in Fig. 12–1): This program merely translates the general requirements specified by the sales department and, like that in Fig. 9–7, it is useful as a basis on which final and detailed schedules can be worked out.

5. *A schedule for breakdown of orders* (such as Fig. 13–1): This is another form of expressing the rate at which the work should progress, and it is particularly useful in decentralized scheduling because it specifies only quantities and the dates on which they should be completed. In other words, it indicates production targets, but it does not suggest how they can be achieved, and the responsibility for planning the activity of the production center related to the appropriate target rests with the production department in question.

DEPT. Subassemblies E — DATE 4/1 — NO. 601

COPIES TO Depts C, D, F; Assembly line; S-A store — ISSUED BY N. A.

			REQUIRED FROM WEEK ENDING ON												
			1	2	3	4	5	6	7	8	9	10	11	12	
ORDER NO.	DESCRIPTION	TOTAL QTY. REC'D	4/8	4/15	4/22	4/29	5/6	5/13	5/20	5/27	6/3	6/10	6/17	6/24	TO NEXT SCHED.
129	Front axles, J-32	600	50	60	60	60	80	60	60	70	100				—
131	Rear axles, J-41	600	50	60	60	60	60	70	70	70	100				—
132	Shafts D-62	400	100	100	100	100									—
140	D-H-Shafts G-61	1200		20	60	60	80	80	100	100	120	120	120	140	200

REMARKS

Figure 13–1. ***Breakdown of a production order into intermediate targets.***

6. *A cycle schedule* (as in Figs. 14–2 and 14–3): If the plant is engaged on batch production in a cyclic fashion, this schedule shows how the cycle time is distributed between the various products.

7. *A detailed schedule* (as in Fig. 13–2): This shows in a planned multiactivity chart how the facilities should actually be employed during the production period.[1]

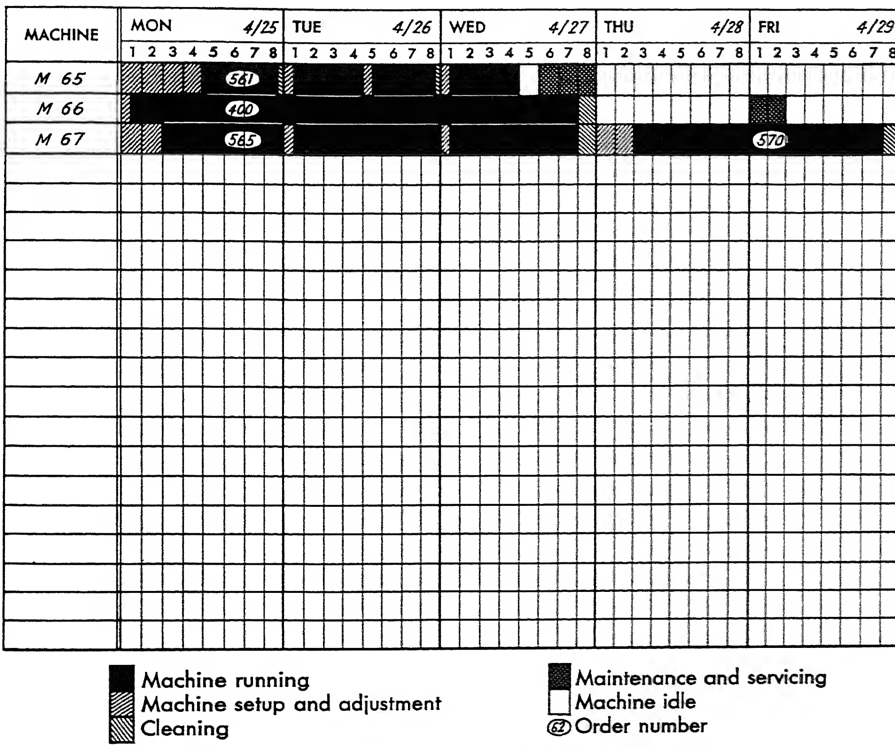


Figure 13–2. ***An example of a detailed schedule.***

Loading and scheduling

The responsibility for loading and scheduling may rest either with the central statistics office of the production planning and control department or with the manufacturing departments, depending on whether production scheduling is a centralized or decentralized function. The tendency to decentralize scheduling

[1] A production period may be defined as that interval of time for which a detailed production program is prepared, followed by another such period for which a separate plan is worked out. This period is usually selected to cover easily handled time units, such as a week, four weeks (which, incidentally, is preferable to a calendar month, since each period can start on a Monday and all periods are equal in length), eight weeks, etc.

in the case of process plant layouts perhaps stems from the fact that production department supervisors are better acquainted with what goes on in their own domains and are therefore more likely than the central planning office to produce a workable schedule. With the use of more mathematical techniques in constructing optimal schedules and with the introduction of computers to central planning offices, it is quite likely that the trend will be reversed and centralized scheduling will become more common, so that production facilities can be made better use of.

Techniques leading to the formulation of production schedules depend on the type of production, type and frequency of jobs, demand patterns, and flexibility offered by available processing time. Scheduling problems vary in character and in complexity and very often several techniques in mathematical programing have to be combined to tackle any particular situation. Furthermore, scheduling problems are often dynamic in character. Situations in which the governing parameters remain unchanged and call for a static schedule are rather rare. As time passes, requirements often fluctuate and vary in character, the facilities and their capabilities change, and with these changing circumstances the problem has to be continuously restated and resolved. An interesting example to illustrate the intricate problems that may arise in production scheduling and inventory control was cited by Cinch Manufacturing Corporation, Chicago. The company reported[2] in 1957 that it made 25,000 different types of electronic and automotive hardware components. Approximately 5,000 orders had to be processed per month (or 250 per day), of which 20 per cent were rush orders (15 per cent received over the telephone plus 5 per cent by cables), 60 per cent were required within one to three weeks, and 20 per cent within one to three months. As some 80 per cent of the sales were to the electronic industry, in which frequent changes in designs and specifications are fairly common, the orders included many variations of the same products, and inventory problems became rather formidable.

Basic scheduling problems

Some of the basic scheduling problems to be found in literature include:

Flow production scheduling for fluctuating demand (sometimes referred to as smoothing problems)
Batch production scheduling, when products are manufactured consecutively
The assignment problem
Scheduling orders with random arrivals
Product sequencing

Let us now examine these problems in some more details.

[2] "Case studies in production forecasting, planning and control," American Manufacturing Association, manufacturing series No. 223, 1957.

Flow Production Scheduling for Fluctuating Demand

When the sales of a certain product are subject to seasonal fluctuations, management may decide to meet the demand in one of the following ways:

1. *Have a static production program*, coupled with an inventory large enough to satisfy the fluctuating demand. The inventory level would fluctuate according to the demand pattern, replenishment being provided by a constant flow from the plant. This method is greatly favored by the production department, since it simplifies planning, ensures higher machine utilization, allows better supervision and control, and promotes a sense of security among the workers. Average stock level is high, however, thus tying up capital and involving high carrying costs.

2. *Have a fluctuating production program*, to cater to the changing demand, and keep a constant inventory level. The purpose of the inventory in this case is to provide a safety cushion between production and marketing. Any change in the demand pattern requires a certain time lag before production can follow suit, and the safety stock enables management to satisfy demand in the interim period. The stock level does not, strictly speaking, remain constant, but the fluctuations and the average stock level are fairly low, compared with the previous method.

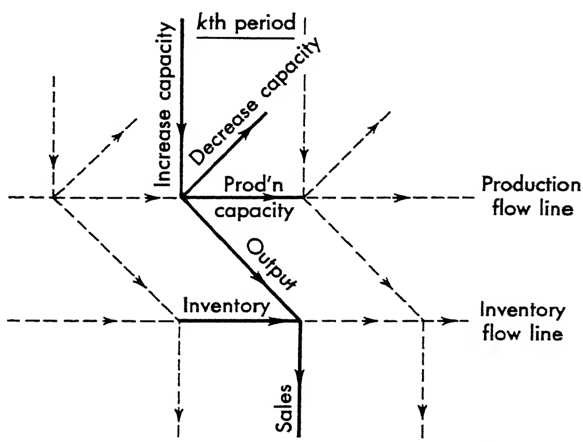


Figure 13–3. ***A graphical presentation of flow production by network flow.***

3. *Have a combination of the two systems*, so as to bring the total costs to a minimum. The problem is, therefore, to achieve a proper balance between the amount of fluctuations in the production program and those of the stock level.

This problem of production for fluctuating demand may be illustrated graphically as a network flow.[3] Sales forecasting and production planning are

[3] Suggested by Te Chiang Hu and W. Prager in their paper "Network analysis of production smoothing" (Naval Research Logistics Quarterly, March, 1959).

broken down into periods (as we have seen in the schedules in Figs. 9–9 and 13–1). The network consists of a chain of identical units, each unit representing one for production and the other for inventory. The flow in the network is as indicated by the arrows in Fig. 13–3 (in which the kth time period is shown), and the flow must always be positive. A certain amount of production capacity is carried over through the productive flow line from the preceding period. At the beginning of the period one can plan to increase the capacity (by adding labor, machines, overtime, subcontracting) or to reduce it (by lowering the labor force, etc.). The output flows into the inventory line, from which a certain quantity is tapped out for sale. For the sake of simplicity it has been assumed in Fig. 13–3 that the output of the kth period reaches the inventory flow line only at the end of the period and that the sales volume of the period also leaves at the end of the period, but modifications to this graphical model can easily be incorporated whenever necessary.

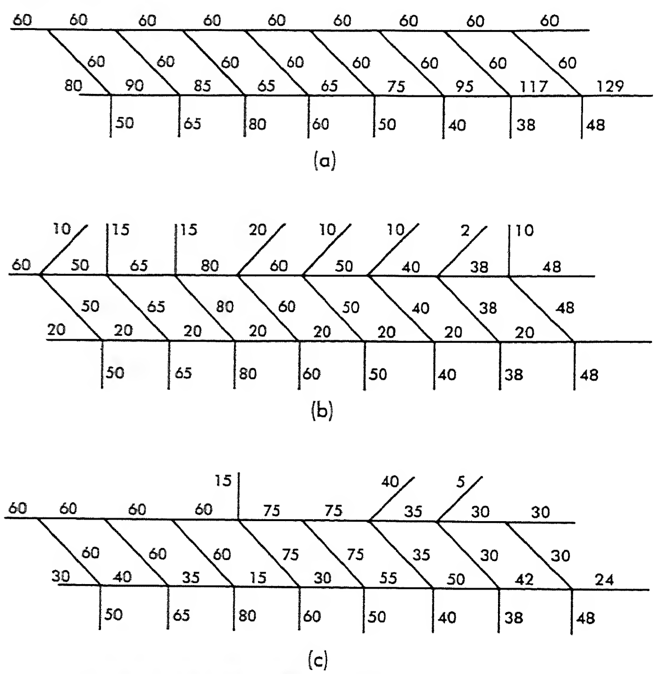


Figure 13–4. ***Policies in analyzing production smoothing problems:***

(a) A static production program.
(b) A static inventory program.
(c) A combination of (a) and (b).

At any point of intersection in the network we must have equilibrium of flow; namely, the flow into the intersection is equal to the flow coming out of it. On the production flow line: (previous capacity) + (any increase) — (any decrease)

= (output) = (capacity carried over to the next period). On the inventory line: (previous inventory) + (output) = (sales) + (inventory carried over to the next cycle).

Bearing these conventions in mind, we can omit any arrows in drawing the network and indicate only the quantities in each branch, as shown in Fig. 13–4. The first policy mentioned above for coping with fluctuating demand (namely, that of a static production program) is illustrated in Fig. 13–4a. The bottom figures indicate the sales volumes, and while the flow in production line remains constant, the inventories fluctuate considerably. Figure 13–4b illustrates the second policy of a constant inventory level, while Fig. 13–4c shows one possible combination.

Production loading[4]

Suppose there are n periods in our planning program, the expected sales volumes during each period being S_1 (for period 1), $S_2, S_3, \ldots, S_n$. The quantities actually produced are $Q_1, Q_2, \ldots, Q_n$. If we assume that a quantity produced during a period (say, one week) becomes available to the sales department only at the end of the period, we must have some initial stock Q_0 to start with, otherwise the demand during the first period cannot be met. The inventory level available is Q_0 at the beginning of the first period, $Q_0 + Q_1$ at the beginning of the second, $Q_0 + Q_1 + \cdots + Q_{i-1}$ at the beginning of the ith period, etc.

If the cost of carrying one unit in stock for one period is c_0, the cost of a unit carried from the initial stock Q_0 to the ith period would be ic_0; the cost of a unit produced in the first period and carried to the ith period would be $(i - 1)c_0$, etc. These unit costs are shown in Table 13–1, where c_1 is the cost of producing one unit, and it was assumed in the table that this does not change from period to period. Suppose now that c_1 is the cost incurred by use of regular time; but if we have to use overtime, the cost would be c_2 per unit, and if we have no more overtime available (or are reluctant to use it), we can still subcontract some of

Table 13–1

Unit Cost

(Carrying Costs and Regular Time)

Sales Periods

	1	*2*	*3*	...	*n*	*Total Capacity*
(Initial inventory)	c_0	$2c_0$	$3c_0$	...	nc_0	Q_0
Production periods						
1	c_1	$c_1 + c_0$	$c_1 + 2c_0$	...	$c_1 + (n - 1)c_0$	A_1
2	—	c_1	$c_1 + c_0$	...	$c_1 + (n - 2)c_0$	A_2
3	—	—	c_1	...	...	A_3
.	.	.	.		.	.
.	.	.	.		.	.
.	.	.	.		.	.
n	—	—	—		c_1	A_n
Requirements	S_1	S_2	S_3		S_n	

[4] The remainder of this section may be omitted at first reading.

the production volume at the cost of c_3 per unit. We can easily construct two more tables (similar to Table 13–1), one for the use of overtime, one for the use of subcontracting.

Table 13–2 shows the quantities obtained from the various sources: Q_{11} is the quantity in period 1 from regular time; Q_{12} is obtained in period 1 from overtime; Q_{13} is still in period 1 but from subcontracting. The cost of producing Q_1 in period 1 is

$$c_1Q_{11} + c_2Q_{12} + c_3Q_{13}$$

Table 13–2

Quantities

Sales Periods

	1	*2*	*3*	...	*n*	*Total Capacity*
(Initial inventory)	Q_0			...		Q_0
Production periods (a) (b) (c)						
1. $Q_{11} + Q_{12} + Q_{13}$	Q_1			...		$A_1 + B_1 + C_1$
2. $Q_{21} + Q_{22} + Q_{23}$		Q_2		...		$A_2 + B_2 + C_2$
3. $Q_{31} + Q_{32} + Q_{33}$			Q_3	...		$A_3 + B_3 + C_3$
.						
.						
.						
n $Q_{n1} + Q_{n2} + Q_{n3}$				...	Q_n	$A_n + B_n + C_n$
Requirements	S_1	S_2	S_3	...	S_n	

Note: (a) regular time; (b) overtime; (c) subcontracting.

The total cost of production is therefore

$$F_{\mathrm{I}} = \sum_{i=1}^{n} \sum_{j=1}^{3} c_iQ_{ij} \tag{13–1}$$

The inventory cost[5] in the first period is $c_0(Q_0 - S_1)$ and in the second period, $c_0(Q_0 + Q_1 - S_1 - S_2)$, the total cost of carrying the inventory being

$$F_{\mathrm{II}} = c_0[n(Q_0 - S_1) + (n-1)(Q_1 - S_2) + (n-2)(Q_2 - S_3) + \cdots + (Q_{n-1} - S_n)] \tag{13–2}$$

where $Q_1 = Q_{11} + Q_{12} + Q_{13}$

$Q_2 = Q_{21} + Q_{22} + Q_{23}$

.

.

.

$Q_n = Q_{n1} + Q_{n2} + Q_{n3}$

We have to find such values for Q_{11}, Q_{12}, etc., that would reduce $F_{\mathrm{I}} + F_{\mathrm{II}}$ to a minimum, but at the same time we have to comply with restrictions of capacity.

[5] It is assumed in this expression that inventory charges are payable only per unit remaining in the store. If, however, the cost is proportional to the time a unit remains in stock, the cost for the first period would be $c_0 \times$ average number of units in stock. For approximately linear consumption during the period, this would be $c_0(Q_0 - \frac{1}{2}S_1)$.

The total amount produced on regular time should not exceed A_1 during period 1, A_2 during period 2, etc. The restrictions on overtime are B_1 for period 1, B_2 for period 2, etc., and on subcontracting it is C_1, C_2, . . . (as indicated in Table 13–2).

This is a linear programing problem. Furthermore, it was pointed out by E. H. Bowman[6] that Table 13–1 is a typical transportation cost table, the values in each cell being the costs to "ship" a unit from a certain production period to a sales period. Some of the cells do not exist (marked by dashes in Table 13–1), since it is impossible to produce something in the second period and sell it in the first. The formulation of the problem as one of the transportation type has great advantages in actual computations, since the normal linear programing problem solved by the simplex method usually requires longer time for calculations.

Multiproduct Scheduling in Batch Production

If the rate of production is higher than the rate of consumption, the plant has to resort to batch production, and if the available time is to be fully utilized, the plant must undertake to produce several products in succession. The procedure becomes as follows: The plant is fully engaged in producing one product until a certain predetermined inventory level is attained, and then it proceeds with the production of another product. When the second product is produced in sufficient quantities, the plant proceeds to the third product, and so on. In the meantime, the stock level of the first product slowly declines owing to the regular consumption rate, until such a level is reached that the plant must start again on that product so that the sales department will be able to meet the demand for it.

The problem may therefore be defined as follows: The plant is producing n products one at a time, and the manufacturing cycle is concluded when all products have been produced, so that the cycle length is determined by the total time required to produce all products in the cycle. The quantities produced must be such that they will cater to this cycle time precisely; otherwise the plant will either run out of stock prematurely or have excessive stocks which have to be carried over to the next cycle. At the same time, the quantities are governed by the batch production theory presented in Chapter 10. The problems that we have to analyze are therefore:

How to go about optimizing the whole schedule, i.e., the whole cycle, rather than one product at a time.

What criterion should be used for optimization?

Does the optimized schedule specify quantities for the individual products that are compatible with the optimal batch sizes computed by the methods in Chapter 10?

If not, how can the two objectives be bridged?

[6] E. H. Bowman: Production scheduling by the transportation method of linear programming (J. Operations Research Society, February, 1956).

Table 13–3

Assignment of n Products to m Machines
Matrix Table of Quantities Produced and Rates of Production

	Product Number							
Machine No.	*1*	*2*	*3*	$\cdots$	*i*	$\cdots$	*n*	*Max. Capacity*
1	$a_{1,1}/Q_{1,1}$	$a_{2,1}/Q_{2,1}$	$a_{3,1}/Q_{3,1}$	$\cdots$	$a_{i,1}/Q_{i,1}$	$\cdots$	$a_{n,1}/Q_{n,1}$	A_1
2	$a_{1,2}/Q_{1,2}$	$a_{2,2}/Q_{2,2}$	$a_{3,2}/Q_{3,2}$	$\cdots$	$a_{i,2}/Q_{i,2}$	$\cdots$		A_2
3	$a_{1,3}/Q_{1,3}$	$a_{2,3}/Q_{2,3}$	$a_{3,3}/Q_{3,3}$	$\cdots$	$a_{i,3}/Q_{i,3}$	$\cdots$		A_3
.								.
.								.
.								.
j	$a_{1,j}/Q_{1,j}$	$a_{2,j}/Q_{2,j}$	$a_{3,j}/Q_{3,j}$	$\cdots$	$a_{i,j}/Q_{i,j}$	$\cdots$	$a_{n,j}/Q_{n,j}$	A_j
.								.
.								.
.								.
m	$a_{1,m}/Q_{1,m}$	$a_{2,m}/Q_{2,m}$	$a_{3,m}/Q_{3,m}$	$\cdots$	$a_{i,m}/Q_{i,m}$	$\cdots$	$a_{n,m}/Q_{n,m}$	A_m
Requirements	Q_1	Q_2	Q_3		Q_i		Q_n	

$a_{i,j}$ = rate of production of the ith product on the jth machine.
$Q_{i,j}$ = quantity produced of the ith product on the jth machine.

These problems, which are an extension to the batch production theory, are fully discussed in Chapter 14 together with a computational example, since space and the somewhat special nature of these problems do not allow their inclusion in this chapter.

A more intriguing and complex situation occurs when products are manufactured on a batch basis, some consecutively and some concurrently, with production time overlapping to a varying degree. This problem calls for a combination of linear programing and the techniques suggested in the next chapter, but this is beyond the scope of the present volume.

The Assignment Problem[7]

In the case of job or batch production an array of tasks is defined at the beginning of a production period, and these tasks have to be performed by use of available processing time of several facilities. The problem is to assign the tasks to the machines or to the operators in such a manner as to minimize the cost of processing time during the period.

Distribution according to capacity

The assignment problem is obviously related to the effective utilization of process capacity, and in that respect it is similar to the problem of determining the best "mix" (i.e., proportion) of products in a production program, which was referred to in the last chapter. However, now the requirements are given, and there is no question of changing the quantities to obtain a better "mix." What we are required to do is assign these given tasks to the available machines. Suppose there are n products to be made in the next production period, the quantities being $Q_1, Q_2, \ldots$, respectively. There are m machines or processes on which these products can be manufactured; hence the rate of production and cost of operating the machines may differ. If product 1 is distributed among the machines so that $Q_{1,1}$ is the quantity produced on the first machine, $Q_{1,2}$ the quantity produced on the second machine, etc., then the total requirement for product 1 is

$$Q_1 = Q_{1,1} + Q_{1,2} + \cdots + Q_{1,j} + \cdots + Q_{1,m} = \sum_{j=1}^{m} Q_{1,j}$$

Similarly, the other products are distributed among the m machines, as shown in Table 13–3. Each column shows the distribution of a certain product to m machines, and each row shows the assignment of n products to a certain machine. Thus, machine 1 will produce $Q_{1,1}$ of product 1; $Q_{2,1}$ of product 2; $Q_{3,1}$ of product 3; etc. The total time loading on machine 1 is

$$\frac{Q_{1,1}}{a_{1,1}} + \frac{Q_{2,1}}{a_{2,1}} + \cdots + \frac{Q_{i,1}}{a_{i,1}} + \cdots + \frac{Q_{n,1}}{a_{n,1}} = \sum_{i=1}^{n} \frac{Q_{i,1}}{a_{i,1}}$$

[7] This section may be omitted at first reading.

where $a_{i,1}$ is the rate at which machine 1 can produce product i. But, since the capacity of the machines is restricted, the total loading should not exceed the maximum capacity A_1 of machine 1, or

$$\sum_{i=1}^{n} \frac{Q_{i,1}}{a_{i,1}} \leqslant A_1 \tag{13–3}$$

and likewise for machine 2,

$$\sum_{i=1}^{n} \frac{Q_{i,2}}{a_{i,2}} \leqslant A_2$$

and in general, for machine j,

$$\sum_{i=1}^{n} \frac{Q_{i,j}}{a_{i,j}} \leqslant A_j \tag{13–4}$$

Furthermore, the quantities produced must be positive:

$$Q_{i,j} \geqslant 0 \tag{13–5}$$

If our object is to minimize the cost of production, and if the cost of producing one unit of product i on machine j is $c_{i,j}$, the function to minimize is

$$C = \sum_{i=1}^{n} \sum_{j=1}^{m} c_{i,j} Q_{i,j} \overset{!}{=} \text{minimum} \tag{13–6}$$

and this minimum must be found subject to the restrictions stated earlier. This is a linear programing problem that may be solved by the simplex method.

Cost of operating production facilities

Very often the cost of operating a machine may be solely dependent on the time it is kept busy, irrespective of which product is actually produced. If c_1 is the cost per unit time (hour or day) of actively using machine 1, the total cost for machine 1 is

$$c_1 \sum_{i=1}^{n} \frac{Q_{i,1}}{a_{i,1}}$$

The total cost function for all the machines is

$$C = c_1 \sum_{i=1}^{n} \frac{Q_{i,1}}{a_{i,1}} + c_2 \sum_{i=1}^{n} \frac{Q_{i,2}}{a_{i,2}} + \cdots + c_m \sum_{i=1}^{n} \frac{Q_{i,n}}{a_{i,n}}$$

or, in short:

$$C = \sum_{j=1}^{m} c_j \sum_{i=1}^{n} \frac{Q_{i,j}}{a_{i,j}} \overset{!}{=} \text{minimum} \tag{13–7}$$

This expression simply means: Find the total time each machine will be engaged on production, multiply this time by the appropriate cost per unit time of using the machines, and add all these costs terms for all the machines to obtain the total costs for the production period in question. Our object is to minimize the total costs.

Example

Four products have to be processed through the plant, the quantities required for the next production period being:

Product 1	2,000 units
Product 2	3,000 units
Product 3	3,000 units
Product 4	6,000 units

There are three production lines on which the products could be processed; the rates of production in units per day and the total available capacity in hours are given in Table 13–4. The cost of using the lines is \$600, \$500, \$400 per day, respectively.

Table 13–4

Assignment of Four Products: Rates of Production in Units Per Day

Production Line	*Product* 1	2	3	4	*Max. Line Capacity (days)*
1	150	100	500	400	20
2	200	100	760	400	20
3	160	80	800	600	18
Total requirements (units)	2,000	3,000	3,000	6,000	

Formulation of the problem: The matrix of quantities produced is

$$\begin{array}{lcccc} & Q_{1,1} & Q_{2,1} & Q_{3,1} & Q_{4,1} \\ & Q_{1,2} & Q_{2,2} & Q_{3,2} & Q_{4,2} \\ & Q_{1,3} & Q_{2,3} & Q_{3,3} & Q_{4,3} \\ \hline \text{Total} & Q_1 & Q_2 & Q_3 & Q_4 \end{array}$$

The total costs function to minimize is

$$\begin{aligned} C = \quad & 600\left(\frac{Q_{1,1}}{150} + \frac{Q_{2,1}}{100} + \frac{Q_{3,1}}{500} + \frac{Q_{4,1}}{400}\right) && \text{Cost of using line 1} \\ & + 500\left(\frac{Q_{1,2}}{200} + \frac{Q_{2,2}}{100} + \frac{Q_{3,2}}{760} + \frac{Q_{4,2}}{400}\right) && \text{Cost of using line 2} \\ & + 400\left(\frac{Q_{1,3}}{160} + \frac{Q_{2,3}}{80} + \frac{Q_{3,3}}{800} + \frac{Q_{4,3}}{600}\right) && \text{Cost of using line 3} \\ & \stackrel{!}{=} \text{minimum} \end{aligned}$$

the restrictions being:

$$Q_{1,1} + Q_{1,2} + Q_{1,3} = Q_1 = 2{,}000$$

$$\ldots \quad = Q_2 = 3{,}000$$

$$\ldots \quad = Q_3 = 3{,}000$$

$$\ldots \quad = Q_4 = 6{,}000$$

$$\frac{Q_{1,1}}{150} + \frac{Q_{2,1}}{100} + \frac{Q_{3,1}}{500} + \frac{Q_{4,1}}{400} \leqslant 20$$

$$\frac{Q_{1,2}}{200} + \frac{Q_{2,2}}{100} + \frac{Q_{3,2}}{760} + \frac{Q_{4,2}}{400} \leqslant 20$$

$$\frac{Q_{1,3}}{160} + \frac{Q_{2,3}}{80} + \frac{Q_{3,3}}{800} + \frac{Q_{4,3}}{600} \leqslant 18$$

Also,

$$Q_{1,1};\ Q_{1,2};\ \cdots;\qquad Q_{2,1};\ Q_{2,2};\qquad \cdots;\qquad Q_{4,3} \geqslant 0$$

The solution may now proceed according to linear programing techniques.

How to determine the costs coefficients c? Certain cost factors may be constant, irrespective of the process or machine used, such as the cost of materials, overhead, and to a certain extent labor costs. These should not be included in c; only the difference between the cost of operating and not operating the machine in question should be specified for this purpose.

Effects of overtime or subcontracting

Sometimes the capacity of a process or a machine may be increased, say, by employing overtime or by subcontracting. But the cost of employing extra facilities may be higher than the cost of using regular time. Thus, process 1 in Table 13–3 has a maximum capacity of, say, A'_1, for which the cost is c'_1 per unit time. If the total capacity A'_1 is used, we may employ overtime up to an additional capacity of, say, A''_1, but this would involve a cost of c''_1 per unit time. If the total capacity $A'_1 + A''_1$ is not adequate to cope with the requirements, we may be able to increase the capacity even further by subcontracting up to a capacity of A'''_1, but the cost per unit capacity would now be c'''_1. This means that the first row in Table 13–3 would be split into three rows as follows:

	Product			
Machine 1	*1*	*2*	...	*Max. Capacity*
Regular time	$Q'_{1,1}$	$Q'_{2,1}$	...	A'_1
Overtime	$Q''_{1,1}$	$Q''_{2,1}$	...	A''_1
Subcontracting	$Q'''_{1,1}$	$Q'''_{2,1}$	...	A'''_1

In fact, what we have done amounts to adding two rows to the matrix table. The three rows may be considered, for all intents and purposes, as three different processes, each with its own capacity limitations and its own cost coefficients.

Since the cost of overtime and subcontracting is higher than that of regular time, the optimal solution will automatically call for A'_1 to be fully used before starting to draw on A''_1 and for A''_1 to be fully used before starting to use A'''_1. The assignment problem in such cases does not, therefore, basically change; only the number of rows in the matrix Table 13–3 increases by the additional alternatives that are presented.

Example

A plant produces two products in flow production, and it can increase the output by use of overtime or by subcontracting. The data for maximum output and costs per unit are given in Fig. 13–5. It is required to show how the products should be scheduled in the forthcoming period in order to reduce costs to a minimum, if 2,000 units of each are to be finished at the end of the period.

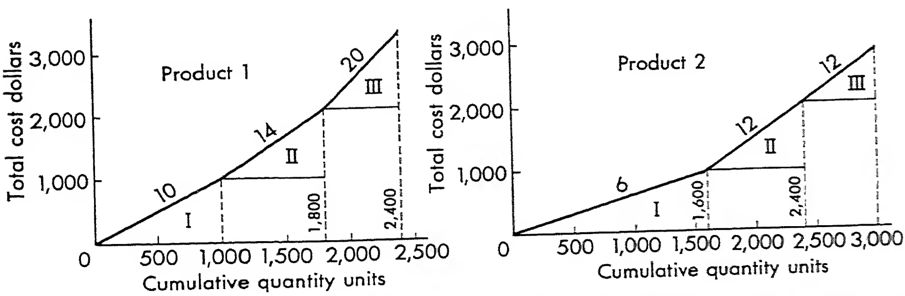


Figure 13–5. ***Two products to be processed on the same production line. (Figures on the broken line indicate the costs per unit by using the following facilities: I, regular time; II, overtime; III, subcontracting.)***

Formulation of the problem: There are, in fact, three facilities; namely, regular time, overtime, and subcontracting. Table 13–5 gives the matrix presentation.

Table 13–5

	Product 1		*Product 2*	
Facility	*Quantity Produced*	*Maximum Capacity*	*Quantity Produced*	*Maximum Capacity*
1: regular time	$Q_{1,1}$	1,000	$Q_{2,1}$	1,600
2: overtime	$Q_{1,2}$	800	$Q_{2,2}$	800
3: subcontracting	$Q_{1,3}$	600	$Q_{2,3}$	600
Total requirements	2,000		2,000	

The cost function to optimize is

$$C = 10Q_{1,1} + 14Q_{1,2} + 20Q_{1,3} \qquad (= \text{total cost of product 1})$$

$$+ 6Q_{2,1} + 12(Q_{2,2} + Q_{2,3}) \qquad (= \text{total cost of product 2})$$

the restrictions being:

$$Q_{1,1} + Q_{1,2} + Q_{1,3} = 2{,}000$$

$$Q_{2,1} + Q_{2,2} + Q_{2,3} = 2{,}000$$

$$\frac{Q_{1,1}}{1{,}000} + \frac{Q_{2,1}}{1{,}600} \leqslant 1$$

$$\frac{1}{800}\,(Q_{1,2} + Q_{2,2}) \leqslant 1$$

$$\frac{1}{600}\,(Q_{1,3} + Q_{2,3}) \leqslant 1$$

$$Q_{i,j} \geqslant 0$$

Scheduling Orders with Random Arrivals

When a plant is engaged on job production or on comparatively small batches, most of which do not recur in any regular fashion, production planning is faced with the problem of scheduling orders with random arrivals. The main difference between this situation and the one described in the preceding problem is that programing in this case is not geared to production periods. There is no array of tasks given at predetermined times to be scheduled during subsequent periods. Orders arrive at random and have to be scheduled on arrival, so that scheduling is a continuous process. When machine time is available, the job is immediately loaded on the machine; otherwise, arriving orders have to wait in a queue until machines become available. If the processes or machines are not identical, the jobs may be arranged in increasing order of operating costs for the products concerned. so that cheaper processes are used first; when the specific process or machine capacity is exhausted, the next process is used, and so on; the same procedure would apply to the use of overtime and subcontracting.

Problems of random-order scheduling

This situation is somewhat similar to conventional problems in queueing; for instance, the problem of assigning k repairmen to look after n automatic machines. When a machine stops, it demands the attention of a repairman, and if one is available, the service is rendered; otherwise the machine has to wait. In our case the orders become customers demanding service, and the machines are the "repairmen," or servers. However, queueing theories have not been applied to solve this scheduling problem, mainly because it is far more complex than the queueing models so far considered. Take, for example, the following important issues:

The machines providing the "service" are usually not identical. Some are technically more suitable and more economical than others; some may be totally unsuitable.

The production time may depend on the machine selected to perform the job.
The sequence in the queue may be reformed by priority rules, depending on the total length of the queue, promised delivery dates, penalties, etc.
Additions to the queue are often dependent on its length, the system having certain servomechanism characteristics.

All these problems make it rather difficult to construct a mathematical model with which the behavior of the system may be adequately described and with the aid of which optimal policies may be formulated. The most effective technique in such cases is that of *simulation* (sometimes referred to in literature as "system simulation" or "monte carlo method").

The simulation technique

The simulation technique is useful in that it allows us to experiment with the system on paper. With the absence of a model describing the behavior of the system, we are not quite sure what outcome to expect if we change its operating conditions. Experimenting with the system itself may prove to be too costly both in money and in time, and indeed in many cases, far too risky. If, for example, an industrial engineer suggests that as an experiment a certain process should be expanded in order to overcome certain scheduling difficulties, and if it turns out that after considerable expense the situation has not improved (or has perhaps even deteriorated), then—well, the prospects of our engineer's future and the frustration of management would better not be put in so many words.

The engineer can, however, experiment with system on paper, without interfering with the system itself in any way. From past history of the system and the frequency of occurrence of events, he can "generate" further history and observe how the system would react to any changes in the parameters. Very often it is possible to generate several years in a matter of a few hours, especially with the aid of a computer, so that one can afford to run several experiments at comparatively little cost, without having to wait many months or years to ascertain the outcome and without running the risk of leading the enterprise to disaster. By systematically changing each parameter at a time, it is possible to obtain a good idea as to how the system behaves, to find the relevant parameters governing its reactions, and to determine what policies should be recommended with the view of improving system performance.

Example

Perhaps the simplest way of explaining how simulation is applied would be by way of an example. Suppose a plant receives orders every day, the number of orders ranging from one to ten. Records in the order book show that the frequency of occurrence for one order arriving per day has been 10 per cent, and so has the frequency of two orders coming in per day, three orders, etc. In short, the distribution of the number of orders per day in the past has been rectangular, with one and ten orders as the two extreme limits. Each order entails a certain number of machine hours; analysis of past records are summarized in Table 13–6.

Table 13–6

		Cumulative	
Machine Hours	*Frequency* (%)	*Machine Hours*	*Frequency* (%)
0–5	6	0–5	6
5.01–10	4	0–10	10
10.01–15	10	0–15	20
15.01–20	12	0–20	32
20.01–25	14	0–25	46
25.01–30	14	0–30	60
30.01–35	14	0–35	74
35.01–40	10	0–40	84
40.01–45	6	0–45	90
45.01–50	6	0–50	96
50.01–55	2	0–55	98
55.01–60	2	0–60	100

The cumulative frequency curve for machine hours per order is plotted in Fig. 13–6. If we assume now that the same probabilities of the number of orders per day and the machine time per order will continue in the future, we can generate future events with the aid of a table of random numbers (given as an Appendix at the end of the book). Each digit in this table is an independent sample from a population in which the digits 0 to 9 have an equal chance to appear; i.e., each digit has a probability of 0.10.

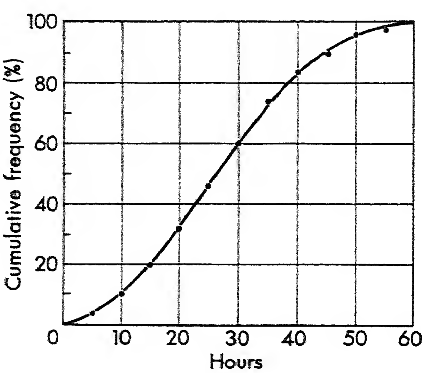


Figure 13–6. ***Cumulative distribution of machine hours per order.***

First, let us find how many orders per day we are likely to obtain in the future. Based on past events, we assume that 1, 2, . . . , 10 orders have a probability of 10 per cent; therefore the digits in the random numbers table immediately tell us the number of orders per day, except that whenever 0 appears, we interpret this event as ten orders per day. Starting off at the beginning of the table (we can, in fact, start wherever we like, since by definition the table is unbiased), we have 2, 10, 1, 7, 4, 2, . . . , orders per day.

And how many hours does each order entail? Again we use the table, but now we take two digits at a time (e.g., 20, 17, 42, 28, etc.). Each number is taken to represent the cumulative frequency in Fig. 13–6, from which the number of

hours can be read off the abscissa. Example: The first order is related to a cumulative frequency of 20 per cent, which (from Fig. 13–6) involves 15 hours; the second, 17 per cent, involving $13\frac{1}{2}$ hours; etc. Our generated history is given in the accompanying table.

Day No.	*No. of Orders*	*Machine Hours*	*Total Hours*
1	2	15; $13\frac{1}{2}$	$28\frac{1}{2}$
2	10	$23\frac{1}{2}$; 18; 16; $13\frac{1}{2}$; $29\frac{1}{2}$; $52\frac{1}{2}$; 22; $30\frac{1}{2}$; $3\frac{1}{2}$; 10	219
3	1	41	41
4	7	10; $26\frac{1}{2}$; 28; $45\frac{1}{2}$; 27; 24; 17	178
5	4	$35\frac{1}{2}$; 26; 5; 26	$92\frac{1}{2}$
6	2	$4\frac{1}{2}$; 5	$9\frac{1}{2}$
7	2	10; $20\frac{1}{2}$	$30\frac{1}{2}$
8	8	$27\frac{1}{2}$; 34; $10\frac{1}{2}$; 28; $25\frac{1}{2}$; 32; $47\frac{1}{2}$; 30	235
9	2	$47\frac{1}{2}$; 26	$73\frac{1}{2}$
10	3	29; 24; $47\frac{1}{2}$	$100\frac{1}{2}$
11	1	34	34
etc.			

The third column specifies the machine hours per order and the last column gives the total number of hours required, i.e., the load that has to be scheduled. It appears that this load varies considerably (the distribution of generated machine hours required every day by incoming orders is shown in Fig. 13–7). The problem is how to determine the best capacity that would cope with this situation. If the capacity is too low, the expected machine utilization would be high, but orders will have to wait until processing becomes possible. In fact, if the process capacity is lower than the average daily load, the queue of orders will continuously increase, and there would be no hope of ever coping with it if orders continued to flow in at the same rate. In our case the average daily load (from Fig. 13–7) is 139 hours. If the process capacity is, say, only 100 hours per day, the queue of waiting orders (measured in machine hours) grows rapidly, as shown in Fig. 13–8.

If, on the other hand, the process capacity is comparatively high, the waiting time of orders will considerably reduce, but machine utilization can be expected to be rather low. As penalties are associated with waiting orders on the one hand and with machine idle time on the other, there is conceivably a point at which the total costs would be at a minimum.

Assuming that orders can be loaded only on the day following their arrival, and that the breakdown of the daily load into orders is of no consequence from the point of view of scheduling, we can proceed to schedule the load generated in Fig. 13–7 on the plant. Two examples, one for a daily capacity of 160 hours and the other for 200 hours, are shown in Fig. 13–9. The simulated schedule has to cover a reasonably long period so that calculations based on mean loads and

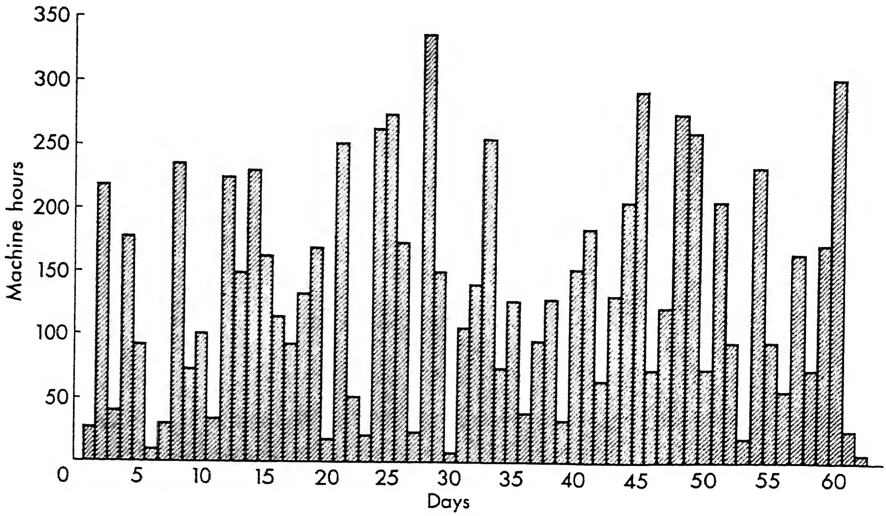


Figure 13–7. ***Generated orders arrival (in machine hours per day).***

waiting times are not greatly affected by the random load arrival. Furthermore, since in our simulated history we started with an idle plant waiting for work, while the plant might actually have been partially loaded, it would be advisable to chop off an initial "running-in" period in the simulated schedule. If in our case the cost of idle machine time is $5.00 an hour and the cost of an order waiting

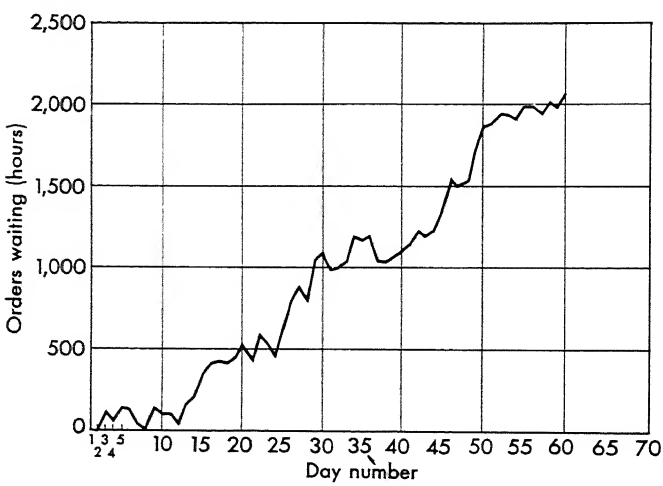


Figure 13–8. ***Queue of waiting orders (measured in machine hours) when capacity is 100 hr.***

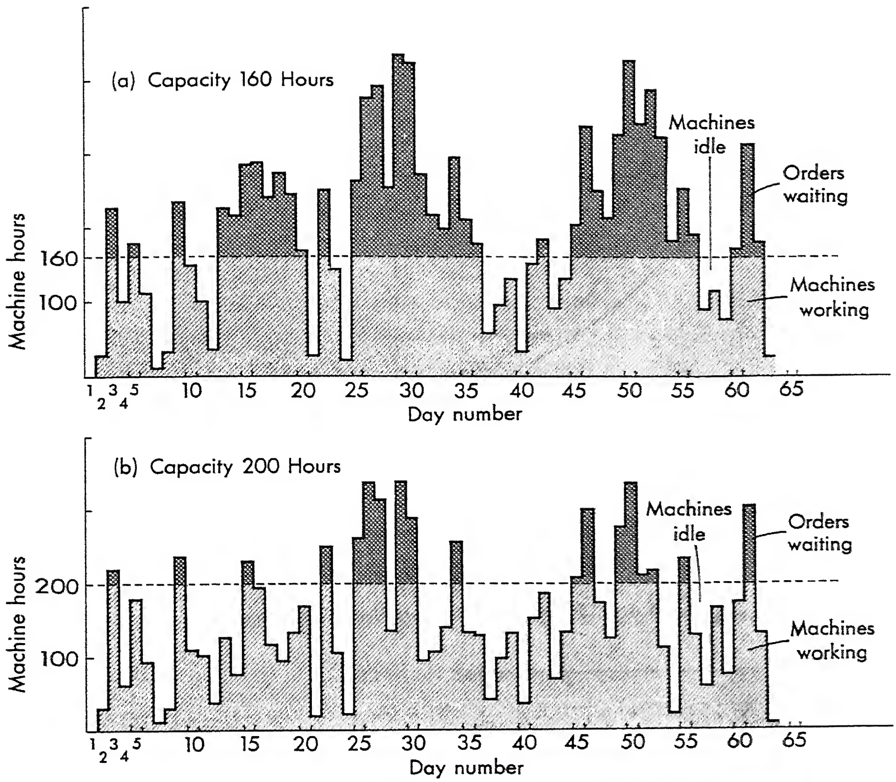


Figure 13–9. ***Scheduling the load in Figure 13-7 for two cases of plant capacity.***
(a) Capacity 160 hours.
(b) Capacity 200 hours.

is \$1.00 per machine hour per day, the results (based on a period of 50 days, from day 11 to 60) are shown in Table 13–7 and Figs. 13–10 and 13–11, from which we find that the optimal capacity lies at 152 machine hours.

Table 13–7

Daily Capacity (hr.)	*Av. Daily Machine Running (hr.)*	*Av. Daily Machine Idle Time (hr.)*	*Av. Daily Orders Waiting Time (hr.)*	*Cost (\$/day)*		
				Idle time	*Orders Waiting*	*Total*
140	135.8	4.2	176.3	21	176	197
145	137.3	7.7	144.5	38	145	183
150	138.6	11.4	116.2	57	116	173
160	138.6	21.4	74.6	107	75	182
170	138.8	31.2	49.8	156	50	206
180	138.9	41.1	35.4	206	35	241
190	138.8	51.2	26.9	256	27	283
200	138.8	61.2	21.3	306	21	327

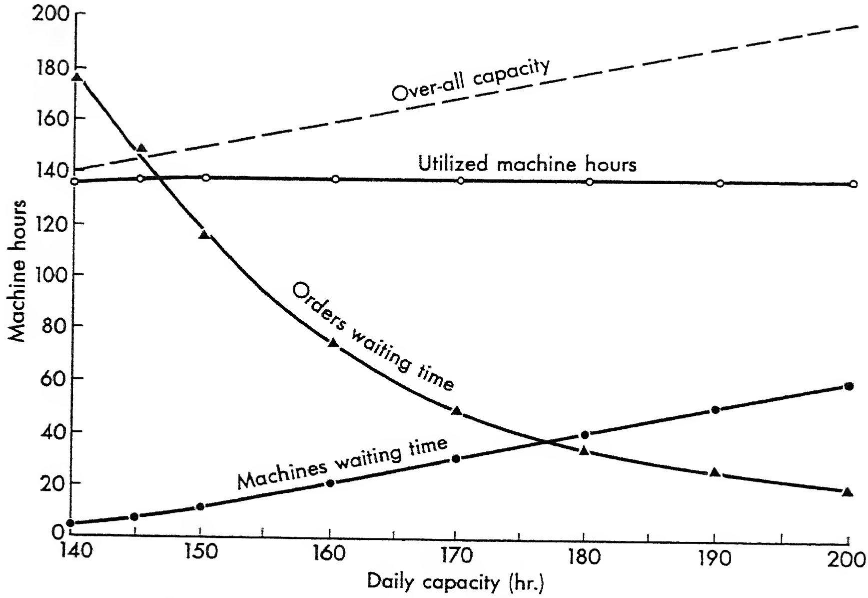


Figure 13–10. ***Effect of altering the plant capacity.***

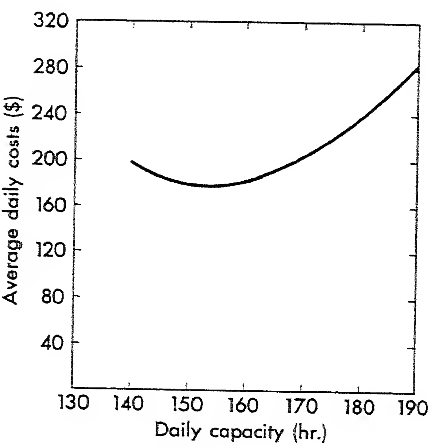


Figure 13–11. ***Effect of plant capacity on average penalty costs per day.***

Other applications of the simulation technique

In the preceding example we dwelt on the effect of over-all machine capacity on the schedule. The same simulation technique can be used to examine several possible alternatives for coping with the arriving orders; for example:

1. Is expansion in capacity from the present one (say, 140 hours) to 152 hours by adding machines (regular time) preferable to use of overtime?
2. If the problem cannot be handled on the basis of over-all capacity (total

capacity being made up of several machines), and if only one order can be loaded on one machine at a time, what is the optimal number of machines required?

3. What effect will priority rules (e.g., orders of less than 20 hours to be placed at the head of the queue) have on the schedule?

The simulation method is a powerful tool. It is both simple to use and effective in demonstrating to management the characteristics of the system, without the use of mathematics. It is capable of handling some very complicated situations and of generating history fairly quickly. The main disadvantage of the method from the analyst's point of view is that most of the time he is working in the dark, especially when the system is complex and governed by several parameters, so that only after a fairly large number of experiments can he draw some conclusions about the proximity of the optimal point. The same technique can be employed to examine many other problems in production planning; for example:

Study of the number of machines allocated to one or several operators
Study of machine balancing when operation times are subject to variations
Scheduling maintenance and repairs in the plant
Determining the optimal number of inspectors in the plant
Determining the number of storeskeepers in a store
Determining the number of clerical staff in an office
Study of the effectiveness of various policies in inventory control

Product Sequencing

In job or batch production we often have at the beginning of a production period several products that have to be manufactured on certain machines according to a predetermined sequence. The data can be presented as, for example, in Table 13–8, and the problem is to determine the best sequence in which the products should be loaded on the machines. There is, naturally, a conflict between performance at maximum machine utilization (implying that there is a queue in front of each machine, so that machine idle time is kept at a minimum) and scheduling to customer satisfaction (i.e., complying with predetermined delivery dates), and a satisfactory compromise has to be struck on the basis of costs ratios.

Table 13–8

	Product									
	A		*B*		*C*		*D*		*E*	
Operation	*Machine*	*Time (hr.)*	*Machine*	*Time*	*Machine*	*Time*	*Machine*	*Time*	*Machine*	*Time*
1	M2	10	M2	5	M1	5	M1	2	M1	2
2	M5	12	M1	5	M2	6	M2	2	M5	5
3	M1	14	M6	5	M3	7	M4	4	M6	6
4	M6	2	M4	8	M4	2	M6	2	M4	7
5	M4	5	M3	2	M5	1	M5	6	M3	15
6	M3	6	M5	6	M6	5	M3	8	M2	2
7	M7	2	M8	1			M8	1	M8	2
8	M8	4					M7	1		

Sequence analysis

Given n jobs to be performed on the same machine, the number of possible sequences for loading the jobs is $n!$, and this number becomes rather formidable as n increases (for $n = 5$, the number of alternatives is already 120; for $n = 6$, it is 720; and for $n = 7$, we have 5,040, etc.). If, however, our aim is to complete all the jobs in the shortest possible time, the selected sequence is of no consequence, since no delay is associated with any particular sequence. Even when

Table 13–9

Operation Times in (days) for Three Products

Machine	*A*	*B*	*C*
1	1	2	3
2	6	2	3
3	5	6	2
Total processing time	12	10	8

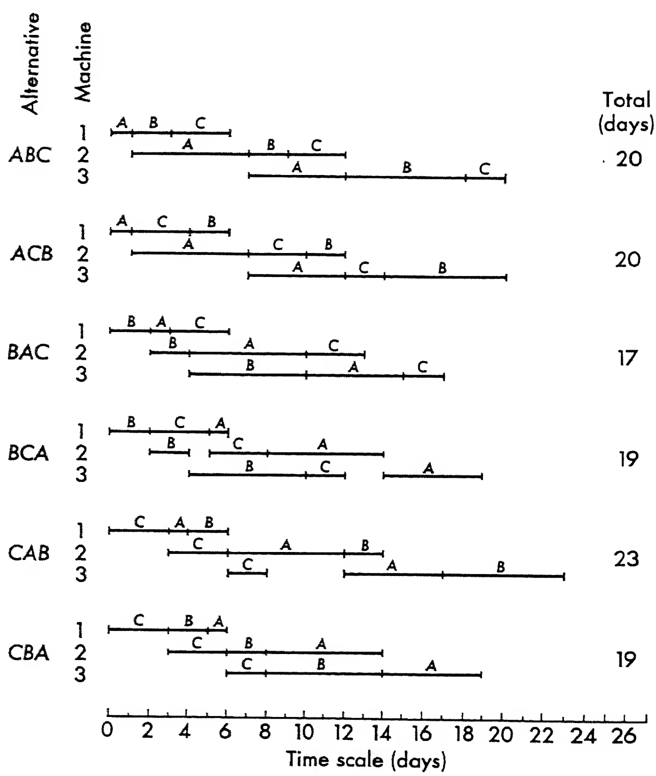


Figure 13–12. ***Alternative schedules of three products on three machines.*** (See Table 13–9)

n products have to be processed on m machines each, the sequence is of no importance, provided processing is carried out on the machines in the same order for all products and provided the operation times on all the machines are the same for all products. But once either the order of operations varies for different products or the operation times vary, the sequencing of products may become a significant problem.

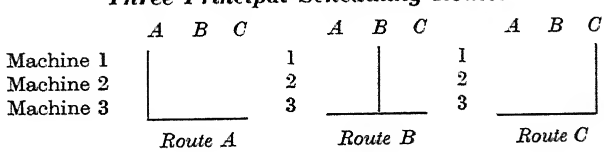
Take, for instance, the example shown in Table 13–9, where three products are processed on three machines. The sequence of operations is the same for all three products, but operation times differ. There are six alternatives in which the products could be scheduled: *ABC*, *ACB*, *BAC*, *BCA*, *CAB*, *CBA*. Which sequence yields the shortest total processing time for all three products? The six alternatives are shown in Fig. 13–12, from which we conclude that the sequence *BAC* provides the answer. It is interesting to note that the processing time for *B* is 10 days, which is neither the longest nor the shortest in Table 13–9. Hence it would seem fallacious to assume (as some schedule planners do assume) that minimum total processing time is achieved by setting priorities either in favor of the longest job or the shortest job. Some schedules involve a certain amount of interim machine-waiting time, such as the sequences *BCA* and *CAB*, and this waiting time may cause an increase in the total processing time.

Minimum processing time

There are three principal scheduling routes, obtained by giving priority to *A*, or *B*, or *C*, as shown in Table 13–10, and if we add the operation times along the routes, we shall get the minimum processing times per route.

Table 13–10

Three Principal Scheduling Routes



The subsidiary routes (principal route *A* has two subsidiary routes, namely, *ABC* and *ACB*) may be longer than the principal routes, if waiting machine time is increased as, for example, in the case of principal route *B*, which is shorter than its subsidiary *BCA*. The shortest possible total processing time is obviously given by the shortest principal route, provided it can be matched by one of its subsidiaries. To discover the shortest principal route, we merely have to find which column in the matrix yields the lowest sum when the last row is removed, since the last row is common to all routes. In the case of Tables 13–9 and 13–10, the row for machine 3 accounts for 13 days processing time, and the columns for the first two operations for products *A*, *B*, and *C* yield 7, 4, and 6 days,

respectively; hence principal route B is the shortest. In general, if we have a matrix of n products (columns) by m operations (rows), such as

$$\begin{array}{cccccc}
t_{11} & t_{21} & \cdots & t_{i1} & \cdots & t_{n1} \\
\vdots & & & & & \\
t_{1j} & t_{2j} & \cdots & t_{ij} & \cdots & t_{nj} \\
\vdots & & & & & \\
t_{1m} & t_{2m} & \cdots & t_{im} & \cdots & t_{nm}
\end{array}$$

where t_{ij} is the time required for the ith product on the jth operation, the shortest principal route is that through product k, provided

$$\sum_{j=1}^{m-1} t_{kj} \leqslant \sum_{j=1}^{m-1} t_{ij} \tag{13-8}$$

where $i = 1, 2 \ldots, n$.

We see from Fig. 13–12 that in order to avoid any machine waiting time, the second operation on the first product must be equal or longer than the first operation on the second product, the third operation on the first product must be equal or longer than the second operation on the second product, and so on. In other words, if the products are now rearranged in the order of scheduling so that the shortest principal route is through product 1, then

$$t_{12} \geqslant t_{21}$$

$$t_{13} \geqslant t_{22} \geqslant t_{31}$$

$$t_{14} \geqslant t_{23} \geqslant t_{32} \geqslant t_{41}$$

and so on. Even if some machine waiting time is incurred, the shortest principal route time is attainable, provided the mth machine has no interim waiting time. This is perhaps clear from the graphical presentation in Fig. 13–12. The schedule BCA could have been reduced to 17 days had the second operation on product A lasted 4 instead of 6 days. This would still have left machine 2 waiting between its first and second tasks, but by eliminating the last machine waiting time, the minimum over-all time could be obtained. If all the subsidiary routes of the shortest principal route incur machine waiting time, it is, of course, possible that each will be longer than the shortest feasible time attainable through some other route. It is still advisable to start with the shortest principal route, since by investigating its subsidiaries, we can easily determine whether this theoretical minimum time is at all attainable. This is particularly useful in product layouts, where the sequence of operations is virtually the same for all products.

Example

Five products have to undergo the following operations:

Product No. 1: 1/12 2/8 3/16 4/5 6/6
2: 2/9 4/16 5/16 7/8
3: 1/12 2/14 6/19 7/20
4: 2/5 3/6 4/9 5/10 6/10
5: 1/9 2/10 3/10 4/5 6/8

If the first number denotes the machine and the second stands for the number of hours, find the shortest principal route.

Solution

Although it is not apparent at first sight, the products are manufactured in the same order. Let us rewrite the data in the form of a matrix, the figures in which will be the operation times in hours.

Machine Number	*Products*				
	1	*2*	*3*	*4*	*5*
1	12	0	12	0	9
2	8	9	14	5	10
3	16	0	0	6	10
4	5	16	0	9	5
5	0	16	0	10	0
6	6	0	19	10	8
7*	0	8	20	0	0
Total time on machines 1–6	47	41	45	40	42

* Total time for this machine = 28.

In other words, we say that each product has seven operations, some of which require zero processing time. The shortest sum of the first six operations is through product 4, the shortest principal route is therefore 40 + 28 = 68 hours.

Situations such as those presented in Table 13–8, where the sequence of operations is not the same for all products, are far more complicated. If one sequence of operations is normally predominant with only few deviations from it, the situation may sometimes be handled successfully by artificially "forcing" all the products into the sequence and numbering afresh the deviating operations. For example, if the sequence is through machines 1, 2, 3, 4, 5, but one product has to be processed through 1, 2, 4, 5, 3, we can force it into the sequence by saying that the product is processed on machines 1, 2, 3, 4, 5, 6, where the time on machine 3 is zero and machine 6 is just another name for machine 3. This may prove to be convenient in studying the possible alternative product sequences. However, no general mathematical treatment has yet been suggested through which the shortest sequence can be located.

Summary

The production planning department is responsible not only for constructing a feasible and realistic schedule but also for analyzing and evaluating all possible alternatives in order to determine which one to adopt. The analysis depends on the circumstances under which scheduling has to be performed and on how easy it is to construct a model that adequately describes the behavior of the system. Of the many scheduling problems known in literature, five basic situations were selected for discussion:

Flow production scheduling, to determine production output and stock levels in cases of fluctuating demand

Batch production scheduling, when products are manufactured consecutively (further analyzed in the next chapter)

The assignment problem, to determine how to allocate an array of given tasks to given available production facilities

Scheduling orders for job or batch production when they arrive at random and have to be loaded on the available facilities on arrival

Product sequencing, to determine in which order to schedule products, each having a given sequence of operations to be performed on given machines

Many scheduling problems are combinations of these basic cases and call for the use of several techniques when optimal solutions are sought.

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Problems

1. For the sales columns indicated in Fig. 13–4, suggest the optimal policy if the cost of increasing or reducing the production capacity is \$20 per unit, whereas the cost of storage is \$10 per unit per period.

2. The sales forecasts for a product are as follows:

Period	*Units*	*Period*	*Units*
1	1,200	6	800
2	1,100	7	800
3	1,000	8	900
4	900	9	1,000
5	850	10	1,000

The production capacity is 1,000 units per period (regular time) and 300 units per period (overtime). Subcontracting can be relied on up to a capacity of 400 units per period.

Cost data: Overtime, \$25 per unit more than regular time; subcontracting, \$20 per unit more than regular time; storage, \$8 per unit per period.

Your Task:

(i) Suggest an optimal production schedule for the ten periods, if the initial inventory is 200 units.
(ii) Find the cost of this program.
(iii) Draw the stock level variations during the ten periods.

3. Four orders have to be assigned to either of two machines or distributed among them. The orders are:

Order No.	*Units*	*Rate of Production, Machine 1*
1	2,000	80/day
2	4,000	140/day
3	800	100/day
4	600	60/day

The rate of production on machine 2 is 20 per cent lower than that of machine 1. If the cost of operating machine 1 is \$80 per day and that of machine 2 is \$60 per day, how would you assign the orders to the machines?

4. Orders can be assigned to three production lines as follows:

		Rates of Production (units per day)		
Product	*Assigned to:*	*Line 1*	*Line 2*	*Line 3*
1	any line	100	100	150
2	lines 1, 2	50	80	—
3	lines 1, 3	80	—	80
4	lines 1, 3	100	—	120
5	any line	120	160	120

The cost of production in dollars per unit is as follows:

Product	*Line 1*	*Line 2*	*Line 3*
1	\$10.00	\$10.00	\$12.00
2	4.00	3.60	—
3	6.00	—	5.00
4	4.60	—	4.00
5	2.40	2.20	2.40

The total requirements are:

Product	*Units*
1	8,000
2	1,000
3	800
4	800
5	1,000

How should the orders be assigned to the three production lines?

5. (i) In the example used in the text to illustrate the simulation technique, find the optimal plant capacity, if the cost of idle machine time is \$5.00 per hour and the cost of an order waiting is \$2.00 per machine hour per day.

(ii) The present capacity of 140 machine hours per day is considered to be too low to cope with the incoming load. Two alternatives are suggested:

(a) Add a machine to increase the capacity to 148 hours; this would entail a flat increase in costs of \$36 per day.

(b) Use overtime, which costs (whether the machines are running or not) \$1.00 per machine hour more than regular time.

Find the optimal number of overtime hours that should be used. Which alternative is cheaper?

6. A shop of 17 semiautomatics (each working for 8 hours regular time per day) has to cope with orders of random arrival. Past records show that the number of machine hours required per order is as in Fig. 13–5, the frequency of arrival being as follows:

No. of Orders per Day	*Frequency* (%)	*No. of Orders per Day*	*Frequency* (%)
0	5	9	9
1	2	10	12
2	3	11	13
3	3	12	13
4	4	13	6
5	5	14	3
6	5	15	2
7	6	16	1
8	8		

The cost of idle machine time is \$5.00 per hour and the cost of orders waiting time is estimated at \$2.50 per hour.

(i) If orders can be scheduled on the day following their arrival, and if only one order can be loaded on one machine at a time, find the optimal number of machines for this shop. Assume all machines to be identical.

(ii) Find the effect of the following priority rule for 17 machines and for the optimal number found in (i): orders on hand are arranged in the order of magnitude (measured in machine hours) and the shortest is always scheduled first.

(iii) The priority rule is modified as follows: orders on hand are classified into two groups, the first consisting of less than 30 machine hours each, the second of more than 30 machine hours each. What effect would you expect when either group is always scheduled in preference to the other?

(iv) Comment on these priority rules. How would you go about determining what optimal priority rules, if any, should be adopted?

7. Three components *A*, *B*, and *C*, have to be produced on four machines, M1, M2, M3, and M4. Sequence of operations and times are given in the following table:

Product	*Operation*	*Machine*	*Operation Time (days)*
A	*A*/1	M1	2
	A/2	M3	4
	A/3	M2	3
	A/4	M4	6
B	*B*/1	M1	4
	B/2	M2	3
	B/3	M3	4
C	*C*/1	M2	4
	C/2	M3	5
	C/3	M4	3

(i) Suggest a schedule by which the production of all three products can be completed in the shortest possible time.

(ii) The cost of operating the machines is given as: M1, \$60 per day; M2, \$100 per day; M3, \$100 per day; M4, \$40 per day. If the cost of idle time is 60 per cent of the above, suggest a schedule by which production costs would be minimum.

8. For a certain bus route the following number of buses are required:

Time	*No. of Buses*
6–8 AM	20
8–10 AM	30
10–12 AM	20
12–2 PM	18
2–4 PM	12
4–6 PM	30
6–10 PM	12
10–12 PM	10
12–6 AM	6

If each driver is working 8 consecutive hours per day, find the smallest number required to comply with these requirements and outline the daily schedule.

14[1]

BATCH PRODUCTION SCHEDULING

The theory of optimal batch computations, as expounded in Chapters 10 and 11, is based on the approach that each product is considered on its own. The justification for such an approach is self-evident: We want each product to be effectively manufactured, so that it can be competitive and contribute to the well-being of the firm. This approach, however, does not take count of the effects of individual products on each other and on the production schedule. Suppose that we have faithfully computed the optimal quantity for each of the products that have to be produced; do these figures constitute a final satisfactory and workable answer to our batch production problem? It appears that our problems now only begin.

First, each quantity requires a certain amount of time for preparation and machine setup and for production. When all these required times for all the products that have to be produced are added together, we have the total load on the plant. Does this load match the available machine time at our disposal? Indeed, it would be a very strange coincidence if it did. If it is too high, the requirements simply cannot be met.

Secondly, each quantity is supposed to last a certain period of time, which we called the *consumption time*. During this consumption period the machines and equipment can be utilized to produce some other products on a batch basis. But if the other products are to be produced at optimal levels too, how can we be sure that when our first product is to be produced again, its stock level would have dwindled precisely to the safety level that necessitates its production in any case? Again, the likelihood that these two events will coincide (and what is more, that they should coincide for every single product in the production program) is extremely small. The computed optimal batch may be too high, so that when the product is produced again, excessive stock remains on hand from the preceding cycle. Or the batch may be too small, in which case the stock will reduce to zero before the product is due for production according to the schedule.

[1] This chapter may be omitted at first reading.

In short: The optimal batches have to be fitted into a schedule, but the schedule imposes certain conditions and limitations on the individual quantities. How can these different claims be reconciled?

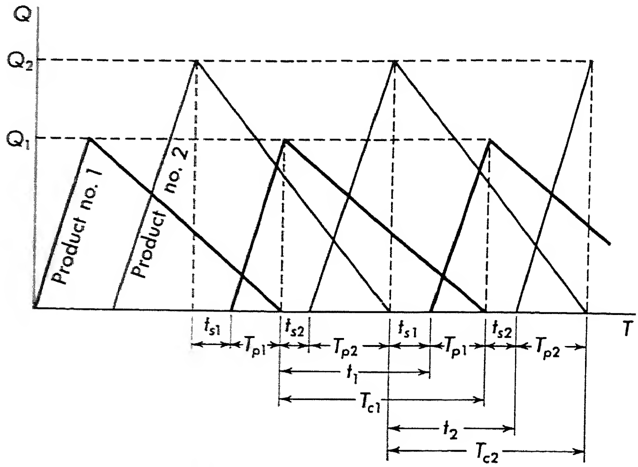


Figure 14–1. *A two-product cycle.*

Sequence of batches

This problem may perhaps be best demonstrated by considering a production schedule for two products manufactured in succession on the same equipment, as shown in Fig. 14–1. The sequence indicated by the schedule would be as follows:

Preparation and setup time for product 1	$= t_{s1}$
Production period for product 1	$= T_{p1}$
Preparation and setup time for product 2	$= t_{s2}$
Production period for product 2	$= T_{p2}$
Preparation and setup time for product 1	$= t_{s1}$

and so on. During the consumption period T_{c1} of product 1, the plant first produces product 2 and then product 1; hence

$$T_{c1} = t_{s2} + T_{p2} + t_{s1} + T_{p1}$$

And during the consumption period T_{c2} of product 2, the plant first produces product 1 and then product 2; hence

$$T_{c2} = t_{s1} + T_{p1} + t_{s2} + T_{p2}$$

$$\therefore \qquad T_{c1} = T_{c2}$$

The interpretation of the equality of the two consumption periods is simply this: If the plant capacity is to be fully utilized (i.e., no idle time allowed), the condition stated above ensures that the two products are always available for issue

and that stocks are replenished at the same level and to the same level at every cycle. But as $T_c = Q/a_c$,

$$\therefore \qquad \frac{Q_2}{Q_1} = \frac{a_{c2}}{a_{c1}}$$

where Q_1 is the batch for product 1, a_{c1} the rate of consumption of product 1, etc. If the quantities Q_1, Q_2 are computed for, say, minimum costs per unit, then by Chapter 10,

$$Q = \sqrt{\frac{s}{K}}$$

$$\therefore \qquad \frac{Q_2}{Q_1} = \sqrt{\frac{K_1 s_2}{K_2 s_1}}$$

Hence, the following condition must be satisfied:

$$\sqrt{\frac{K_1 s_2}{K_2 s_1}} = \frac{a_{c2}}{a_{c1}}$$

The ratio on the right is determined by circumstances prevailing in the market, while the factors on the left are mainly dependent on internal conditions in the plant. Thus, if optimal quantities are to be produced and at the same time the schedule is supposed to be devoid of idle time, to ensure availability of the products and to avoid excessive stock build-up, we are faced with an unreasonable condition to satisfy. It is evident that some deviations from the optimal batch sizes are inevitable when quantities have to be fitted into a production schedule; in other words, we have to resort to a compromise.

Optimizing the Production Schedule

Before we can attempt to outline any acceptable compromise, we must first seek the ideal solution when the whole production schedule is optimized. Supposing there are n products to manufacture and these are produced in succession on the available equipment. In other words, the production schedule would read as follows:

$$\underbrace{t_{s1}, T_{p1}, t_{s2}, T_{p2}, \cdots, t_{si}, T_{pi}, \cdots, t_{sn}, T_{pn},}_{\text{one production cycle}} t_{s1}, T_{p1}, \cdots$$

Thus the equipment is assumed to be fully utilized and no idle time is included in the schedule. The total length of the production cycle is

$$T_0 = \sum_{i=1}^{n} (t_{si} + T_{pi}) \tag{14-1}$$

and at the end of the cycle, product 1 comes up again for production and the cycle is repeated.

Maintaining stock level

As stated before, one of our basic objectives is to ensure that the product is always available and that the pattern of stock level variations repeats itself from cycle to cycle and that there is no accumulation of stock due to excessive residuals at the end of each cycle. In order to plot these stock level variations on a time scale, we have to superimpose the individual patterns (as shown in Fig. 10–2) on each other with appropriate time displacements. The peaks in the stock level graph for product 2 would be displaced by $t_{s2} + T_{p2}$ from those of product 1, and the peaks of product 3 would be displaced by $t_{s3} + T_{p3}$ from those of product 2, and so on. The result would be somewhat similar to Fig. 14–1, except that n graphs instead of two would be superimposed on each other. The consumption period for each product is given by the horizontal (time scale) distance between two successive peaks of the stock level graph for that product, and it is not difficult to ascertain (precisely in the same way it was shown for two products) that, for perfect superimposition as described above, the consumption period for each product must be

$$\sum_{i=1}^{n} (t_{si} + T_{pi})$$

hence

$$T_1 = T_2 = \cdots = T_i = \cdots = T_n \tag{14–2}$$

where T_i is the consumption period for the ith product. But since

$$T_i = \frac{Q_i}{a_{ci}} \tag{14–3}$$

$$\therefore \qquad Q_i = \alpha_i Q_1 \tag{14–4}$$

where $\alpha_i = a_{ci}/a_{c1}$ is the ratio of consumption rate of the ith product to that of the first product. Thus it is important to note that as soon as the products are put into a production schedule within the framework of conditions set above, there must be a quantitative relationship between the batch sizes of these products. This relationship is determined by the relative values of the rates of consumption of the products.

Specifying batch sizes

What batch sizes for the n products should be specified in order to secure maximum effectiveness of the whole schedule?

First, however, we must define how this effectiveness is to be measured, and immediately the four criteria considered in Chapter 10 (where each product was considered on its own) spring to mind:

Minimum costs per unit

This criterion is meaningless in a multiproduct schedule, since the units of the different products are not identical. The basic costs of materials and labor, the

carrying charges, and the setup costs may be different for different products, and there is no common denominator to which they can be reduced in order to clarify the term *minimum costs per unit.*

Maximum profit for the whole schedule

If the total cost per unit for product i is Y_i and the sales price is Y'_i, the total profit for the schedule would be

$$Z = \sum_{i=1}^{n} Q_i(Y'i - Y_i) \tag{14–5}$$

For convenience we may denote the sum $\sum_{i=1}^{n}$ by Σ. Substituting

$$Y_i = c_i + \frac{s_i}{Q_i} + K_i Q_i$$

and the relation 14–4, we obtain an expression for the profit in terms of only one variable, namely, the batch size for product 1 (the suffix i is also omitted for convenience of writing):

$$Z = Q_1 \, \Sigma(Y' - c)\alpha - \Sigma s - Q^2{}_1 \, \Sigma K\alpha^2$$

This profit becomes maximum when

$$\frac{dZ}{dQ_1} = 0 = \Sigma(Y' - c)\alpha - 2Q_1 \, \Sigma K\alpha^2$$

$$\therefore \qquad Q_{1P} = \frac{\Sigma\alpha Y' - \Sigma\alpha c}{2\,\Sigma K\alpha^2} \tag{14–6}$$

where Q_{1P} is the optimal batch size for product 1 when the schedule is optimized for maximum profit. If a nondimensional ratio P'_1 is defined as

$$P'_1 = \frac{\Sigma\alpha Y' - \Sigma\alpha c}{2Q_{1M}\,\Sigma K\alpha^2} \tag{14–7}$$

where Q_{1M} is the optimal batch size for product 1 when the schedule is optimized for maximum return (see the analysis of the next criterion), then

$$Q_{1P} = P'_1 Q_{1M} \tag{14–8}$$

which is analogous to the expression derived when only one product is considered (Chapter 10, Eq. 10–27).

Maximum return to the whole schedule

Return was defined as the ratio of profit to the cost of production, and we saw that in the case of a single product, the optimal solution provided for production at minimum costs per unit. The return for the whole schedule would be

$$\eta = \frac{\text{profit}}{\text{cost of production}} = \frac{\Sigma Q(Y' - Y)}{\Sigma QY} = \frac{\Sigma QY'}{\Sigma QY} - 1$$

Substitute Eq. 14–4; therefore

$$\eta = \frac{\Sigma\alpha Y'}{\Sigma\alpha Y} - 1 \tag{14–9}$$

Now, this ratio becomes maximum when $\Sigma\alpha Y$ is minimum, or when

$$\frac{d\,\Sigma\alpha Y}{dQ_1} = 0 = \frac{d}{dQ_1}\left(\Sigma\alpha c + \frac{\Sigma s}{Q_1} + Q_1\,\Sigma K\alpha^2\right)$$

$$\therefore \qquad -\frac{\Sigma s}{Q_1^{\,2}} + \Sigma K\alpha^2 = 0$$

hence

$$Q_{1M} = \sqrt{\frac{\Sigma s}{\Sigma K\alpha^2}} \tag{14–10}$$

or

$$Q_{1M} = g_1 Q_{1m}$$

where

$$g_1 = \sqrt{\frac{\Sigma s/s_1}{\Sigma K\alpha^2/K_1}} \tag{14–10a}$$

and the quantities for the other products are obtained by Eq. 14–4, namely,

$$Q_{iM} = \alpha_i Q_{1M}$$

Maximum rate of return for the whole schedule

Rate of return was defined as the ratio of profit to cost of production per unit time, or

$$R = \frac{\text{return}}{\text{cycle time}} = \frac{\Sigma(QY' - QY)}{\Sigma QY}\,\frac{1}{T_c} \tag{14–11}$$

Substitute

$$T_c = \frac{Q_1}{a_{c1}}; \quad Q = \alpha Q_1$$

Hence

$$R = \frac{a_{c1}}{Q_1}\left(\frac{\Sigma\alpha Y'}{\Sigma\alpha Y} - 1\right) = a_{c1}\left(\frac{\Sigma\alpha Y'}{\Sigma\alpha Q_1 Y} - \frac{1}{Q_1}\right)$$

The rate of return is thus expressed in terms of the batch size for product 1 and its maximum is found at

$$\frac{dR}{dQ_1} = 0$$

$$\therefore \qquad \frac{\Sigma\alpha Y'}{Q^2_{\,1}\Sigma^2\alpha Y}\,\frac{d}{dQ_1}\left(\Sigma\alpha Q_1 Y\right) = \frac{1}{Q^2_{\,1}}$$

or

$$\Sigma\alpha Y'\,\frac{d}{dQ_1}\left(\Sigma\alpha Q_1 Y\right) = \Sigma^2\alpha Y \tag{14–12}$$

where $\Sigma^2\alpha Y$ stands for $(\Sigma\alpha Y)^2$.

We know that

$$\alpha Q_1 Y = \alpha Q_1\left(c + \frac{s}{\alpha Q_1} + K\alpha Q_1\right) = \alpha c Q_1 + s + K\alpha^2 Q^2{}_1$$

$$\therefore \qquad \frac{d}{dQ_1}\left(\Sigma \alpha Q_1 Y\right) = \Sigma \alpha c + \Sigma 2K\alpha^2 Q_1 = \Sigma \alpha c + 2Q_1\,\Sigma K\alpha^2$$

Also

$$\Sigma^2 \alpha Y = \left[\Sigma\left(\alpha c + \frac{s}{Q_1} + K\alpha^2 Q_1\right)\right]^2$$

$$= \Sigma^2 \alpha c + \Sigma^2 \frac{s}{Q_1} + \Sigma^2 K\alpha^2 Q_1 + 2\,\Sigma\alpha c\,\Sigma\frac{s}{Q_1} + 2\,\Sigma\frac{s}{Q_1}\,\Sigma K\alpha^2 Q_1 + 2\,\Sigma\alpha c\,\Sigma K\alpha^2 Q_1$$

$$= \Sigma^2 \alpha c + \frac{\Sigma^2 s}{Q_1} + Q_1\,\Sigma^2 K\alpha^2 + \frac{2\,\Sigma\alpha c\,\Sigma s}{Q_1} + 2\,\Sigma s\,\Sigma K\alpha^2 + 2Q_1\,\Sigma\alpha c\,\Sigma K\alpha^2$$

Substitute these into Eq. 14–12 and multiply by $Q^2{}_1$; then

$$Q^4{}_1\,\Sigma^2 K\alpha^2 - 2Q^3{}_1(\Sigma\alpha Y' - \Sigma\alpha c)\,\Sigma K\alpha^2 + Q^2{}_1(\Sigma^2\alpha c + 2\,\Sigma s\,\Sigma K\alpha^2 - \Sigma\alpha Y'\,\Sigma\alpha c) + 2Q_1\,\Sigma\alpha c\,\Sigma s + \Sigma^2 s = 0$$

This equation of the fourth order for Q is similar to the one obtained in Chapter 10 when only one product was considered. The equation can be simplified if divided by $\Sigma^2 s$ and if the following relations are substituted:

$$\frac{\Sigma s}{\Sigma K\alpha^2} = Q^2{}_{1M} \qquad \text{(by Eq. 14–10)}$$

and

$$q_1 = \frac{Q_{1E}}{Q_{1M}} \tag{14–13}$$

where Q_{1E} is the optimal batch size for product 1 for maximum rate of return, the following expression is obtained:

$$q^4{}_1 - 2\frac{\Sigma\alpha Y' - \Sigma\alpha c}{\Sigma s/Q_{1M}}\,q^3{}_1 + \left(2 - \frac{\Sigma\alpha Y' - \Sigma\alpha c}{\Sigma s/Q_{1M}}\cdot\frac{\Sigma\alpha c}{\Sigma s/Q_{1M}}\right)q^2{}_1 + 2\frac{\Sigma\alpha c}{\Sigma s/Q_{1M}}\,q_1 + 1 = 0$$

By using the following two nondimensional ratios

$$P'_1 = \frac{\Sigma\alpha Y' - \Sigma\alpha c}{2Q_{1M}\,\Sigma K\alpha^2} = \frac{\Sigma\alpha Y' - \Sigma\alpha c}{2\,\Sigma s/Q_{1M}} \qquad \text{(from Eqs. 14–7 and 14–10)}$$

$$U_1 = \frac{\Sigma\alpha c}{\Sigma s/Q_{1M}} \tag{14–14}$$

the equation for q_1 is reduced to

$$q^4_1 - 4P'_1q^3_1 + 2(1 - U_1P'_1)q^2_1 + 2U_1q_1 + 1 = 0 \tag{14-15}$$

This equation is similar to Eq. 10–34, which was obtained for one product, except that the ratios P'_1 and U_1 differ from p'_1 and u_1 by definition. This means that in order to find the economic batch size for product 1 (i.e., the value of q_1), the same methods used for the solution of Eq. 10–34 may be employed, namely:

1. Since Eq. 14–15 is nondimensional, its solution can be presented by a series of curves, as shown in Fig. 10–12. These curves can therefore be used both for multiproduct and single-product problems. For the solution of a multiproduct schedule, use P'_1 as the ordinate (instead of p'), U_1 as a parameter (instead of u), and the relation $Q_{1E} = q_1Q_{1M}$.

2. The analysis of Eq. 10–34 has shown that the solution lies between

$$q = \frac{1}{p'}$$

and

$$q = \frac{p^2}{p'} \tag{14-16}$$

Likewise, the solution of Eq. 14–15 lies between the values

$$q_1 = \frac{1}{P'_1}$$

and

$$q_1 = \frac{p^2}{P'_1} \tag{14-17}$$

which may be used as good approximations for q_1.

The procedure for determining the economic batches would be as follows:

(i) Start with a data table in which the values of c, s, K, α, and Y' for each product are specified.

(ii) Find by Eqs. 14–10 the batch size for product 1 for a maximum return multiproduct schedule (Q_{1M}).

(iii) Evaluate P'_1 by Eqs. 14–14.

(iv) To find Q_{1E}, use either the approximations, Eqs. 14–17, or the solutions provided by Fig. 10–12, in which case U_1 should first be evaluated by Eqs. 14–14.

(v) Find the other batch sizes by Eq. 14–4.

The solutions for optimal batch size determination, using the different criteria, are summarized in Table 14–1. The similarity between the solutions derived for

a single product and the corresponding ones for a multiproduct schedule is striking but perhaps not wholly unexpected. The solutions for a single product may be considered a special case of those for the multiproduct schedule (obtained when the schedule consists of only one product).

A discussion of the criteria used for the optimal solutions is given in Chapter 10, and generally the same remarks apply when these criteria are evaluated in the light of a whole schedule. The concept of maximum profit per cycle appears to be misleading here as well because by adopting such a yardstick for measuring the effectiveness of the schedule, the batches are increased, thereby lengthening the cycle and tying up the financial resources of the firm in unduly large stocks. Since maximum rate of return ensures lower average stock levels and high turnover, it would appear to be a better criterion to adopt than that of maximum return, unless we may expect to derive some benefit from improved production methods (and thereby reduce the costs) through the processing of larger batches.

Deriving a Realistic Solution

Chapter 10 presented optimal solutions for each product when it is considered on its own. In this chapter optimal solutions for the whole schedule were derived. But we have seen that the two are not compatible, since the schedule optimization sets a rigid relationship between the quantities of the various products included in the schedule, and quantity ratios are determined by the relative rates of demand and not by the cost parameters that play an important role in optimal batch size determination when each product is analyzed individually. It is also quite clear that neither approach can afford to disregard the other. It is ridiculous to consider the products on an individual basis without any relation to the schedule, first because if in this way the schedule will consist of a large proportion of idle time, the whole basis for costing the individual products may be seriously disrupted, and secondly because we cannot ensure that the products will always be available in stock and be scheduled for production at the right time.

It is risky to consider the schedule alone and forget about the individual products, since each one of these must stand on its own in the market, it must be competitive, and therefore it must be produced in such quantities that a certain total production ceiling cost is not exceeded. We must also ensure that the total load imposed on the plant by the computed schedule is compatible with the facilities at our disposal.

Procedure

It is evident that some deviations from the ideal computed batch sizes are inevitable, and a compromise solution must be worked out in a form of a realistic schedule. This is where the production range becomes very useful. The maximum total production costs specified by management (conveniently defined by the

Table 14–1

Summary Solutions

Objective	Single Product: Function to optimize	Single Product: Solution	Multiproduct: Function to optimize	Multiproduct: Solution
Minimum costs per unit	$Y = c + \frac{s}{Q} + KQ$	$Q_m = \sqrt{\frac{s}{K}}$	not applicable	
Maximum profit per batch or cycle	$Z = Q(Y' - Y)$	$Q_p = p'Q_m$	$Z = \Sigma Q(Y' - Y)$	$Q_{1P} = P'_1 Q_{1M}$
Maximum return per batch or cycle	$\eta = \frac{Y'}{Y} - 1$	$Q_m = \sqrt{\frac{s}{K}}$	$\eta = \frac{\Sigma\alpha Y'}{\Sigma\alpha Y} - 1$	$Q_{1M} = \sqrt{\frac{\Sigma s}{\Sigma K\alpha^2}}$
Maximum rate of return	$R = \left(\frac{Y'}{Y} - 1\right)\frac{1}{T_c}$	$q^4 - 4p'q^3 + 2(1 - up')q^2 + 2uq + 1 = 0$ Solution of this equation: (a) Fig. 10–12 (b) Approximations, Eqs. 14–16	$R = \left(\frac{\Sigma\alpha Y'}{\Sigma\alpha Y} - 1\right)\frac{1}{T_c}$	$q^4_1 - 4P'_1 q^3_1 + 2(1 - U_1 P'_1)q^2_1 + 2U_1 q_1 + 1 = 0$ Solution of this equation: (a) Fig. 10–12 (b) Approximations, Eqs. 14–17
Summary of nondimensional ratios	$p' = \frac{Y' - c}{2KQ_m} = \frac{Y' - c}{2s/Q_m}$ $u = \frac{c}{KQ_m} = \frac{c}{s/Q_m}$ $q = \frac{Q_e}{Q_m}$		$P'_1 = \frac{\Sigma[\alpha(Y' - c)]}{2Q_{1M}\Sigma K\alpha^2} = \frac{\Sigma[\alpha(Y' - c)]}{2\,\Sigma s/Q_{1M}}$ $U_1 = \frac{\Sigma\alpha c}{Q_{1M}\Sigma K\alpha^2} = \frac{\Sigma\alpha c}{\Sigma s/Q_{1M}}$ $q_1 = \frac{Q_{1E}}{Q_{1M}}$	

factor p) is translated into production range limits for each product. The proposed procedure for deriving a realistic solution is as follows:

1. *Compute the production range* or the economic production range (depending on the selected criterion) for each product.
2. *Find the ideal optimal solution* for a multiproduct schedule.
3. *Test the solution by:*
   (i) Subjecting it to what may be termed the *p test*, which simply implies that it is necessary to ascertain that the batches included in the ideal solution lie within the limits of the production range of the respective products.
   (ii) Subjecting it to the *cycle test*. Since the proposed schedule involves a certain production cycle time (which can be computed when the rates of production are known), it is necessary to examine whether this time matches the consumption cycle.

If by any chance the solution passes the two tests, the individual approach and the schedule approach are evidently not incompatible, and the ideal solution can be adopted without further ado. If, however, the solution does not pass either test, it can be modified as follows:

Failure to pass the p test

When the batch size for a certain product lies beyond the limits of the production range, there may be two possibilities:

1. The batch is too large (i.e., above the upper limits of the range), in which case it can be divided into two or more sub-batches. Each sub-batch should lie within the range (we saw in Chapter 10 that even with comparatively low values for p, the production range becomes wide enough to allow for such a flexibility). This means that the product will be produced more than once in the cycle.

2. The batch is too small (i.e., below the lower limit of the range), in which case the quantity can be doubled or trebled and produced only once every two or three cycles, the idea being that the new quantity thereby derived would fall within the range and pass the p test. In this way nonidentical production cycles are formed: long cycles, which include all the products on the schedule, and short cycles, which include only some of them. This would necessitate reconsidering each cycle on its own and the batch size of the product that is not produced every cycle will have to be modified accordingly.

Failure to pass the cycle test

Again we may have either of the following two alternatives:

1. The production cycle is shorter than the consumption period; in other words, when the nth product is completed, there is still extra time left before the first one is due again for manufacture. If this time is to be utilized to advantage, we have to seek methods to step up the rate of consumption of some

of the products and use the available time to produce more of these, or we have to consider the introduction of an additional product into the schedule.

2. The production cycle is larger than the consumption period; in other words, the schedule presents too heavy a load compared with the available facilities. Possible courses of action are:

(i) Increase the plant capacity by use of overtime, by purchase of new or additional machines, or by subcontracting some of the orders

(ii) Reduce the commitments either by dropping certain products from the schedule or by relaxing the restriction that all the products must always be available

The ideal optimal schedule can thus be modified, so that the final proposed solution will comply with the various restrictions imposed on the system. One may, of course, ask (1) whether this solution may be regarded as a satisfactory one in the light of the criterion used for optimization, and (2) whether this method can be easily applied. The answer to the first question is that by starting from the ideal solution and by modifying it to comply with the restrictions, we stand a reasonable chance of retaining its basic features while not being too far from that theoretical optimum that could perhaps be derived had we been able to incorporate these restrictions into the mathematical model and had we been able to find an optimal solution to it. The answer to the second question depends on the relaxations that management is prepared to allow for the factor p. Since a small increase in p results in a large increase in the production range, the problem becomes comparatively easy to solve if reasonable values for p are allowed.

Multischeduling Six Products: An Example[2]

The application of the multischeduling method may best be illustrated by an example. Data of six products which have to be manufactured in a plant are given in Table 14–2.

Preliminary data and first solution

The six products have been arranged in the order of increasing consumption rate, and they vary considerably in their production rates, setup costs, and constant costs per piece. It is assumed that the products are not being manufactured simultaneously and that during the setup period, no production takes place. Usually, among six products, it may be found that at least one has such a low variable-costs content that it can be produced within a very wide range with little effect on the total costs per piece, and this "slackness" greatly simplifies the problem of scheduling. In this example, however, strict limitations are imposed (shown in the last row in Table 14–2) for the allowable increase in variable costs above those which are incurred when the minimum-cost batch

[2] This example and Figs. 14–2 and 14–3 have been taken from S. Eilon: Scheduling for batch production (J. Institution of Production Engineers, London, August, 1957).

size is produced. The increase in variable costs by 5 per cent for products 1,2,3,5,6, is likely to cause an increase in total costs of about 2 per cent. Because of the high constant-costs content and comparatively low setup costs of product 4, it is felt that 10 per cent increase in variable costs may be allowed for this product. The actual increase in total costs for each product is computed at the end of this example, after the final production schedule has been outlined.

Table 14–2

Date of Six Products To Be Manufacturcd

	Product					
	1	*2*	*3*	*4*	*5*	*6*
Consumption rate a_c (units/day)	20	24	30	36	40	50
Production rate a_p (units/day)	100	150	200	110	400	280
Constant costs c/piece (in dollars)	4.0	1.6	6.0	12.0	16.0	6.0
Storage costs B/piece/day (in dollars)	0.001	0.001	0.0015	0.00125	0.002	0.00225
Setup costs s (in dollars)	3,000	1,800	3,600	1,500	6,000	30,000
Setup time t_s (days)	4.0	2.4	4.8	2.0	4.0	8.0
Allowable increase in variable costs	5%	5%	5%	10%	5%	5%

Determining the production range

From these strict limitations the allowable range of batch sizes, within which production has to be confined, can be determined. The carrying costs, the minimum-cost batch sizes for each product, and the allowable ranges of production have been computed, and the results are shown in Table 14–3. The interest rate is taken as 12 per cent; i.e.,

$$i = \frac{12}{100} \times \frac{1}{300} = 4 \times 10^{-4} \text{ per day}$$

(assuming 300 working days in a year).

Let us assume that maximum return was selected as an appropriate objective and that our task is to formulate a production schedule in accordance with this criterion. A safety allowance has to be taken into account to cover possibilities of delays, maintenance, breakdowns, and other stoppages. The appropriate figure for such an allowance is determined by previous history of the machinery involved, and other circumstances, and in our example it is assumed to be 5 per cent of the total production time. The first attempt at a solution is illustrated in Table 14–4. The batch size for product 1 is computed by Eqs. 14–10a.

Table 14–3

Minimum-Cost Batch Sizes and Production Ranges for the Six Products

	Computed by:	Product 1	2	3	4	5	6
Interest charges/piece/day, I	ic	1.6×10^{-3}	0.64×10^{-3}	2.4×10^{-3}	4.8×10^{-3}	6.4×10^{-3}	2.4×10^{-3}
Ratios: consumption, α	a_c/a_{c1}	1.0	1.2	1.5	1.8	2.0	2.5
production, β	a_p/a_{p1}	1.0	1.5	2.0	1.1	4.0	2.8
γ	a_c/a_p	0.2	0.16	0.15	0.326	0.1	0.179
setup	s/s_1	1.0	0.6	1.2	0.5	2.0	10.0
carrying costs factor, K	$\dfrac{I(1+\gamma)+2B}{2a_c}$	0.98×10^{-4}	0.571×10^{-4}	0.96×10^{-4}	1.23×10^{-4}	1.38×10^{-4}	0.733×10^{-4}
carrying costs factor	$K\alpha^2/K_1$	1.0	0.839	2.205	4.07	5.63	4.67
Min. cost batch size, Q_m	$\sqrt{s/K}$	5,540	5,610	6,120	3,490	6,590	20,230
Var. costs factors, p		1.05	1.05	1.05	1.10	1.05	1.05
Production range		4,040–7,600	4,090–7,690	4,470–8,390	2,230–5,450	4,810–9,030	14,800–27,700

Table 14–4

First Solution for a Production Plan

	Computed by:	Product 1	2	3	4	5	6	Remarks
Batch size, Q		5,050	6,060	7,570	9,090	10,100	12,630	Consumption period $T_c = Q/a_c$ = 252 days
p test		√	√	√	×	×	×	
Production time, T_p (days)	Q/a_p	50.5	40.4	37.8	82.6	25.2	45.0	*Totals:* $\Sigma T_p = 281.5$
Setup time t_s (days)		4.0	2.4	4.8	2.0	4.0	8.0	$\Sigma t_s = \underline{25.2}$
Total production time (days)	$T_p + t_s$	54.5	42.8	42.6	84. 6	29.2	53.0	306.7
Production cycle, T_0	add 5%	Length of production cycle = 306.7 × 1.05 = 322 days						

From Table 14–3,

$$\frac{\Sigma s}{s_1} = 15.3$$

$$\frac{\Sigma K\alpha^2}{K_1} = 18.41$$

Hence $$g_1 = \sqrt{\frac{\Sigma s/s_1}{\Sigma K\alpha^2/K_1}} = \sqrt{\frac{15.3}{18.41}} = 0.911$$

$\therefore$ $$Q_1 = g_1 Q_{1m} = 0.911 \times 5{,}540 = 5{,}050 \text{ pieces}$$

and $$Q_2 = \alpha_2 Q_1 = 1.2 \times 5{,}050 = 6{,}060 \text{ pieces (etc.)}$$

Testing the solution

The batch sizes quoted in the Table 14–4 ensure that throughout the consumption period, $T_c = 252$ days, all products are available and that the suggested solution provides for maximum return, not for each of the products individually but for the program as a whole. It is now necessary to apply the two tests to ensure that the solution is compatible with the given data and can be carried out.

The p test

Each quantity is compared with the production range set in Table 14–3. The results for the first three products comply with this limitation (denoted by $\surd$), while the quantities for the last three fall outside the prescribed respective ranges marked in the table by $\times$. In the case of products 4 and 5 the quantities are considerably above those at minimum cost. This can be rectified by splitting the batches into lots, the size of which will fall within the production range. Hence products 4 and 5 could be produced in two lots each of 4,045 pieces and 5,050 pieces, respectively, the lots being more or less evenly distributed in the production cycle. Overshooting the higher limit of the production range does not present a serious difficulty. The reason for this mainly lies in the fact that the allowable relaxation factor on the available costs, $p = 1.05$, is usually large enough to provide the necessary flexibility. The limits of the range in this case are 73 to 137 per cent of the minimum-cost batch size.

If the figure emerging from the solution is, say, 150 per cent, the batch can be divided by two, each part being 75 per cent in magnitude, to satisfy the requirements. Even when the batch size falls between the limits of 137 to 141 per cent, the resultant increase in p above the 1.05 mark is so slight, that in most cases it can probably be tolerated (at 141 and 71 per cent, the value of p is 1.06). The division of the batch into lots having a smaller size than the minimum-cost one is even beneficial from the point of view of rate of return, but it must be remembered that the production cycle is thereby lengthened, owing to additional setup time for the new lots. Another problem is to schedule these lots in the

production cycle in such a way as to ensure that the products are available in the interval period between them and that not too high a stock is carried from the first lot when the time for producing the second lot arrives.

More complicated is the case when the solution indicates that the quantity to be produced is below the allowable range, such as product 6. If the quantity shown in Table 14–4 is adopted, the increase in variable costs will be 11 per cent. The only possible course to follow—short of dropping product 6 altogether from our schedule—is to produce a quantity that will satisfy the demand during the period of two cycles. Thus the production plan will have to provide for two modified cycles, different in length, one including product 6 and one without it.

The cycle test

The length of the production cycle is worked out as shown in Table 14–4, and a comparison is made with the consumption period. This test is a more crucial test than the p test because it immediately shows what relation the load of the schedule has to the capacity of the plant. In the solution shown in Table 14–4, the plan provides for a consumption period of 252 days, but the length of the production cycle is 322 days. Hence it is obvious that the proposed solution is incompatible with the capacity of the plant, the plan to produce the six products being far too ambitious. Had the production cycle been found to be shorter than the consumption period, extra plant capacity would have been available and additional production could have been undertaken.

Methods leading to modified solutions

The double check as described above provides an answer to the question whether a satisfactory schedule is possible. Having shown that the solution in Table 14–4 is unacceptable, a modified solution may be sought on one of the following lines:

1. Increase the quantities, so that a longer consumption period is covered, the increase being such that the new consumption period will be equal to the production cycle. This method will be shown to be impracticable.
2. Re-examine the production range of the products and determine whether they can be relaxed.
3. Increase the capacity of the plant by adding or substituting some of the equipment, or by using overtime (and thereby increase the rate of production).
4. Attempt a more modest production plan by modifying the requirements that (i) all products must be produced, or (ii) that all products must always be in stock, or both.

The question of which alternatives should be taken can be decided when the circumstances of the case have been fully analyzed, but it may be useful to add a few comments on each method and outline the procedure that will lead to a final solution.

Increase the consumption period

The objective of increasing the consumption period is to arrive at a situation that will satisfy the cycle test. Since such an attempt involves an increase in quantities, it is reasonable to assume that the p test could also be met.

To increase the consumption period T_c from 252 days in Table 14–4 to the level of the production cycle of 322 days, it is necessary to increase the quantities by 28 per cent. To achieve this increase, the production time for each product will have to be lengthened by 28 per cent, and as the setup time is likely to increase owing to splitting of batches into lots, the total production time will probably rise by 28 per cent or more. The new plan is therefore no better than the original one of Table 14–4 because the gap between the consumption period and the production cycle has not been bridged or even diminished. Hence this method of trying to obtain a satisfactory plan by increasing the cycle time does not lead to a solution and must be abandoned.

That this method is impracticable can be demonstrated by the following considerations. We know that

$$\Sigma T_p + \Sigma t_s > T_c$$

This expression may be replaced by

$$\Sigma T_p + \Sigma t_s = T_c + G$$

where G is the gap between the production cycle and the consumption period.

Suppose we wanted to increase T_c by a factor e in such a manner that the cycle test would be satisfied, thus leading to the production time being increased by the same amount. Then

$$e\,\Sigma T_p + \Sigma t_s = eT_c$$

$$\therefore \qquad e = \frac{\Sigma t_s}{T_c - \Sigma T_p}$$

or

$$e = \frac{\Sigma t_s}{\Sigma t_s - G}$$

If the denominator is negative, the factor e has no meaning, and the objective cannot be attained (this is in fact the case in our example in Table 14–4). If the denominator is positive, its magnitude compared with that of the numerator may be such as to cause the factor e to be very large, leading to a very long cycle. This may introduce additional serious complications, such as the determination of the position of the large number of subdivided lots in the production cycle. In such cases the problem may be far more easily resolved by alternative 3, namely, by increasing the rate of production.

Relaxation of the production ranges

The advantage to be derived from this method is obvious; the wider the production range, the bigger is the chance for the calculated batch size to comply

with the p test. This may both eliminate the necessity to split a large quantity into lots on the one hand and the introduction of a two-cycle system—owing to one quantity being too small—on the other. Scheduling is greatly simplified, and if the production ranges are wide enough to allow the first interim solution to be accepted, the procedure is indeed a straightforward one. Suppose in the example analyzed above a variable costs factor, $p = 1.10$, was allowed for products 4 and 5, and suppose $p = 1.11$ was tolerated for product 6; the first solution presented in Table 14–4 would then be acceptable, and only the quantity for product 4 would have to be split into two lots. Products 5 and 6 could be produced in the stated quantities every cycle and only once per cycle.

These advantages warrant a careful examination of the allowable increase in the variable costs. In many cases it may be found that a relaxation of this condition would not cause a serious increase in the total costs per piece, and may therefore be justified. The effect of the relaxation mentioned above on products 5 and 6 is as follows:

	Product 5	*Product 6*
Min. variable costs	1.82	2.97
Min. total costs	17.82	8.97
Total costs for $p = 1.05$	17.91 (0.5% increase)	9.12 (1.7% increase)
Total costs for $p = 1.10$	18.00 (1.0% increase)	9.27 (3.3% increase)
Total costs for $p = 1.11$	—	9.30 (3.7% increase)

It is evident, therefore, that an allowable increase in p has little effect on the costs of product 5, and it may be concluded that its batch size need not be subdivided. Whether the increase in total costs of product 6 can be allowed is a question that can be decided only when the full facts of the case under consideration are known.

It should be pointed out, however, that relaxation of the p limitation constitutes only a half-measure in reducing the difficulties arising from the solution given in Table 14–4. The p test has nothing to do with the production capacity of the plant, and the cycle test has yet to be met. Hence a critical examination of the production ranges should be carried out in conjunction with a method that tackles the problem of cycle compatibility by either increasing plant capacity or reducing the production loading, two methods which will now be examined in some detail.

Increase the capacity of the plant

If the complete production program as prescribed is to be maintained, i.e., all products must be manufactured and all must always be available, an increase of the output capacity of the plant is unavoidable. Having failed to increase the consumption period to the level of the production cycle, this method aims at reducing the production cycle to the level of the consumption period. This increased capacity of the plant may be achieved either by adding or improving existing machines or by employing overtime or additional shifts. The exact method by means of which this increased capacity can be attained is a matter

for management decision after the merits of the case have been properly scrutinized. Provided the p test is satisfied, the first step in our example would be to reduce the production cycle to 252 days.

The new production rate can then be computed in the following manner:

	Time Quoted in Table 14–4 (days)	*New Time (days)*
Consumption period	= 252	252
Production cycle	= 322	252
Setup time	= 25.2	25.2
Setup time and allowance	= 25.2 × 1.05 = 26.5	25.2 × 1.05 = 26.5
Hence total production time + allowance	= 322 − 26.5 = 295.5	252 − 26.5 = 225.5
Total production time	= 281.5	$\frac{225.5}{1.05} = 214.8$

Assuming that the increased rate of production will be kept throughout, the production rate for each product should be increased by the factor 281.5/214.8 = 1.31, i.e., by 31 per cent, and the production time for each product will then be reduced by the same factor.

The new values of production rates will cause the ratios γ in Table 14–3 to decrease, thus leading to a slight increase in the batch sizes. But this effect is not likely to be marked. For product 1, for example, the new value of γ_1 is 0.2/1.31 = 0.153, resulting in $K_1 = 0.963 \times 10^{-4}$, $Q_{1m} = 5{,}570$, and the production range being 4,070 − 7,650. Unless the change in the ratios γ is considerable, there is no need to recalculate these quantities, and the solution presented in Table 14–4 may be left as it is.

Of course it is possible to adopt a policy whereby the rate of production is increased only during specific intervals in the cycle, by using overtime or shift work. For instance, one could visualize a solution by doubling the production rates of products 3 and 4, thus bringing down the production cycle to the desired level. However, when only some of the products are to be manufactured at an increased rate, the effect on γ can be large and the batch size may have to be rechecked. Whether a uniform increased rate of production or a concentrated effort in an intermittent fashion should be adopted is again a question of policy which management has to decide on.

This method of increased production rates—though sometimes a nuisance and not even always feasible for various reasons—is a useful and a comparatively simple tool from the point of view of scheduling, when a solution is sought for an ambitious production plan such as the one shown in Table 14–4.

Problems presented by a product for which the computed batch size is too small can also be tackled by increased production. The quantity quoted for product 6 in Table 14–4 does not pass the p test, but, as already mentioned, it could be produced in a larger quantity (nearer to the minimum-cost batch size) once every two cycles, so that the consumption period of the product will be

equal to the sum of the two production cycles. The production plan can be constructed by the following steps:

1. Find the length of the consumption period of the short cycle (excluding product 6).
2. Add the two consumption periods and compute the quantity for product 6, to satisfy the demand throughout the whole period.
3. Apply the p test, to determine the number of lots per batch.
4. Adjust the production rates in the two cycles to bring the production cycles in line with the consumption periods, and modify the production plan accordingly.
5. Check whether this adjustment will have any serious effect on the batch sizes, and if so, recalculate the quantities and repeat steps 1, 2, and 3. Even if the quantities are greatly affected, it will be found that one repeated calculation will be sufficient.
6. Apply the p test and the cycle test as a final check.
7. Determine the final production schedule.

In the procedure outlined above, a decision has to be taken before step 4, on the question of intermittent or uniform increase rates of production. Let us now follow this procedure in an attempt to suggest a reasonable solution to our problem.

Step 1

In order to find the length of the shorter cycle, only the batch size of product 1 needs to be computed, but it may be just as well to give the complete tentative solution, similar to Table 14–4, as it provides additional information which will be needed later. Since product 6 is not included in this cycle, the data regarding this product in Table 14–3 may be ignored. Hence

$$\frac{\Sigma s}{s_1} = 5.3$$

$$\frac{\Sigma K\alpha^2}{K_1} = 13.74$$

$$g_1 = \sqrt{\frac{\Sigma s/s_1}{\Sigma K\alpha^2/K_1}} = \sqrt{\frac{5.3}{13.74}} = 0.621$$

leading to a batch size for product 1:

$$Q_1 = g_1 Q_{1m} = 0.621 \times 5{,}540 = 3{,}440 \text{ pieces}$$

It is evident at this point that the value of g_1 is too low and that some of the quantities in the solution will be smaller than the lower limits of the respective production ranges and will not satisfy the p test. Instead of proceeding with the complete tentative solution for the short cycle, a short cut can be taken by

stating that the batch size of product 1 has to be increased at least to the lower limit of the production range by selecting $g_1 = 0.73$. This is the lowest value for g_1 we dare take, as a lower value would violate the p test. It is evident that by increasing g_1, the return of profit per investment will be lower than the one that could be attained by $g_1 = 0.621$, as found above. If, however, the conditions as stated do not allow any increase in the price of the product above a certain level, and if the product must always be available, it must be understood that a solution for maximum return of profit cannot be achieved.

It should also be stated that in general the point of maximum return of profit is attained when the first tentative solution passes the p test and the cycle test. But when these conditions are not satisfied by the first solution (and it may be reasonably expected that in most cases they are not), any modifications introduced into this solution, such as division of batches into lots or boosting one quantity to the production range level, may cause the solution to depart from the point of maximum return. The amount of such a departure and the question whether it will be acceptable are subjects of an additional check that can be carried out when the final solution has been suggested.

Assuming that in our case the condition limiting the increase in costs per piece is prevalent, and selecting $g_1 = 0.73$, then

$$Q_1 = g_1 Q_{1m} = 0.73 \times 5{,}540 = 4{,}040 \text{ pieces}$$

$$Q_2 = \alpha_2 Q_1 = 1.2 \times 4{,}040 = 4{,}850 \text{ pieces}$$

and so on. The interim solution of the short cycle is given in Table 14–5, in which it is concluded that the consumption period is 202 days. As in the case of Table 14–4, it appears that the production plan for the short cycle is too ambitious and overtime will have to be used, as shown later.

Table 14–5

Interim Short-Cycle Plan

	Product 1	2	3	4	5	*Remarks*
Batch size, Q	4,040	4,850	6,060	7,280	8,090	Consumption period $T = \frac{Q}{a_c} = 202$ days
p test	$\surd$	$\surd$	$\surd$	$\times$	$\surd$	
No. of lots	1	1	1	2	1	
						Totals
Production time, T_p (days)	40.4	32.3	30.3	66.2	20.2	$\Sigma T_p = 189.4$
Setup time, t_s (days)	4.0	2.4	4.8	4.0	4.0	$\Sigma t_s = 19.2$
Total production time (days)	44.4	34.7	35.1	70.2	24.2	208.6
Production cycle	Length of production cycle $= 208.6 \times 1.05 = 219$ days					

Step 2

The total length of the two cycles can now be found by adding the consumption periods: $252 + 202 = 454$ days. The quantity for product 6 that should be produced in the long cycle is $454 \times 50 = 22{,}700$ pieces. The modified version of Table 14–4 is shown in Table 14–6.

Table 14–6

Interim Long-Cycle Plan

	Product 1	2	3	4	5	6	*Remarks*
Batch size, Q	5,050	6,060	7,570	9,090	10,100	22,700	Consumption period $T_{c1-5} = 252$ days; $T_{c6} = 452$ days
p test	√	√	√	×	×	√	
No. of lots	1	1	1	2	2	1	*Totals*
Production time, T_p (days)	50.5	40.4	37.8	82.6	25.2	81.1	$\Sigma T_p = 317.6$
Setup time, t_s (days)	4.0	2.4	4.8	4.0	8.0	8.0	$\Sigma t_s = 31.2$
Total production time (days)	54.5	42.8	42.6	86.6	33.2	89.1	348.8
Production cycle	Length of production cycle $348.8 \times 1.05 = 366$ days						

Step 3

The application of the p test to Tables 14–5 and 14–6 shows that none of the quantities in the two interim solutions is below the appropriate production range. Product 5 will have to be produced in two lots in the long cycle, while the quantities for product 4 can be divided into either two or three lots in both cycles. In both cases two lots per cycle were selected in order to avoid excessive setup times.

Step 4

Assuming a uniform increase in production rate is to be adopted for all products, the increase can now be calculated. For the long cycle (see Table 14–6), the total production time (including the safety allowance) is

$$366 - 31.2 \times 1.05 = 333.2 \text{ days}$$

This period should be reduced to

$$252 - 31.2 \times 1.05 = 219.2 \text{ days}$$

The increase should be by the ratio

$$\frac{333.2}{219.2} = 1.52$$

For the short cycle (see Table 14–5), the total production time (including the safety allowance) is

$$219 - 19.2 \times 1.05 = 198.8 \text{ days}$$

This period should be reduced to

$$202 - 19.2 \times 1.05 = 181.8 \text{ days}$$

Hence the increase is by the factor

$$\frac{198.8}{181.8} = 1.09$$

Now that the amount by which production should be boosted has been found, the modified version of the solution can be stated, as shown in Table 14–7. Note that this modification does not affect the batch sizes, the only change being in the production times.

Step 5

The main effect of increased plant capacity on the batch sizes is expected in the long cycle, where production rates have been increased by 52 per cent. Since the suggested two-cycle plan in Table 14–7 is based on the normal production rates quoted in Table 14–2, it is necessary either to ascertain that this plan is still valid or to modify it in the light of the new data. Normally a check on one product may be enough to indicate whether the effect is likely to be marked. The minimum-cost batch sizes and production ranges for the case under consideration have been recalculated, and when these (shown in Table 14–8) are compared with the figures in Table 14–2, it is evident that the changes are very slight, and hence the proposed solution in Table 14–7 is unaffected.

Step 6

The p test and the cycle test have been applied with satisfactory results, as shown in Table 14–7. All the batch sizes fall within the production ranges, and the production cycle is geared to the demand.

Step 7

Having now satisfied all the requirements and limitations that were imposed at the outset, the only problem left is the formulation of the final production schedule. When only one cycle has to be scheduled, during which each product is produced only once, the solution is straightforward, since the gearing of production to consumption is ensured when the formula for optimizing the return of the profit in a multiproduct program is applied. The order in which the products should be manufactured is immaterial as long as the same order is kept in every cycle.

However, when cycles are not identical, and especially when some products have to be produced more than once in one cycle, the scheduling may present some difficulties. The main problem is then to ensure that while each product should always be in stock, this stock should not be excessive when manufacture

Table 14–7

A Two-Cycle Production Plan (Increased Plant Capacity)

	Product 1	2	3	4		5		6	*Remarks*
				Long Cycle					
Lot size, Q	5,050	6,060	7,570	4,545	4,545	5,050	5,050	22,700	$T_{c1-5} = 252$ days
p test	√	√	√	√	√	√	√	√	$T_{c6} = 454$ days
Production rate, a_p (units/day)	152	228	304	167		608		426	
Production time, T_p (days)	33.2	26.6	24.9	27.2	27.2	8.3	8.3	53.3	$\Sigma T_p = 209.0$ days
Setup time, t_s (days)	4.0	2.4	4.8	2.0	2.0	4.0	4.0	8.0	$\Sigma t_s = 31.2$ days
Safety allowance (days)	1.9	1.4	1.5	1.5	1.5	0.6	0.6	3.1	
Total production time (days) (including allowance)	39.1	30.4	31.2	30.7	30.7	12.9	12.9	64.4	$T_0 = 252.3$ days
Cycle test		$T_0 = T_{c1-5}$							
				Short Cycle					
Lot size, Q	4,040	4,850	6,060	3,640	3,640	8,090			$T_c = 202$ days
p test	√	√	√	√	√	√			
Production rate, a_p (units/day)	109	164	218	120		436			
Production time, T_p (days)	37.1	29.6	27.8	30.4	30.4	18.5			$\Sigma T_p = 173.8$ days
Setup time, t_s (days)	4.0	2.4	4.8	2.0	2.0	4.0			$\Sigma t_s = 19.2$ days
Safety allowance (days)	2.1	1.6	1.6	1.6	1.6	1.1			
Total production time (days) (including allowance)	43.2	33.6	34.2	34.0	34.0	23.6			$T_0 = 202.6$ days
Cycle test		$T_0 = T_c$							

Table 14–8

The Effect of Increased Plant Capacity on the Production Ranges

	Product 1	2	3	4	5	6
Long cycle						
γ	0.132	0.105	0.099	0.214	0.066	0.118
Min.-cost batch size	5,610	5,650	6,190	3,600	6,660	20,400
Production range	4,100–7,700	4,130–7,750	4,520–8,480	2,310–5,610	4,870–9,130	14,900–28,000
g_1	0.911					
Short cycle						
γ	0.184	0.147	0.138	0.299	0.092	
Min.-cost batch size	5,550	5,630	6,140	3,510	6,610	
Production range	4,050–7,610	4,110–7,720	4,480–8,410	2,250–5,480	4,830–9,060	
g_1	0.617*					

* But $g_1 = 0.73$ has to be selected.

of the product is resumed; otherwise, high carrying costs will be incurred. In some cases it may be necessary to introduce some modifications in the batch sizes, mainly where quantities which have to be divided into several lots are concerned. It has been decided, for instance, to manufacture product 4 in two equal lots, but this decision was an arbitary one and it is possible to divide the quantity for this product into two unequal lots, if thereby a more satisfactory schedule can be obtained.

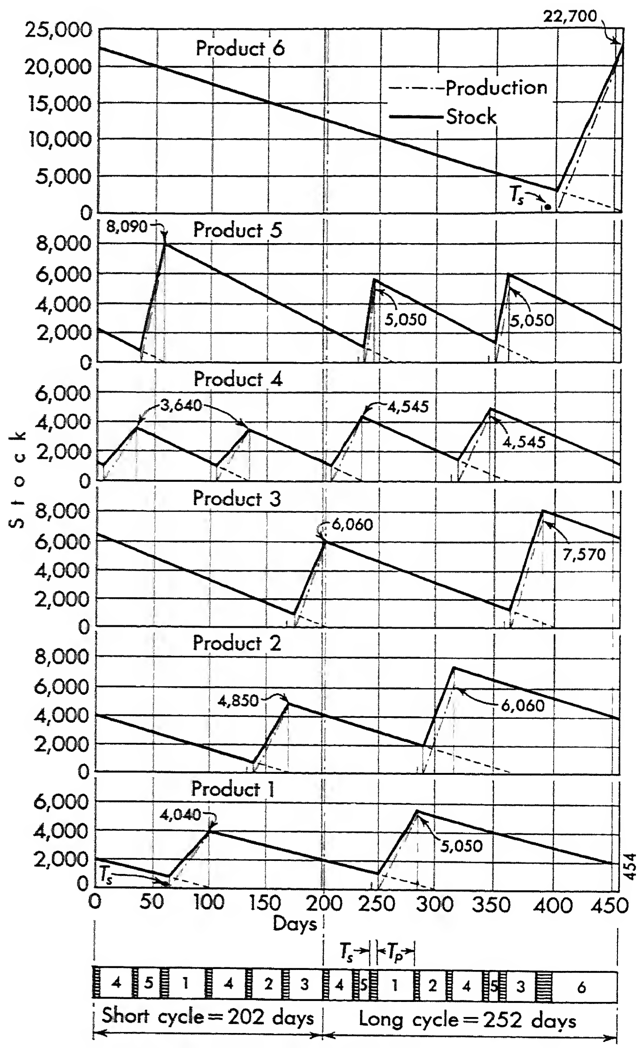


Figure 14–2. ***Production schedule: increased plant capacity. (Figures indicate batch sizes)***

The final schedule for the plan as suggested in Table 14–7 is shown in Fig. 14–2. The setup time and the safety allowance are represented together as total setting period T_s (e.g., for product 1 in the short cycle, $T_s = 4.0 + 1.9 = 5.9$ days). The change in stock is plotted for each product in order to facilitate control of stock levels throughout the cycle.

Thus it has been shown that the method of increased plant capacity offers a means to master a production load that would be otherwise impossible to meet with the existing facilities. It must, however, be borne in mind that increased capacity may affect the given data of machine setup costs and constant costs per piece. Introduction of new equipment or special-purpose machines and increased labor charges for overtime are only two factors that are likely to affect the issue and which may have to be taken into account.

Modify the requirements

If it is impossible to add or replace machinery to increase the output capacity of the plant, or if it is undesirable for any reason to employ overtime or shift work, the preceding method cannot be applied, and we are faced then with the problem of tackling the demands with the production rates specified in Table 14–2. It has already been stated that existing machines and facilities are inadequate to cope with the rate of consumption, and therefore the load as presented by the first solution in Table 14–4 will have to be relaxed. This can be done in one or both of the following methods:

1. Delete one or more of the products from the production plan in order to shorten the production cycle; the gap between the consumption period and the production cycle in Table 14–4 is 70 days, and it would appear that elimination of product 4 or two other products will be necessary.

2. Modify the requirement that all the products must always be in stock; it would be unreasonable to assume that a relaxation of this requirement could be applied to all products (in which case the cycle test is immaterial and the production plan would consist of producing each product in turn at its minimum-cost batch size), but even if it can be tolerated for one or two products, the plan could be more easily formulated. From Table 14–4, however, it is evident that the production cycle is so long in relation to the production period, that reducing the production level of one or two products is not going to help.

The question of which products are absolutely indispensable to the organization, and which may be deleted if need be, is obviously connected with a great many factors such as competition in the market, legal or moral obligations by the firm to produce specific articles, technical or consumer relation between products, profitability of the products, future trends of consumption indicated by market research, and future policies of the firm for specialization or expansion.

Suppose in our case it was decided better to dispense with product 4 rather

than with two other products. An interim solution replacing Table 14–4 can be worked out. Using the data in Table 14–3 and excluding product 4,

$$\frac{\Sigma s}{s_1} = 14.8$$

$$\frac{\Sigma K\alpha^2}{K_1} = 14.34$$

$$\therefore \qquad g_1 = \sqrt{\frac{14.8}{14.34}} = 1.015$$

The modified plan is given in Table 14–9.

The p test. Products 1 and 2 comply with this test. The quantity for product 3 deviates so little from the production range (by 50 pieces) that it does not seem practical to divide it; indeed in this case a division to two lots will cause a more marked deviation. Product 5 has to be manufactured more than once during the cycle. In view of the arguments put forward earlier, the production range can probably be relaxed in such a way that only one lot per cycle need be produced. For the sake of illustration we shall, however, prescribe two lots, in strict accordance with the production range cited in Table 14–3. The quantity for product 6 is again far too small and a two-cycle system is required.

Table 14–9

Modified Production Plan Excluding Product 4

	Product 1	2	3	5	6	Remarks
Batch size, Q	5,620	6,750	8,440	11,250	14,050	T_c = 281 days
p test	√	√	√	×	×	
No. of lots	1	1	1	2	Q too small	
Production time, T_p (days)	56.2	45.0	42.2	28.1	50.2	ΣT_p = 221.7 days
Setup time, t_s (days)	4.0	2.4	4.8	8.0	8.0	Σt_s = 27.2 days
Total production time (days)	60.2	47.4	47.0	36.1	58.2	248.9 days
Production cycle, T_0		248.9 × 1.05 = 261 days				

The cycle test. It is clear from Table 14–9 that elimination of product 4 has served the purpose of bridging the gap between the consumption period and the production cycle. In fact, we have now 20 extra days available for production.

A two-cycle plan. The application of the p test to product 6 has shown the necessity for a two-cycle plan. The 20 available days in the cycle suggested in Table 14–9 could be used to manufacture more of product 6 to satisfy the demand for it during two cycles, the short cycle excluding product 6 altogether. The procedure to be followed is somewhat similar to that of the increased plant capacity method:

1. Find the consumption period of the short cycle.

2. Add the two consumption periods and compute the quantity required for product 6.

3. Determine if the additional required amount complies with the available production capacity in the long cycle. If it does not, transfer partial production to the short cycle, if capacity for the purpose is available, or modify the short cycle.

4. Outline the production plan.

5. Apply the p test and determine the number of lots per batch. Apply the cycle test.

6. Introduce adjustments into the plan, if necessary.

7. Determine the final production schedule.

We shall now follow this procedure in an attempt to define an acceptable solution.

Step 1

From Table 14–3, it can be found that for the short cycle (in which products 1,2,3,5 are produced) $g_1 = 0.705$. As this factor will yield for product 1 a quantity that is smaller than the production range, the value $g_1 = 0.73$ has to be selected, leading to quantities as shown in Table 14–5 (disregarding product 4) and a consumption period of 202 days.

Steps 2 and 3

The consumption periods of the two cycles amount to $281 + 202$ days $= 483$ days. Hence $483 \times 50 = 24{,}150$ pieces of product 6 have to be manufactured in the long cycle, an addition of 10,100 pieces, corresponding to 36.1 days of production. Since only 20 days are available in the long cycle, (actually, only 19 if the 5 per cent safety allowance is to be accounted for), production of product 6 will have to continue for 17.1 days in the short cycle. From Table 14–5 it is clear that if product 4 is missing from the plan, the production cycle will be shorter than the consumption period, and hence such an overflow into the short cycle, using some of its available production capacity, is possible.

Step 4

The production plan is now outlined in Table 14–10. Since the manufacture of product 6 in the two cycles is, in fact, carried out in one continued period, the setup time was taken only once into account.

Steps 5 and 6

Application of the p test and the cycle test, as shown in Table 14–10, gives satisfactory results. The production time interval in both cycles does not exceed the consumption period and, in fact, in the short cycle we have 38.8 days in hand, which is available for meeting additional orders. This time may be used, for example, to produce product 4, but it is evident that the quantity that can be produced will be sufficient to satisfy only a fraction of the demand

for this product. Accounting for the safety allowance and the setup time, the available time for production is 35.0 days, during which 3,850 pieces of product 4 can be produced, and this figure passes the p test. It will, however, cover a demand period of only 107 days, i.e., 22.1 per cent of the total demand during the two cycles. At this stage no adjustment to the plan is necessary because it complies in all details with our modified conditions.

Step 7

Scheduling is attempted on the principles discussed previously, and as in the case of Fig. 14-3, it is found easier to start with the short cycle. The

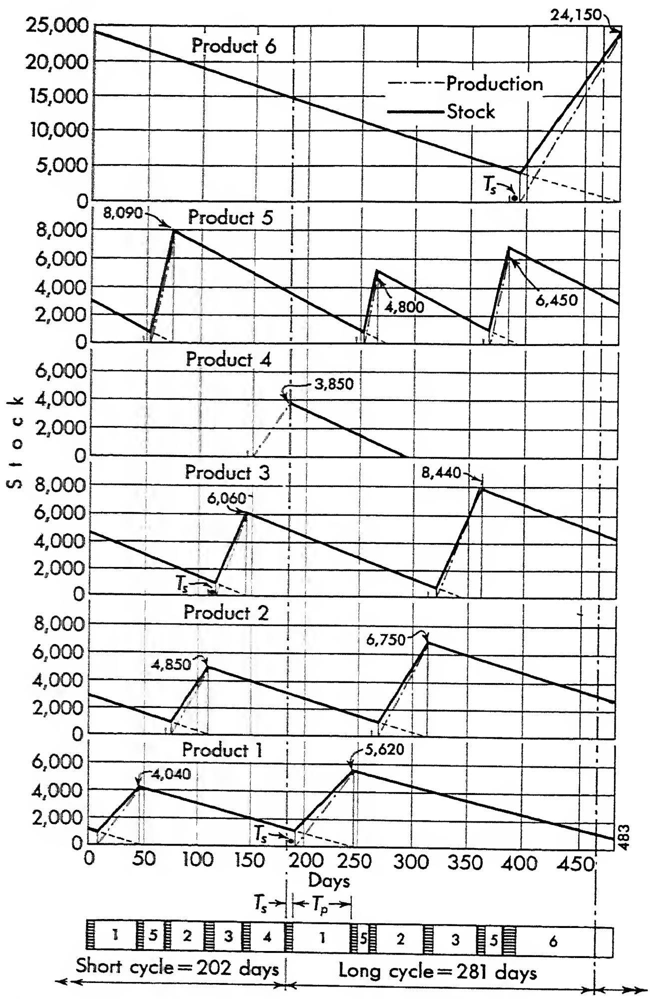


Figure 14-3. ***Production schedule: modified requirements. (Figures indicate batch sizes)***

position of product 4 in this cycle is immaterial because it is not bound by a consumption period to what occurs in the long cycle, and this fact offers a certain amount of flexibility. Another arbitary decision that may be modified at this stage is the division of the quantity relating to product 5 to lots.

Suppose we determine that two equal lots (5,625 pieces each, as shown in Table 14–10) should be adopted and that production should proceed by the following order: short cycle, 6 (finish) 1,5,2,3,4; long cycle, 1,5,2,3,5,6 (start). If we plot the change in stock levels, in a similar way to Fig. 14–2, we shall find that too much stock is carried over to the last production of product 5. Although this does not result in stocks for product 5 ever exceeding the ceiling of the production range, it would be better to divide the quantity for this product into unequal lots (say, 4,800 and 6,450 pieces, respectively) and thus reduce considerably the stock carried over. These figures, as well as the respective production times for the new lots, are shown in brackets in Table 14–10 and are taken as the basis for Fig. 14–3, in which the final schedule is given.

Figure 14–3 suggests that the distinction between two separate cycles in the production plan is somewhat artificial, since the overflow of production from one cycle to the other makes their boundaries difficult to define. Nevertheless the outline as given in Table 14–10 is useful, since it can serve as a basis for planning any changes that may be envisaged in the production schedule.

The flexibility of this method is obviously tied up with the nature of the relaxations of the original requirements that can be made, and the looser these are, the simpler the procedure and the less time taken to arrive at a satisfactory plan. It has, however, been shown above that even with the imposition of many restrictions, the method is workable and clearly indicates when and what kind of decisions have to be taken.

The schedules that are finally defined by the use of the two methods described above are strikingly similar, as shown in Figs. 14–2 and 14–3. The main difference between them is the treatment of product 4, which was sacrificed in the second method as being the least important in the production plan.

As we have been trying to produce the articles at the point of minimum costs per piece, it is interesting as a final check to see how the two proposed schedules deviate from this objective. The comparison between the actual production costs and the minimum costs is made in Table 14–11, and it appears that the costs do not increase by more than 0.6 per cent in the case of product 1 and that the increase is appreciably smaller for the other products.

Relaxation of p for two-cycle production plans

In a two-cycle production plan, if a quantity is placed in the extreme production ranges in one cycle, the effect of the corresponding quantity in the other cycle would be to damp the average increase in cost during the whole schedule. This is a good argument in favor of relaxing in some cases the p test in a two-cycle system because, even if in one cycle too high variable costs are incurred,

Table 14–10

A Two-Cycle Production Plan (Modified Requirements)

	Product 1	2	3	5		6	*Remarks*
			Long cycle				
Batch size, Q	5,620	6,750	8,440	5,625 (4,800)	5,625 (6,450)	24,150	$T_{c1-5} = 281$ days; $T_{c6} = 483$ days
p test	√	√	√	√	√	√	
Production time, T_p	56.2	45.0	42.2	14.1 (12.0)	14.1 (16.1)	69.0*	*for 19,350 pieces
Setup time, t_s	4.0	2.4	4.8	4.0	4.0	8.0	
Safety allowance	3.0	2.4	2.3	0.9 (0.8)	0.9 (1.0)	3.8	
Total production time (including allowance)	63.2	49.8	49.3	19.0 (16.8)	19.0 (21.1)	80.8	$T_0 = 281.1$ days
Cycle test		$T_0 = T_{c1-5}$					
			Short cycle				
Batch size, Q	4,040	4,850	6,060	8,090			$T_{c1-5} = 202$ days
p test	√	√	√	√			
Production time, T_p	40.4	32.3	30.3	20.2		17.1†	†for 4,800 pieces
Setup time, t_s	4.0	2.4	4.8	4.0		‡	‡production continued from long cycle
Safety allowance	2.2	1.7	1.7	1.2		0.9	
Total production time (including allowance)	46.6	36.4	36.8	25.4		18.0	$T_0 = 163.2$ days
Cycle test		$T_0 < T_{c1-5}$ Available time: $202 - 163.2 = 38.8$ days					

Table 14–11

Total Costs per Piece ($)

	Product 1 Y	1 *Increase* %	2 Y	2 *Increase* %	3 Y	3 *Increase* %	4 Y	4 *Increase* %	5 Y	5 *Increase* %	6 Y	6 *Increase* %
Minimum costs (Y_m)	5.08		2.24		7.18		12.86		17.82		8.97	
Increased capacity method (Table 14–7)												
Long cycle	5.09	0.2	2.24	—	7.20	0.3	12.89	0.2	17.88	0.3	8.99	0.2
Short cycle	5.14	1.2	2.25	0.4	7.18	—	12.86	—	17.86	0.2	—	
Average for schedule	5.11	0.6	2.24	0.2	7.19	0.1	12.88	0.2	17.87	0.3	8.99	0.2
Modified requirements method (Table 14–10)												
Long cycle	5.09	0.2	2.25	0.4	7.24	0.8	—	—	17.86	0.2	} 9.01	
Short cycle	5.14	1.2	2.25	0.4	7.18	—	12.86	—	17.86	0.2		0.4
Average for schedule	5.11	0.6	2.25	0.4	7.21	0.4	12.86	—	17.86	0.2	9.01	0.4

the average throughout the schedule may still be within the original prescribed limits. The costs of product 1, for instance, will reach \$5.14 per piece in the short cycle. This is the limit imposed by the initial condition that the variable costs should not increase by more than 5 per cent (selection of the batch size for product 1 in this cycle has in fact been governed by this limitation, when g_1 was ascertained). However, the average costs per unit for the whole schedule is \$5.11, resulting in an increase of variable costs of only 2.8 per cent. Hence a lower value for g_1 could be acceptable, and the production cycle could be shortened without violating the initial p limitations.

Stock Level Variations

The ideal multiproduct schedule should be planned in such a manner that when a batch of a product is completed and transferred to the stores, the former batch has been consumed. This situation is shown in Fig. 10–2, in Fig. 14–1, and again in Fig. 14–4. When production starts, we have $Q_0 = T_p a_c$ units in stock. When production ends, the stock level consists of Q units, which is the

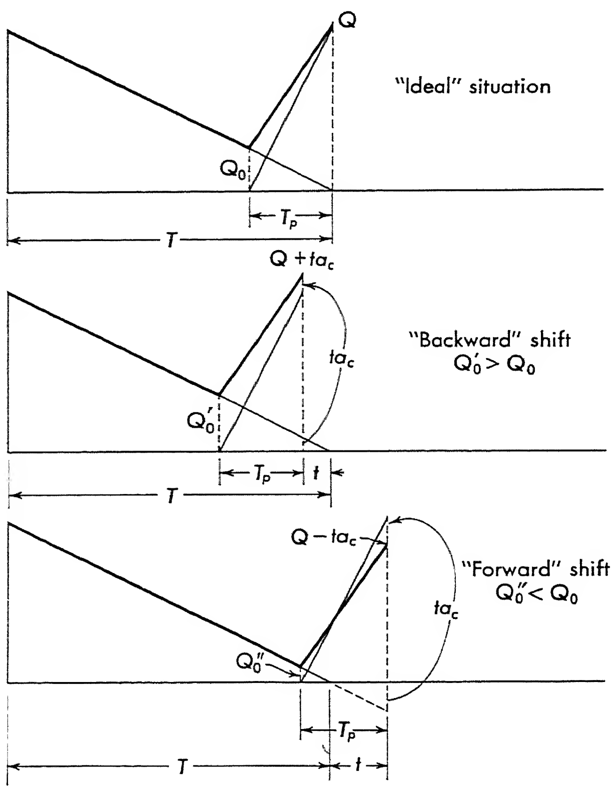


Figure 14–4. ***Scheduling effects on stock level variations.***

amount that has just been produced. Thus the stock level would fluctuate between Q_0 and Q under these conditions, Q_0 being the safety stock that ensures against fluctuations in rate of demands or delays in the schedule.

As soon as we start modifying the initial multiproduct schedule when it does not comply with the p test, the sequence of the products in the schedule may cause the stock level changes to deviate from the "ideal" situation. Two situations may arise (Fig. 14–4):

1. A "backward" shift in the schedule, when production starts by t units of time too soon; at the point where production starts, the stock level is

$$Q'_0 = (T_p + t)a_c = Q_0 + ta_c$$

and when production is completed, the stock level is $Q + ta_c$. The effect of backward shift is, therefore, to raise the whole level of the stock level diagram by ta_c. The stock will fluctuate between $Q_0 + ta_c$ and $Q + ta_c$, and the average stock held in the stores is increased by ta_c.

2. A "forward" shift in the schedule, when production starts too late, production starts when the stock level is

$$Q''_0 = Q_0 - ta_c$$

and when production stops, the level is $Q - ta_c$.

We may sometimes want to introduce such shifts quite deliberately. If the system is fairly deterministic, we can afford to reduce the safety stock Q_0 by having a forward shift. If, on the other hand, comparatively large fluctuations may be expected, the stock level can be increased by a backward shift. When a batch has to be divided into two or more lots during the cycle, the actual lot size can often be used as a regulating mechanism through which desirable or tolerable shifts can be included in the schedule. Thus, as we have already pointed out in the multiproduct example above, the division of the batch need not be into equal lots, and if the production range is wide enough, a considerable amount of flexibility in specifying the individual lot sizes is ensured. In outlining the production schedule, it is often convenient to construct a skeleton consisting of the more sensitive products and then to try and fill in the gaps by batches of insensitive products.

Application of the Theory to Assembly Work

The theory of multiproduct batch-size determination may prove to be very useful in cases where the final product is an assembly made of several components, which must also be produced on a batch basis. The components often involve different setup costs, materials costs, production rates, etc. If each component were considered on its own, each would have its corresponding optimal batch size, and in most cases these optimal values would be different

for the different components; furthermore, they would be different from the optimal batch size for the final product. This could easily lead to complications in the production schedules and result in a cumbersome inventory control system—a situation only too familiar in plants engaged in batch production. Since some of the components have to be carried in stock over a long period, there are many cases where changes in designs or models make some of the stocks obsolete, causing considerable losses to the firm.

These difficulties are greatly alleviated by the use of the theory of multi-product batch size determination. The consumption rate a_c of the final product is, in fact, the rate at which the components are required for the assembly line, namely,

$$a_c = a_{c1} = a_{c2} = a_{c3} = \cdots = a_{ci} = \cdots$$

where a_{ci} is the consumption rate of the ith component in units per unit time (a unit of a component is defined here as the number of pieces required per one final assembly). But as the consumption period T_c is the same for all, it follows that the quantities are the same:

$$Q = Q_1 = Q_2 = \cdots = Q_i = \cdots$$

where Q is the batch size of the final product and Q_i is the batch size of component i. The term *quantity* in this case prescribes the number of units required per Q units of the final product. Thus, if two pieces, say, of component 4 are required for each assembly, and if $Q = 1{,}000$ units, then $Q_4 = 1{,}000$ units $= 2{,}000$ pieces.

The procedure for determining the batch sizes would be as follows:

1. Calculate the optimal batches for the product and for the components when each is considered on its own.

2. Find the production range for each.

3. Select a quantity Q that would fall within the production range of the final product and within the respective ranges of most of the components.

4. These components, for which Q is beyond the limits of their ranges, will be treated by the "long cycle" approach and will be produced once every two or more cycles. With a realistic selection of the p factor for each component, the ranges would usually be so wide that resorting to the long cycle would be the exception rather than the rule.

5. The batch size Q has to be re-examined in the light of the production schedule.

The advantages of this method are only too obvious. The selection of a batch size Q within the limits of the production range of the final product ensures that the costs per unit will remain below a predetermined level, while the

uniformity of batch sizes for all the components greatly simplifies production planning and inventory control procedures; it also provides protection against obsolescence.

Summary

In addition to batch size optimization for individual products, it is desirable to seek an optimal solution for the whole production schedule. It was assumed in this chapter that n products are manufactured in succession, using the same machines and equipment, and although this model has its obvious limitations in that it does not cover the case where several products are manufactured concurrently side by side, it is nevertheless a useful guide and may often be used as an adequate approximation for a mixed concurrent-successive schedule.

Since, however, these two objectives need not be compatible, the batch sizes for the optimal schedule may differ from the respective batches required for individual product optimization, and in order to reconcile the two, a compromise production schedule has to be constructed. This is where the production range becomes very useful, since it specifies the allowable deviations from the individual optimums and thereby provides a certain amount of flexibility in the construction of the schedule.

The model in this chapter is basically a deterministic one. Fluctuations in consumption are accounted for by specifying appropriate safety stock levels, but changes in average consumption and trends in consumption may affect the batch sizes and the schedule. The solution must therefore be constantly re-evaluated in the light of changing circumstances, in order to ensure that the plant continues to comply with the restrictions imposed on the system and to operate according to the best schedule that can be currently defined.

References

Eilon, S.: Scheduling for batch production (J. Institution of Production Engineers, August, 1957).

Eilon, S.: Economic batch size determination for multiproduct scheduling (Operational Research Quarterly, U.K., December, 1959).

Problems

1. For the following two products:

	No. 1	*No. 2*
s (\$)	200	400
K (\$)	1.0×10^{-3}	5.0×10^{-3}
a_p (units/day)	100	860
a_c (units/day)	50	200
p	1.08	1.04

(i) Specify the batch sizes for each product for maximum return of the schedule.

(ii) Check whether these batches are in the respective production ranges.

(iii) Find the available machine time for additional production.

2. Show that when a schedule of n products is optimized for maximum return, the batch size for product 1 is

$$Q_{1M} = \frac{\Sigma \alpha \alpha_m K p'}{\Sigma K \alpha^2} Q_{1m}$$

where

$$\alpha_{im} = \frac{Q_{im}}{Q_{1m}}$$

3. Data relating to three products in a production schedule are given below:

	Product 1	2	3
a_c (units/day)	10	40	100
a_p (units/day)	200	200	200
K ($)	1.0×10^{-4}	1.0×10^{-4}	1.0×10^{-4}
Q_m (units)	4,000	6,000	6,000

If the allowable relaxation factor $p = 1.08$, draw out a production schedule for (i) maximum return; (ii) maximum rate of return.

4. Suggest a schedule aimed at maximum return for the following three products, when $p = 1.05$ is allowed:

	Product 1	2	3
a_c (units/day)	20	24	36
a_p (units/day)	40	150	80
c ($)	4.0	1.8	12.5
B ($)	0.001	0.001	0.001
s ($)	3,000	1,800	1,500
t_s (days)	2.0	1.4	1.2

Assume: $i = 8\%$, 300 working days in a year.

5. Repeat the calculations for the example given in this chapter, but assume no safety allowance need be taken into account.

6. A plant has two production lines. Some of its products can be manufactured only on one line; some can be loaded on both. How would you proceed to determine an optimal schedule, if data for eight products are as given below:

	Product 1	2	3	4	5	6	7	8
a_c (units/day)	50							
α	1.0	1.0	1.2	1.2	1.5	1.6	2.0	4.0
a_p (units/day)	500	500	500	500	800	800	1,000	1,000
s ($)	500	500	500	500	1,000	1,000	1,000	1,000
K ($\$ \times 10^{-3}$)	1.0	1.0	1.0	1.0	0.5	0.5	0.5	0.2
p	1.05	1.05	1.05	1.05	1.10	1.10	1.10	1.10
Producible on line	I	I, II	I, II	I	II	II	I, II	II

Assume: (i) $t_s = 1$ day for any product.
(ii) Optimize for maximum return.
(iii) Products are manufactured in succession.
(iv) Use overtime if necessary.
(v) p for product 3 may be relaxed to 1.08, if so desired.

15

ELEMENTS OF CONTROL PROCEDURES

The efficiency of a process or a machine is measured by the ratio of output to input. Ideally, this is the criterion that ought to be used in defining the efficiency of a production organization. The input includes all the resources and potential capabilities of the plant in men, materials, and machines, as well as in organizational abilities; the output is simply the results realized within a certain period of time. But it is obvious that "input" and "output" are rather difficult to define in accurate terms in this case, especially if we are to measure them quantitatively in so many units. Hence the only way in which we can assess the efficiency of the production plant is by comparing actual performance with targets specified in the production program, provided this program is planned in good faith and provided the plant is capable of coping with it.

It is erroneous, however, to conclude that production control is only concerned with an academic detached and somewhat passive assessment of the plant performance. Production control must be an active function; ideally, it must operate like an automatic self-regulating mechanism that registers events and reacts to them by adjusting relevant parameters in the production centers. In addition to that, it must be self-learning, so that it can gain from its experience and learn from past mistakes.

Stages and Activities of Control

Control begins as soon as the production operations begin; in fact it begins slightly earlier by the actual issues of the production orders to the shop floor. The four stages of control are:

1. Triggering off the operations; observing progress and recording it.
2. Analyzing the data by comparing progress with the plans and with achievements of competitors.
3. Taking immediate action to modify plans and redirect activities in order to attain the target.
4. Post-operation evaluation, feeding back information and conclusions to the planning section in order to improve future planning.

Each of these stages is important, each is a link in the control procedure, but each is worthless and indeed pointless without the others. There is no need to make observations and collect data if they are not being used for analysis. Any analysis is equally pointless if its results are disregarded, and when corrective action is undertaken but confined to immediate patching up, the control procedure lacks the vital feedback that is possible only after evaluation, and without which the system is deprived of the opportunity of learning from mistakes and experience.

Table 3–1 gives a summary of planning and control activities as a whole, but in production control we are mainly concerned with four specific fields.

Control of processes and production activities

This is one of the major responsibilities of production control, which begins by the dispatching function (releasing of production orders, observation of activities and recording) and proceeds immediately with expediting (analysis of progress, comparison with schedules, taking immediate corrective actions for chasing orders, materials, etc.). In the final evaluation stage, the collected data are used to reassess process capacities, on the lines described in Chapter 12, and to replan maintenance schedules.

Quantity or inventory control

Stocks are held at various stages of the production line, from raw materials and bought-out components to semifinished products and final products ready for sale. Inventory control is so closely connected with the control of production activities that it is very often included within the responsibilities of the production planning and control department (see discussion in Chapter 4). Inventory control follows the same pattern of control procedure listed above:

1. Recording of stock levels, such as shown in Chapter 9.
2. Analysis of the distribution of demands, trends, fluctuations, etc.
3. Immediate action may be required in the form production or procurement orders.
4. Final evaluation is important, to compute optimal reorder quantities, economic safety margins, optimal periods for placing procurement orders, and inventory systems to facilitate effective control and replenishment.

Quality control

This function, again, is vital for feedback of information on how well the processes are running, but quality control begins, in fact, with the inspection of raw materials before they are admitted to the stores. The control procedure involves the following successive steps:

1. Through various control charts, the performance of key processes is recorded.

2. Analysis of these charts reveals the process capabilities from the quality

aspect; in other words we can state the accuracy and quality characteristics that are reasonable to expect from the processes. Also, trends in performance can often be detected.

3. Immediate action can take two forms: First, if the process is not accurate enough, there may be a need for 100 per cent inspection, in order to sort out all the components that do not conform to the specification; secondly, as soon as trends in the processes are found, there is a need to determine at what stage the machine ought to be adjusted.

4. Finally, the familiar post-mortem: Are the specifications realistic? Should the process be changed? What effective inspection plans can be devised on the basis of past experience?

Cost control

The same control procedure applies here: Data collection is carried out in conjunction with dispatching, the costs are computed and compared with the estimates, the sales price is adjusted (if possible), and the discrepancies between estimates and actual cost figures are analyzed for future reference in order to develop the estimating methods to as high a degree of accuracy as possible.

The four elements of control procedures are summarized in Table 15–1. The control of production processes, namely, the function of dispatching and expediting, and progress control methods will be discussed in this chapter and in Chapter 16; inventory, or quantity control, in Chapters 17 and 18. Chapter 19 is devoted to quality control and Chapter 20 to cost control.

Table 15–1

Elements of Control Procedures

	*Processes**	*Inventory**	*Inspection*	*Cost*
Observation	Active processes: output vs. time Idle processes: machine idle time; breakdowns	Records of stock level	Process control Control charts	Collect cost data
Analysis	Compare progress with plan	Distribution of demand Trends Seasonal fluctuations	Process capabilities Trends	Compute costs and compare with estimates
Immediate action	Expedite	Issue production and procurement orders	Initiate 100% inspection Adjust processes	Adjust sales price (if possible)
Evaluation	Process capacity; maintenance schedules	Replenishment policies Inventory systems	Reassessment of specifications Process improvement Inspection procedures	Economic evaluation of processes Preparing better data for future estimates

* Production control responsibilities.

Dispatching and Expediting

When the production order is complete and the schedules planned, everything is ready for the word "go." This, in short, is the responsibility of the dispatching function, i.e., to trigger off the flow of information and instructions and with them the issue of materials, tools, production aids, drawings and specifications, inspection orders, etc. The sequence of dispatching activities is shown in Fig. 15–1. If the product is made of several parts, the production order is broken down accordingly, and for each operation there is a materials and tools issue, a job card that details the task and the methods to be used, an inspection order and

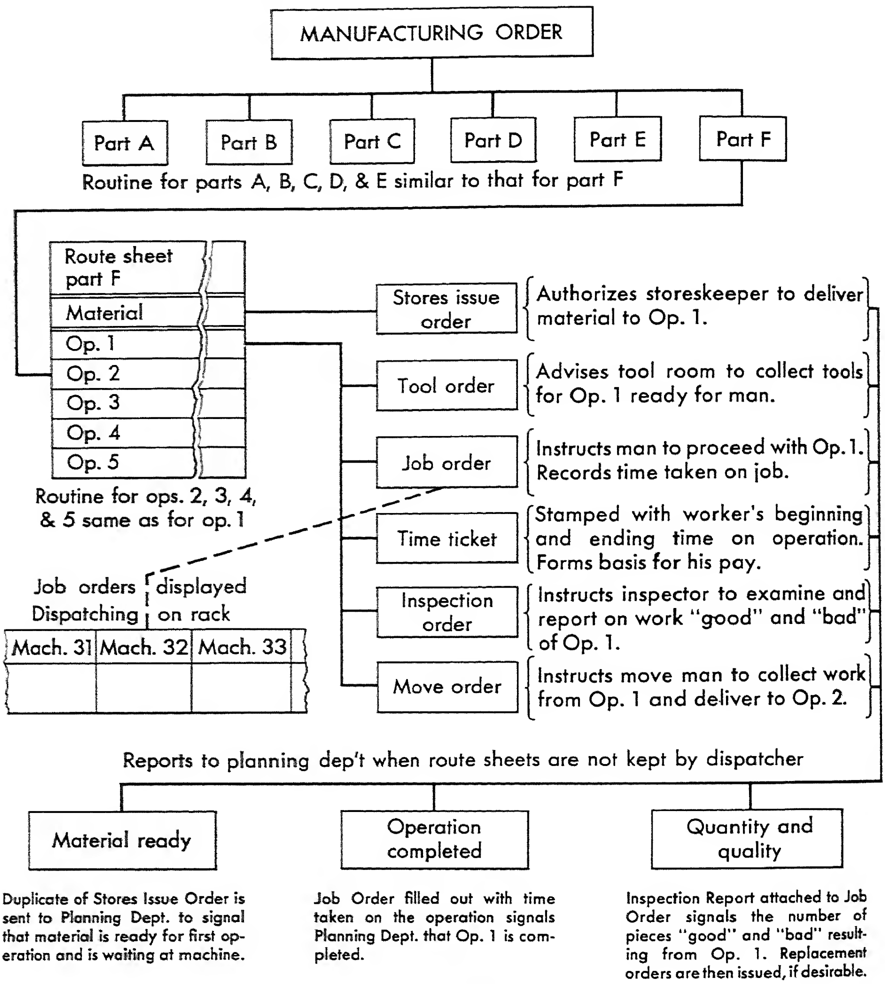


Figure 15–1. ***Sequence of dispatching activities.*** (Reproduced with permission from *Production Handbook,* The Ronald Press, 1958, edited by G. B. Carson)

information about the next destination of the component (see Chapter 9). Finally the dispatching function is responsible for keeping records of actual operation times, idle man and machine times, length and causes of breakdowns, and any other relevant information about reality keeping within schedule or deviating from it. These records can then be used for:

Calculating wages, if the wage scheme in the plant is dependent on output

Examining the cost of the job and the product and comparing with the planned budget and the initial cost estimates

Comparing operation times with the estimated times and verifying whether the discrepancies are consistent with past experience, so that future estimates can be made more accurate

Locating weak points in the production line, so that work study can be undertaken to explore ways of alleviating the difficulties

Studying how much idle time was caused because of schedule interference (i.e., overlap in operation times) due to unaccountable delays or too tight a production schedule

Studying how much delay was caused by machine breakdown and whether the maintenance and emergency repair facilities were compatible with the requirements of the situation (this analysis could lead to an optimal number of repairmen required, the amount of effort that should be devoted to preventive maintenance and to an optimal maintenance schedule)

Analyzing the delays caused by tool breakdown, regrinding and resetting, to ensure that the specified speeds are compatible with reasonable amount of tool wear

Analyzing to what extent both queues and delays are caused by inadequate materials handling systems.

The method of exercising the dispatching function depends very much on the type of production and on the size of the plant. In the case of repetitive production, when the same product is frequently produced either with no variations or with only minor ones, the flow of information and instructions becomes a routine matter. Everyone concerned is already trained to perform the task, to use the tools and identify them, and to recognize faults in the produced components. The interpretation of the production order becomes a straightforward matter, and in many cases only details about quantities, dates of delivery, types of models, or colors need be mentioned in the production order, while job cards with detailed instructions how to perform the task may become almost standard orders that need rarely be consulted by the trained personnel. The dispatching effort, after releasing the orders, would then be mainly devoted to maintaining the records, and this can be done effectively by dispatching stations located along the production line, each being responsible for recording progress at certain production centers.

The expediting function consists of a close study of the gathered data and comparison of actual progress of the work with the planned schedule. One could

almost liken the expeditors or progress men to policemen, who keep a benevolent eye on the flow of traffic and whenever anything happens to obstruct a smooth flow, they endeavor to locate the source of trouble and take action to remove it.

For example, a batch of components fails to arrive at a certain production center on the date scheduled. The expeditor learns this fact from progress charts or other records at the dispatching station, and he proceeds to investigate the cause for this failing and to decide in what way he can help put things right. If any difficulties arise through ambiguities or misunderstanding of the instructions in the production order, the expeditor should clarify all such points. In other words, it is through the expeditor that interpretation of the production order is sought, and when he is unable to cope with a problem on his own, he turns to the production planning office for assistance.

Since the expeditor is so much concerned with keeping up to schedule and with flow of materials, semifinished products, and final products from the stores and from one production center to another, it is common practice to entrust him with the responsibility for moving the materials by putting suitable facilities (equipment and personnel) at his disposal. If the responsibility for removal of the goods to the next working station is left to the production centers, they can either let the skilled operators do the removing or ask a plant removal pool to do it for them. By relieving the production centers from this concern and transferring it to the expediting function, there is a clear demarcation between the responsibility for what goes on within the production centers and responsibility for circulation and flow, and in many cases this division has obvious organizational advantages.

It should perhaps be stressed again that expeditors are concerned with fulfilment of setout schedules, and hence they are concerned with the present and with immediate remedies if anything goes wrong, not with lengthy studies of intricate cause-and-effect systems. As a general rule, therefore, expeditors should not be asked to carry out detailed post-mortems and evaluation analyses. They are usually not equipped with the qualifications or the patience that such analyses require. Naturally they can often contribute to the analysis by adding up-to-date facts, and therefore they should be told of any conclusions and action contemplated after the study is completed, but the actual expediting function and the evaluation function should be kept separate.

The above concepts are difficult to apply, however, because a clear-cut demarcation does not exist between dispatching and expediting, and it is not often easy to state exactly where dispatching ends and expediting starts. Theoretically it is possible to say that dispatching consists of flow of information, while expediting is concerned with flow of materials and components; dispatching records events, expediting tries to adjust them whenever necessary. However, since in practice the two functions are not chronologically separate but exist side by side, one complementing the other, an overlap in responsibilities is often unavoidable. The actual detailing of staff to the two functions will therefore

depend on the amount of recording, materials handling, and chasing that have to be carried out. Many firms find that it is futile to separate the two and that a combined dispatching-expediting organization as a single production control department is very effective.

To summarize, the purposes of the active functions of production control (i.e., dispatching and expediting) are:

1. To release the production orders at the appropriate time and facilitate effective flow of information
2. To record the flow of materials and tools and adjust whenever necessary
3. To record progress of production operations and adjust whenever necessary
4. To record amount of work in process and verify its effect on the schedule
5. To record quantities produced and compare with required quantities
6. To record amount of faulty work and scrap and issue orders for production of replacements
7. To record machine idle time and check the reason for it
8. To record stoppages and holdups and classify them according to:
   (i) Lack of drawings or instructions
   (ii) Lack of materials
   (iii) Lack or failure of tools
   (iv) Work held up by previous operation
   (v) Machine breakdown
   (vi) Operator missing or not available
   (vii) Waiting for inspection to approve work or machine setting

Recording Progress

Watching the progress of production ensures that up-to-date information is always available for effective expediting. How should progress be recorded? Let us examine three methods in common use in industry.

Gantt charts

These were developed by Henry L. Gantt about a half-century ago. Their purpose is to provide an immediate comparison between schedule and reality, and this is achieved simply by marking on the schedule the actual progress of the work. There are several variations of Gantt charts, which can be adjusted to the specific circumstances prevailing in the plant. The following symbols are common to most Gantt charts:

┌ denotes the time when work is supposed to begin.

┐ denotes the time when work is supposed to end.

|4 | the figure indicates that the work load in the planned interval is equivalent to four work periods (days, weeks, number of pieces, etc.).

| 12| the figure indicates that the work load to be completed by the specified date amounts to 12 work periods (or number of pieces).

the top line indicates planned work; the bottom line, actual work. Where discrepancies between plan and reality occur, it is often useful to indicate the reason for future reference. The following abbreviations are sometimes employed:

A – operator absent from work
G – "green" operator
I – instructions lacking or deficient
L – slow operator
M – holdup due to materials failing to arrive or failing in quality
R – machine or tools in need of repairs
T – tools lacking or inadequate
V – holidays

An example of a Gantt chart for machine activities is shown in Fig. 15–2. Similar charts can be used for controlling operators' activities, product progress, master schedules, etc.

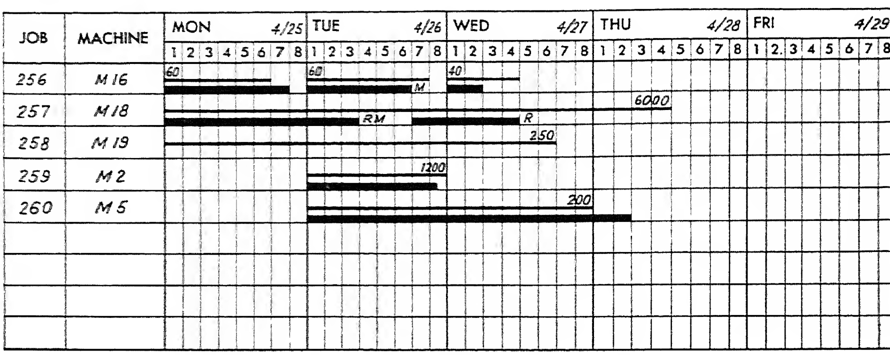


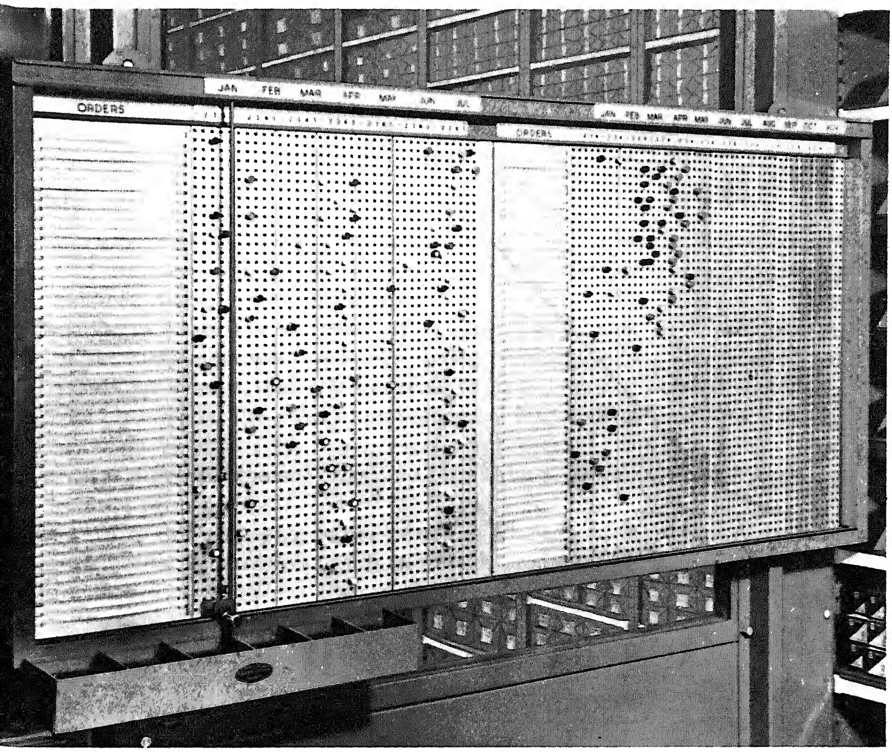
Figure 15–2. ***An example of a Gantt chart.***

Visual charts

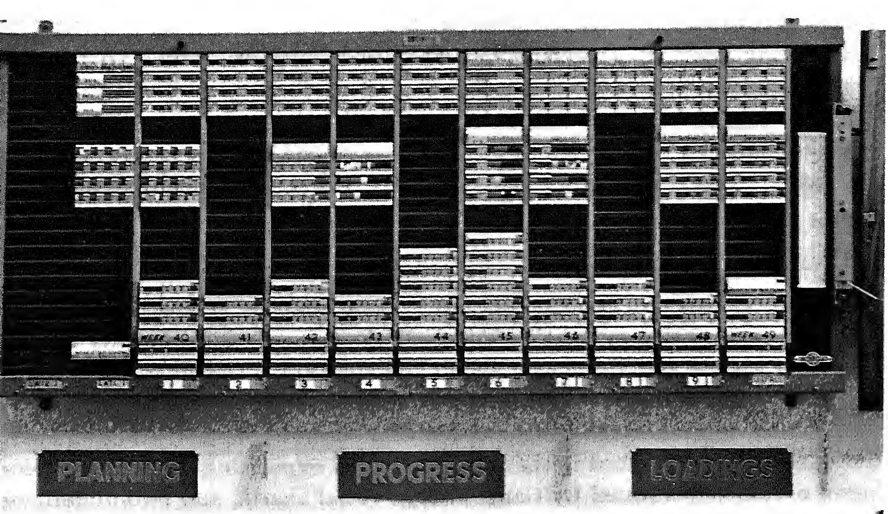
These are in a sense a variation of the Gantt chart, their purpose being more temporary in nature. They present a current picture of the situation but leave no record of past events. Visual charts often make use of colored bars, strings, moveable rulers, meters with numbers, etc. Two examples of visual charts are shown in Figs. 15–3 and 15–4.

Cumulative and weekly charts

Both the Gantt charts and the visual charts are, in fact, one-dimensional graphical presentations of output activity, plotted in terms of time. Their main virtue is simplicity of presentation of current position, and this simplicity originates from their one-dimensional quality. Their drawback, however, is



gure 15–3. ***A visual control chart using a pinboard.*** (Courtesy Constructors Ltd., rmingham, England)



gure 15–4. ***A visual control chart for recording figures, which employs revolving sks set in mobile cases.*** (Courtesy Dacron Plan-O-Matic Ltd., London)

that they provide no record of progress itself. In other words, by looking at one of these charts, we cannot tell whether output increased or decreased during a certain span of time in the past unless we introduce various symbols and colors, which often confuse rather than clarify the picture.

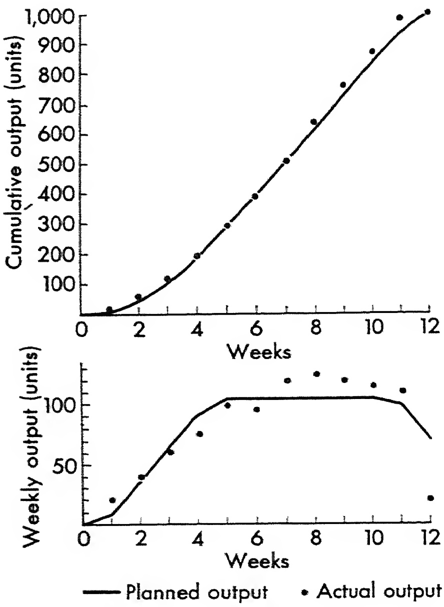


Figure 15–5. ***Comparing progress of actual with scheduled output.***

Recording progress, especially for evaluating purposes, is effective in two-dimensional charts, as in the example in Fig. 15–5, where a cumulative chart and a weekly output chart are demonstrated. The cumulative chart shows how much we have managed to produce so far, but it is somewhat insensitive to weekly fluctuations, and if these are worthy of study, the weekly output chart is very useful.

Summary

The four elements of control are: observation, analysis, immediate corrective action, post-operation evaluation. All these elements are vital for any effective control procedure and Table 15–1 shows how they apply to control of processes, inventory control, inspection, and cost control. In production control, the first three control elements (releasing of orders and observation, analysis, and corrective actions) are embodied in the dispatching and expediting functions. Recording of events is facilitated by Gantt charts, visual charts, and two-dimensional cumulative and weekly output charts.

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Problems

1. Differentiate between dispatching and expediting, and point out to what extent this difference should lead to a clear and rigid demarcation of responsibilities between dispatchers and expeditors.
2. Describe in what way the elements of control procedure are related in the fields of production control, inventory control, inspection, and cost control; in other words, to what extent is it desirable to have not only vertical feedback (from one element to the other) but horizontal feedback as well (from one field to another).
3. Can you suggest a control procedure for a sales department, based on production control principles?
4. Surveys in industry suggest that cases of average machine downtime of 50 per cent are fairly common in job and batch production plants. Suggest a method for recording and controlling machine utilization in a machine shop.

16

COMPUTER-ASSISTED PRODUCTION CONTROL[1]

Norbert Wiener has suggested that control is essentially communication that has been made effective.[2] If a control mechanism is conditioned to respond to any given set of parameters, its usefulness is determined by the flow of information that define these parameters, and for this flow to be effective, the system must be:

Quick, in order to provide information which truly represents the current situation, and to ensure that any corrective action taken is relevant to the prevailing parameters and does not suffer from serious time lags

Accurate, so that the records provide unambiguous data and computations of quantities are free from errors

Comprehensive enough, i.e., it must include all the relevant details required for analysis and decision making.

Since the purpose of production planning and control is to match the sales requirements with production facilities, and since both are subject to constant variations, the problem of formulating optimal solutions for machine loading has to be continuously restated and resolved. Whenever a relevant parameter changes, materials and manpower requirements may have to be recalculated, machine loading may have to be readjusted, and schedules for completing operations, assemblies, or deliveries may have to be modified. Thus the data must be fed and classified in such a way that the risk of excluding an important parameter would be rather small. Furthermore, information that necessitates corrective action should be prominently displayed so as not to be "lost in the crowd." This requires an intelligent sorting mechanism that evaluates the significance of the recorded changes or the possible effect of their combinations, and

[1] The author is indebted to International Business Machines Corporation (New York), and International Computers and Tabulators, Ltd. (London) for providing him with papers and catalogs, on which some of the material in this chapter is based; also to Mr. W. J. Kease of A.E.I.-Hotpoint, Ltd. (London) for reading the chapter and for his helpful comments and suggestions.

[2] Wiener, N.: *Cybernetics—or control and communication in the animal and the machine* (John Wiley & Sons, Inc. 1948).

evidently this task becomes more involved as the amount of data grows. Also, since we are dealing with a dynamic situation, the readjustment of schedules and quantities necessitates repetitive computations, which must both be accurate and carried out at high speeds.

Computer requirements

This is where the computer can be of assistance in production control. It should have a storage unit, capable of holding a large amount of data, which is easily accessible and which can be quickly brought up to date. It should also have a sorting facility, with the aid of which the data can be classified, an arithmetic unit to carry out the computations quickly and reliably, and a printing unit to produce the results in such a form that the various departments concerned can immediately interpret their significance and take action if and when required.

It cannot, however, be overemphasized that a computer is merely a tool; it does not constitute control—it assists control by making it more effective, simply by saving time in the laborious task of processing the data. It cannot in any way be a substitute for defining policies for production, defining targets, setting criteria for measuring effectiveness, and formulating methods through which optimal performance can best be achieved. These are the essentials without which production control cannot possibly hope to succeed, and these are part of the responsibility of the production management, not the computer. Furthermore, the control procedure, including the routes of flow of information and assignment of responsibilities for actions, should be clearly defined and implemented; otherwise time would be saved by the computer on storage and calculations but wasted later if nobody quite clearly understands what the output of the computer means and what should be done with it.

Computer types and operation

Computers are normally classified into two types:

1. *The digital (or numerical) computer*, which operates on the principle of counting digits at a very high speed; the computing and storage media are concerned with numbers, and the data are therefore translated into a digital form by some sort of suitable code, which is employed initially from the input stage. The digital computer can be easily adapted to a wide variety of applications, provided the data can be translated into coded numerical form.

2. *The analogue computer*, in which the production system governed by definable variables is simulated by a physical model; changes in the variables of the system are fed into the model by appropriate adjustments of the corresponding variables in the model, and the expected effect is physically measured by "analogy."

The digital computer is suitable for applications in production control, where descriptions of materials, specifications of products and machines, recording of

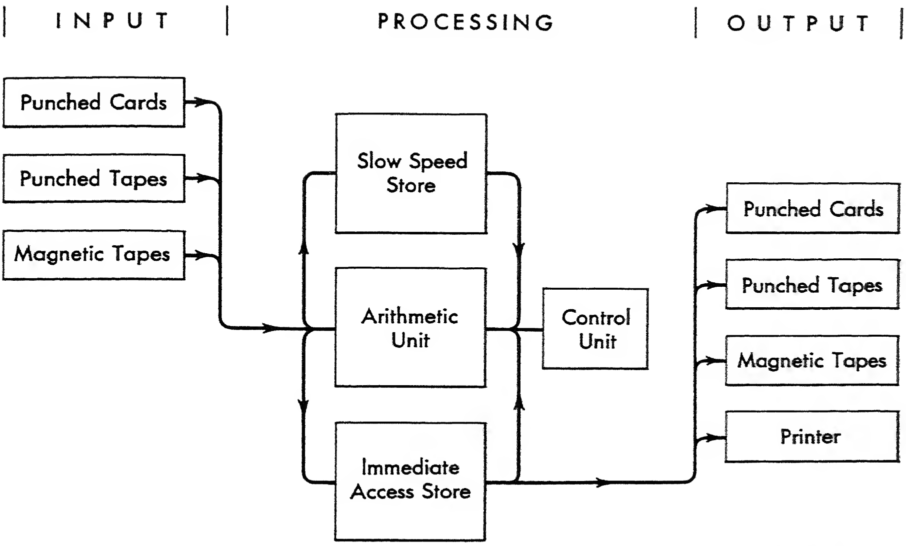

Figure 16–1. ***A schematic diagram showing how a computer works.***

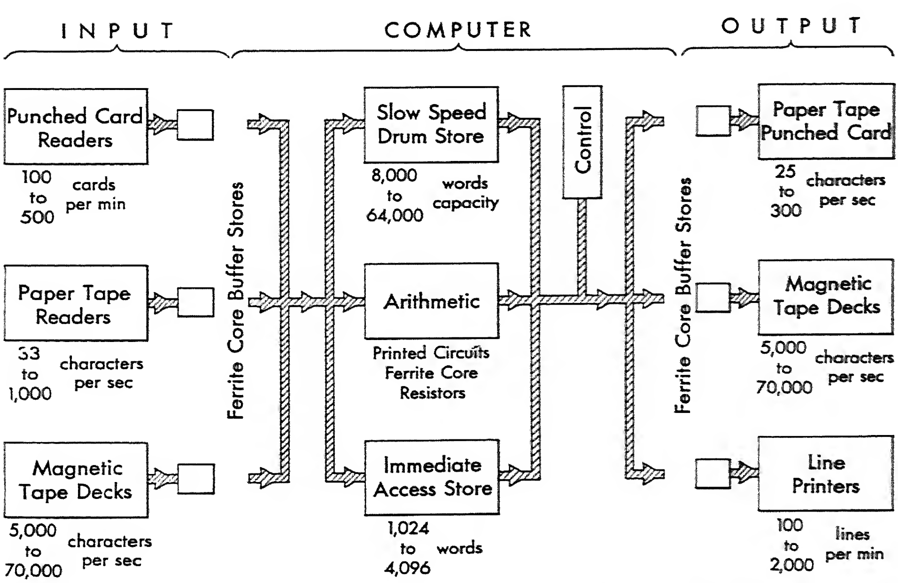

Figure 16–2. ***Range and capacities in computers used for production control.*** (Contributed by W. J. Kease)

equipment available capacity, etc., can easily be presented in numerical language.

Figure 16-1 shows schematically how such a computer operates. The information is fed at the input stage by means of magnetic tapes, punched cards, or punched tapes. Data processing consists of the library (storage) units and arithmetic unit. A control unit prescribes to the computer what should be done with the information, how to manipulate it, and in what sequence to perform calculations. At the output stage the processed data appear again in the form of magnetic tapes, punched cards, or punched tapes, and a printing unit translates the data from these media and puts them in printed form.

Although the computer is limited in performing a restricted range of operations, its versatility and capacity can evidently be increased by adding certain operating units; for instance by having several channels for input and output, several arithmetic units, several transfer functions (to transfer information from the internal storage unit to various registers or from one register to another), shift functions (to move data up or down the value scale), and several sorting units for data classification. Each of these units has its own operating speed, the

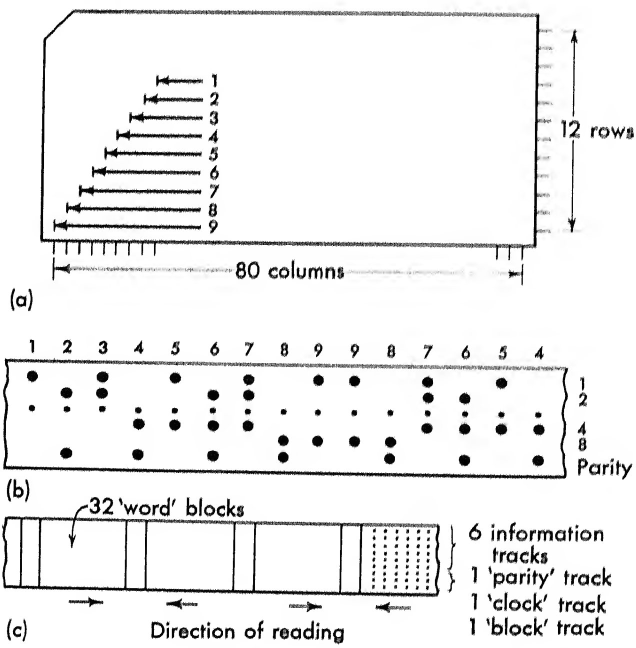

Figure 16-3. ***Three types of input-output media.***
(Contributed by W. J. Kease)

(a) Punched card
(b) Punched tape
(c) Magnetic tape

range of capacities of speeds being rather wide (as suggested in Fig. 16–2), so that the problem of coordinating their performance is one that system analysts and programers have to consider very carefully.

Media for Recording Data

Let us now examine more closely the various media most commonly used for recording data (see Fig. 16–3).

Punched cards

Series of holes are punched in a card, usually designed as a matrix of 10 rows and 80 columns. Two additional rows are available, bringing the total number of rows to 12, which is useful for recording data for 12 months. Examples of 80 column cards are shown in Figs. 16–5, 16–6, and 16–7.

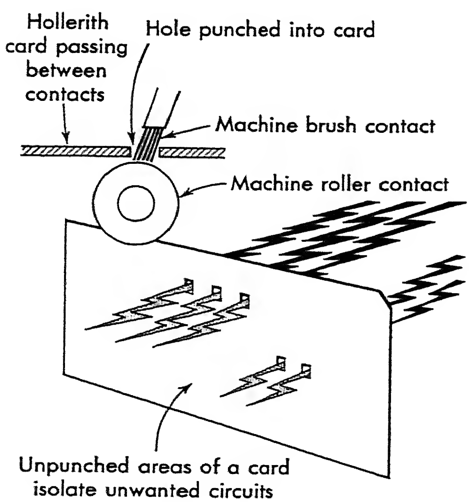

Figure 16–4. ***Actuating a punched card machine by electrical control.*** (Courtesy International Computers and Tabulators Ltd., London)

Actuating the machines for sorting cards is carried out by closing electric circuits through the punched hole, as shown in Fig. 16–4. By using this basic principle, the punched hole can serve for the purposes of addition, subtraction, multiplication and division, selection, classification, filing, listing, reproduction, etc. The punched card system is highly versatile and widely used as a recording medium. The actual recording is carried out by punching the holes in appropriate positions on the cards; there are two methods of notation:

1. *The decimal notation system*, using the rows 0–9 on the cards; this is a two-dimensional system, the location of a hole being determined by two coordinates (row number and column number).

2. *The binary notation system*, in which numbers are expressed by relation to the base 2, uses only the figures 0 and 1. The number 110 in the binary system means: the first 1 is $(2)^2 = 4$; the second 1 is $(2)^1 = 2$; the 0 is simply 0; hence 110 in the binary system is 6 in the decimal system. A conversion table from the decimal to the binary system would read as follows:

Decimal	*Binary*
0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	111
8	1000
9	1001
10	1010
etc.	

In punching tapes by the binary system, the figure 0 is represented by the absence of a hole and the figure 1 by the presence of a hole; the system is very useful in recording information of the yes–no type, in which case it becomes one-dimensional, the location of a hole being defined by its distance along one axis. The translation of information from the decimal system to the binary system, whenever this is required, is often carried out automatically by the computer itself.

The 80-column card allows for a great deal of information to be recorded on it for a wide variety of purposes. Several examples are given below.

Sales information

Sales information can be punched as shown in Fig. 16–5, where (see bottom part of the card):

Column	*Records*
1	the code number
2–6	date
7–12	order number
13–15	quantity
16–18	model number
19–24	bill of materials
25–49	customer's name or product description
50–52	schedule period
53–55	cards to make
56–80	design details of product

Production control

The International Computers and Tabulators, Ltd., London, used a computer to control the production of three of its factories, which included several

departments concerned with specialized equipment, a machine shop, an assembly shop, a special orders department, inspection, and stores. The total number of products amounted to approximately 600, and the information punched on the cards used both the decimal and binary systems as follows:

Column	*Data*	*System*
1–6	part number	decimal
7–8	factory location	decimal
9–10	(not used)	
11–50	quantities	binary
51–60	(not used)	
61–70	drum location	binary
71–76	(not used)	
77–78	card serial number	decimal
79	class of card	decimal
80	computer designation	decimal

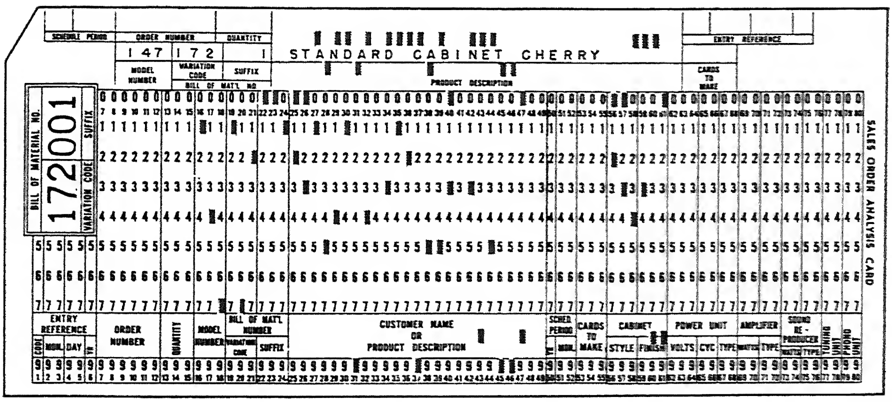

Figure 16–5. ***A punched card for sales records.*** (Reprinted by permission from "IBM Manufacturing Control," International Business Machines Corporation, 1954)

Payroll and assignment

A wages or job card, used for recording operators' earnings, can include details about the job to be carried out, work done, earnings and bonus, as shown in Fig. 16–6.

Materials control

Control of materials movements is facilitated by a card as in Fig. 16–7. One portion includes a location record and the other (which, when detached, becomes a 38-column card) can be used as a stock identification tag.

Punched tape

On the tape, too, the data are recorded as a pattern of punched holes, the principle of storage of information and sensing devices being similar to the

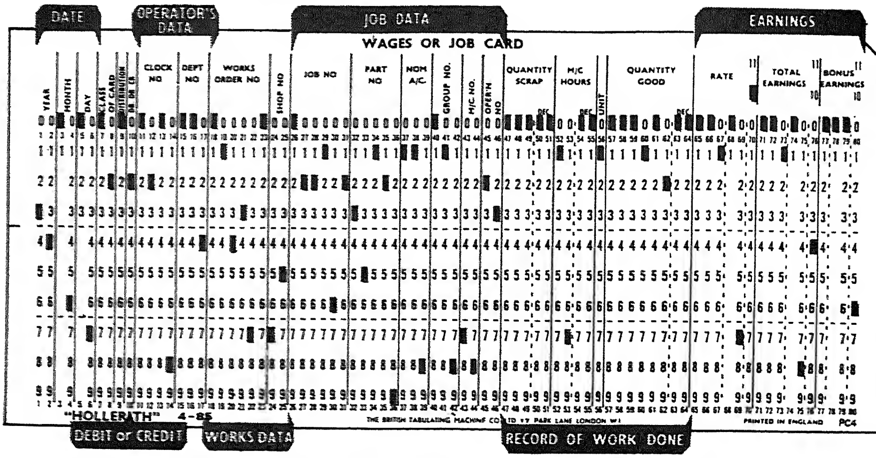

Figure 16–6. ***A wages or job card, punched and showing the large amount of data which can be recorded.*** (Courtesy of International Computers and Tabulators, Ltd., London)

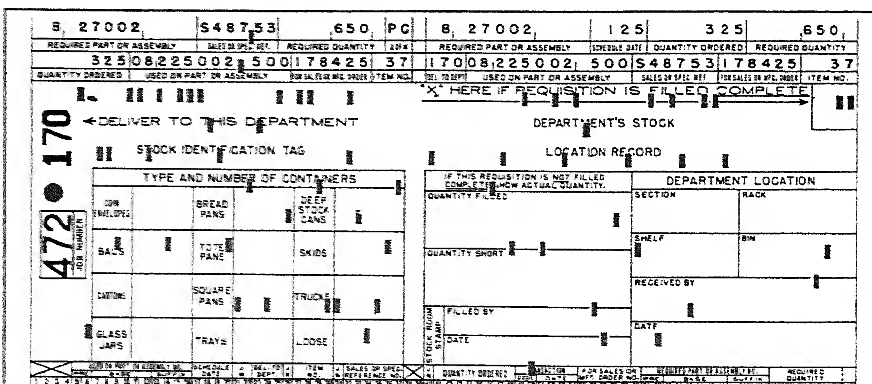

Figure 16–7. ***Control of materials movements by a punched card.*** (Reprinted by permission from "IBM Manufacturing Control," International Business Machines Corporation, 1954)

punched card system. The punched tape is a far cheaper medium to use, but being a continuous strip, it does not lend itself to convenient sorting of data as the punched cards do.

Magnetic tape

The data are superimposed on the material in the form of magnetic strains, as in a tape recorder. The tape is very economical in storage space and allows a great deal of information to be recorded on one reel. In one case study it was

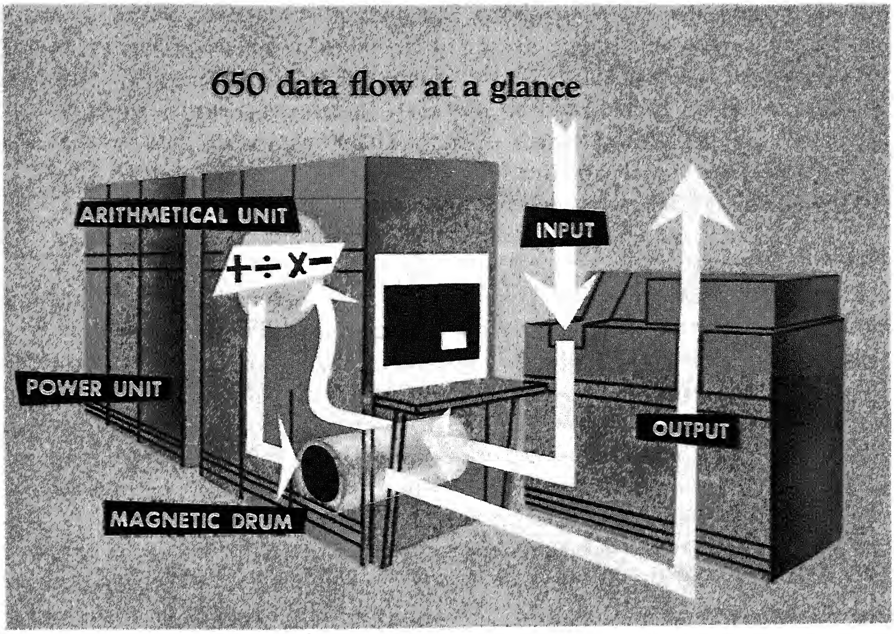

Figure 16–8. ***The magnetic drum IBM 650 computer.*** (Reprinted by permission from "IBM Presents the 650 Magnetic Drum Data Processing Machine," International Business Machines Corporation, 1955)

reported[3] that information involving 350,000 cards was transferred to seven reels of magnetic tape; another library of 30,000 cards for piece parts was converted into one reel; and stock files for bought-out finished materials containing 45,000 cards could be recorded on one reel. Information can be recorded on magnetic tape and read at speeds of up to 15,000 characters per second. But while the storage capacity of this medium is very high compared with that of punched cards, it is far more expensive.

Magnetic drum

This is used for internal storage of information, whereas the three preceding methods are media for the input and output stages. The magnetic drum, a nickel–cobalt-coated cylinder (in the IBM 650 computer it is 4 inches in diameter, 16 inches long, and it rotates at 12,500 rpm), stores the information and feeds it to the arithmetic unit, as shown in Fig. 16–8. The transfer of information from the input media (such as punched cards) is carried out by feeding the cards between contact rolls and a set of 80 "sensing" brushes. In this way each card is read and the data are transformed into electric impulses and stored on the surface of the drum as a pattern of magnetized spots (see Fig. 16–9). The information is stored in bands on the drum, each band being divided into sections having specified locations (see Fig. 16–10). The IBM 650 computer has 40 bands, each comprising 50 locations, so that it has a capacity of 2,000 locations, each location taking up to 10 digits and a sign. The instructions to the computer can be recorded on the drum in very much the same way (Fig. 16–11).

Installation of a Computer-assisted Production Control System

From the foregoing remarks it is fairly evident that the main benefits of computers in production control are derived in plants dealing with a wide variety of products, with a great diversification of specifications of materials, products, and components, and (last but not least) with a highly dynamic schedule which has to cope with changing customers' requirements. Such a situation would often arise in large batch and job production plants. It would appear that the type of production in itself is not the decisive factor as to whether or not the computer can be usefully employed in production control. Many so-called flow production firms are, in fact, engaged in batch-producing components and assemblies for their final flow assembly line; they are conscious of scheduling problems in their batch shops, as well as production-smoothing problems associated with their final flow lines (see Chapter 13), and therefore find the computer useful.

Experience in recent years has shown that the computer has potential economic applications in both large and medium size enterprises. Small plants, which are restricted by their limited financial resources from acquiring expensive computers (and for which in any case the utilization of the computers would be

[3] Kease, W. J.: Electronic data-processing for production control (1957, a private publication).

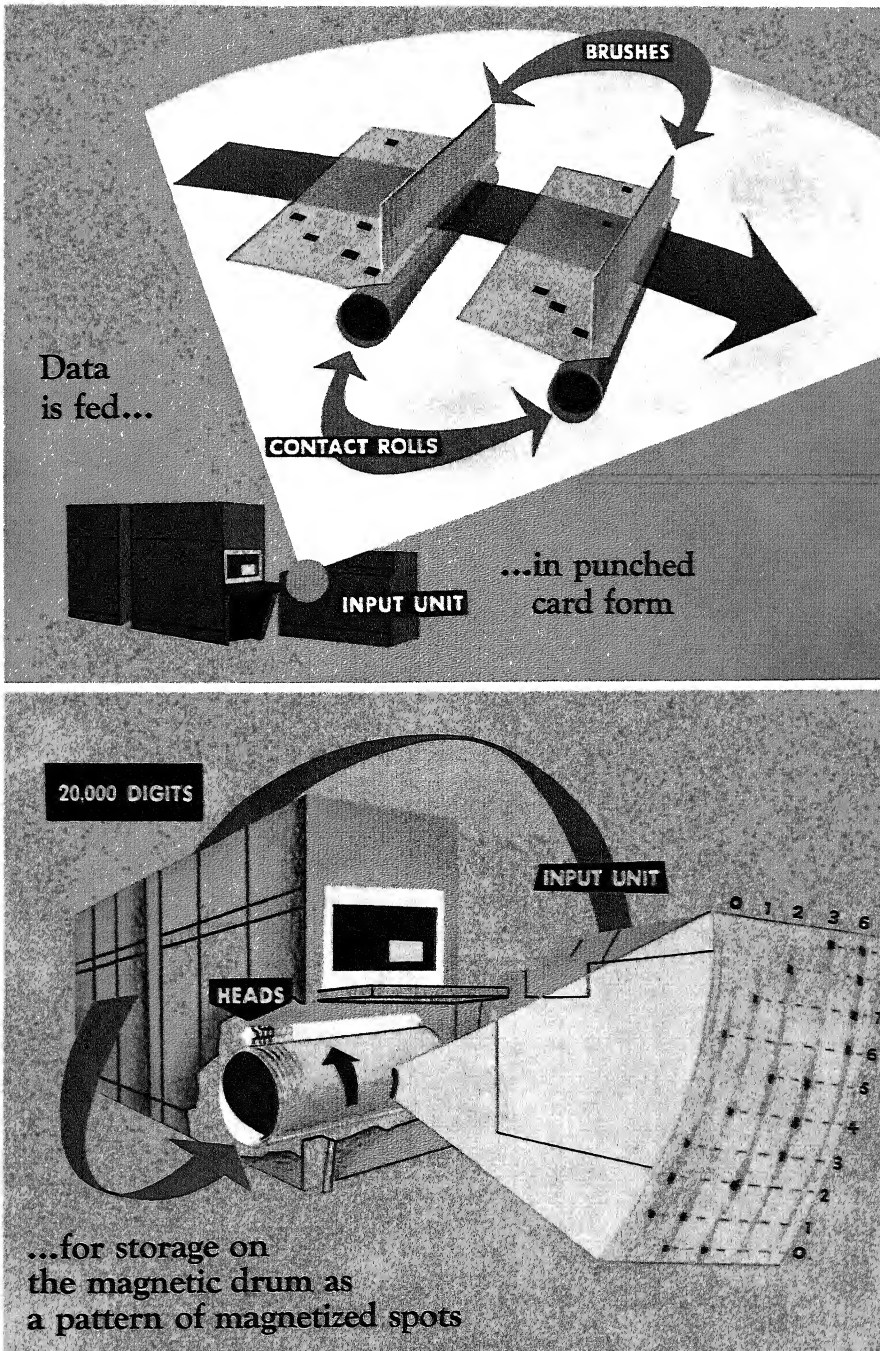

Figure 16–9. ***Recording information on a magnetic drum.*** (Reprinted by permission from "IBM Presents the 650 Magnetic Drum Data Processing Machine," International

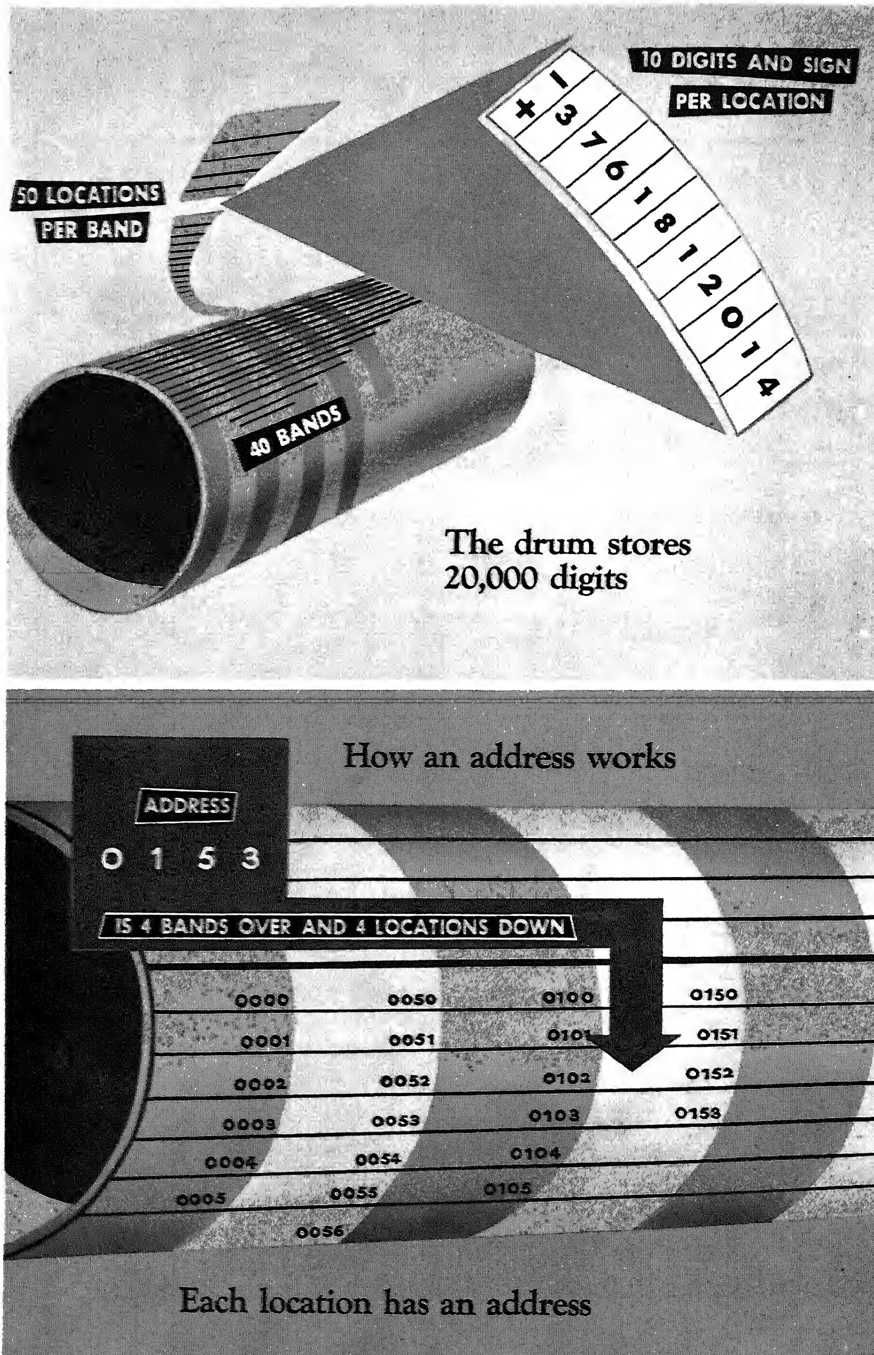

Figure 16–10. ***The magnetic drum.*** (Reprinted by permission from "IBM Presents the 650 Magnetic Drum Data Processing Machine," International Business Machines Corporation, 1955)

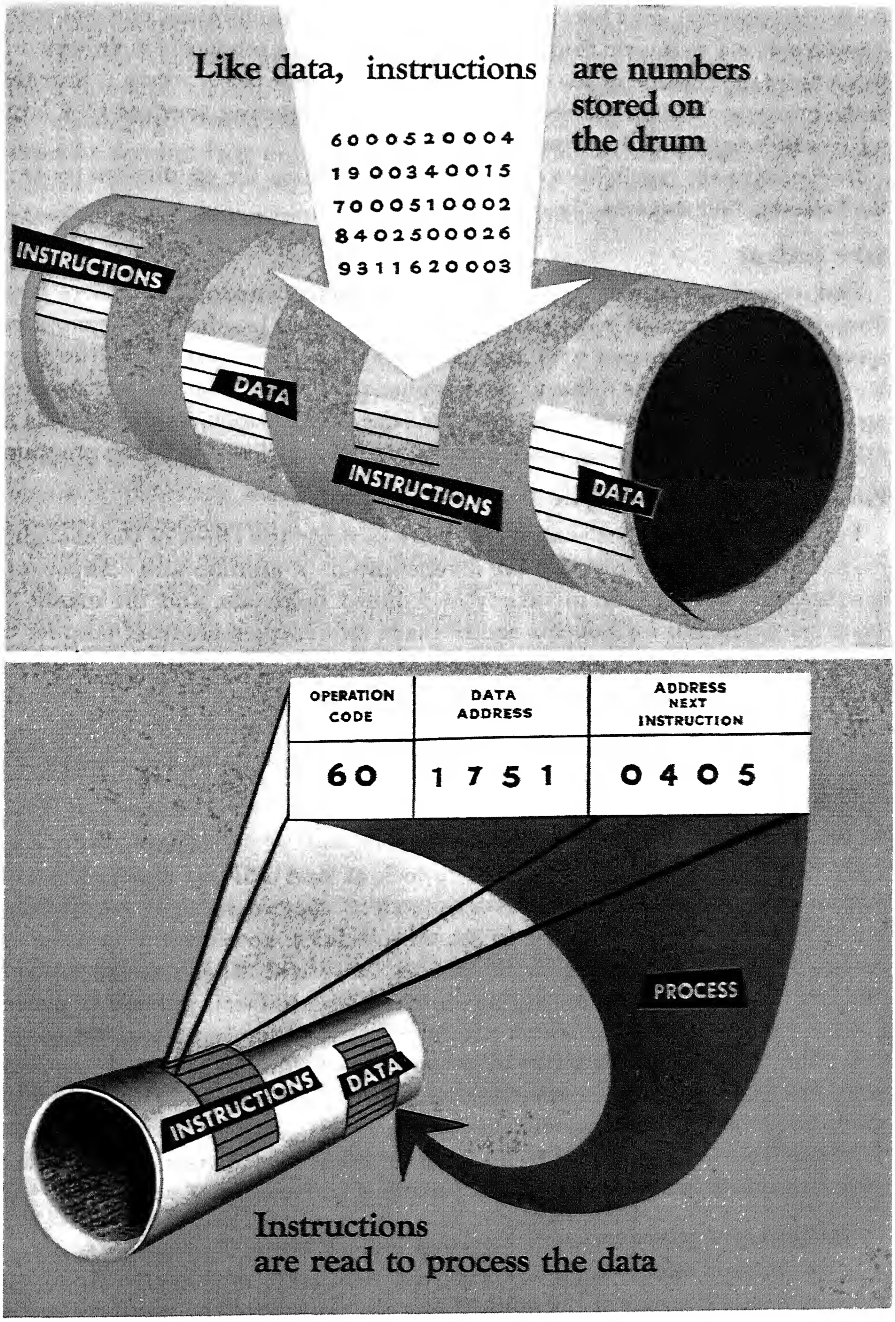

Figure 16–11. ***Recording instructions on a magnetic drum.*** (Reprinted by permission from "IBM Presents the 650 Magnetic Drum Data Processing Machine," International Business Machines Corporation, 1955)

far below capacity), may benefit by cooperating with each other, one computer offering services to several firms simultaneously. Such a scheme is in no way different from the position of the computer in a large enterprise, where it provides central storage facilities for recording data and computing services to a large number of independent departments.

In exploring the possibilities of computer installation for production control, the following four steps may be considered.

Sales analysis

Past records of the firm will show to what extent customers' orders have diversified in character, what percentage of the work has been to customers' specifications as compared with standard products, and what quantities have been associated with the different models over the years. From market research and sales forecasts we have to express our sales expectations, analyze trends in existing models, and consider the possible effects of a variety reduction program.

Product and materials analysis

Following from the sales analysis, we have to study the effect of the changing characteristics of past orders on the diversification of product and components specifications, the increase in variety of required materials, and the extent to which materials and components can be made interchangeable from product to product and model to model. These effects should then be compared with expected trends in the future, based on our sales forecasts, in order to conclude whether any trends of past events may be reasonably expected to proceed and also decide whether it would be advantageous to arrest them and reformulate fundamental managerial policies.

Schedule analysis

Another interesting study would be to look at past delivery and production schedules in order to determine to what extent they increase in complexity, what time and effort is required to set up schedules, how often they need be changed because of changing circumstances, how tight schedules can be constructed, what freedom for modifying centralized schedules is granted to manufacturing departments, and where responsibility for complying with schedules rests. The purpose of this study is merely to ascertain whether the present procedure is adequate to handle the complexity of the situation. Presumably, the system was outlined several years ago, and the questions are: Does it still work as efficiently as conceived by its designer? What unofficial modifications had been introduced because of change of personnel or pressure of work?

Designing a new procedure

If the present system is far from being satisfactory, we have to modify or redesign it, and it is only at this stage that an assessment of the possible benefits of a computer-controlled system can be made. "The successful installation of an

electronic computer depends fundamentally on one simple thing: that the system of which the computer forms a part is the finest that can be devised."[4] The notion that a computer should be employed on existing systems is therefore completely erroneous, since such installations are not likely to yield the best possible economic results. It is only through a critical analysis and redesign of existing methods, with unflinching meticulous passion for detail, that the computer can successfully become an integrated part of the system.

Programing

The operation of a computer essentially consists of basic elements, each involving one action. This action may be computational (such as addition) or making a decision of the yes–no type (in comparing data with predetermined criteria). These basic actions are combined when the computer follows any sequence of operations, such as: searching for specific data, reading, comparing, sorting or arranging, checking, duplicating or listing, counting, and performing a series of calculations.

In programing the operation of a computer, it is therefore necessary to break down complicated processes to basic elements, arrange the elements in the proper sequence in order to define clearly to the computer at what stages the operations are required, and finally to determine rules which will guide the computer in making its yes–no decisions in the course of sorting data.

Hence the first step is the *operational flow chart*, which (like the ordinary flow charts described in Chapter 9) is designed to present an over-all graphical picture of the sequence of operations that constitute the control procedure. This is where the critical analysis of an existing procedure lies, and this is where the new procedure begins to take shape. The flow chart is the basis on which the detailed programs for the computer will be worked out, and it is therefore essential to bear in mind possible implications of the circuits in the chart while it is being designed.

In outlining an operation flow chart, the basic framework shown in Figs. 16–1, 16–2, and 16–12 is naturally adherred to. This procedure is somewhat analogous to the task of a designer of a complicated assembly: The detailing of components and subassemblies can be undertaken after the master assembly drawing has been finished, but when designing the main assembly, it is necessary to bear in mind the possible effect on the geometry of the components, their relation to each other, the possible methods of making these components, and the question of accessibility for assembly and maintenance.

The flow chart leads to the preparation of program sheets, in which the operations of the computer are broken down into elements and translated into a numerical coded form. This is a specialist's job, requiring meticulous detail and understanding of the computer's anatomy and capabilities. Programing is a

[4] *Ibid.*

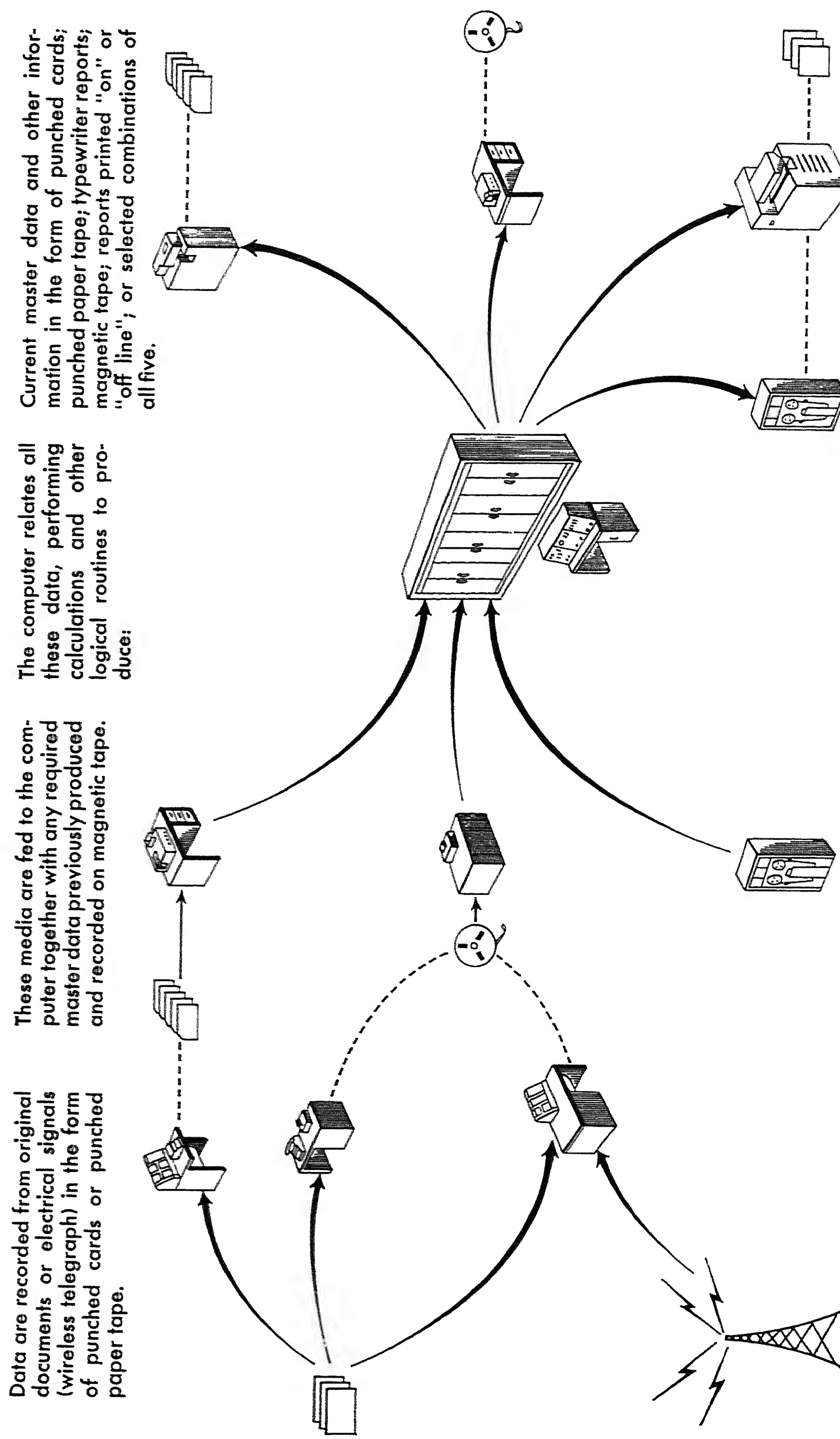

Figure 16–12. ***Data processing with a computer.*** (Contributed by W. J. Kease)

time-consuming job and sometimes it has been found economical to install machines to work out the details of these programs in a computerized language. These are called "compilers" and they are constructed in such a manner as to convert problems presented to them, in a form of comparatively short statements, into programs for the computer.

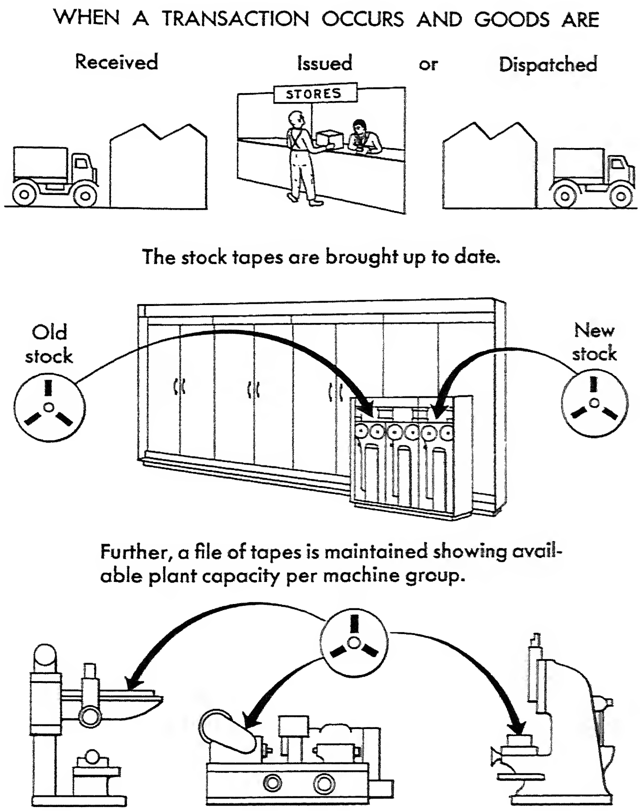

Figure 16–13. ***Bringing stock records up to date.*** (Contributed by W. J. Kease)

Examples of Application

Many thousands of computers are now used in industry for production and inventory control, and the number of computers on order virtually exceeds the number already in use. Numerous case studies of computer applications have been recorded in literature, providing useful details and hints for anybody who contemplates the introduction of a computer to a production control system. The following examples are very briefly described for the purpose of illustration

Stock control

Figure 16–13 shows schematically what happens to stock record cards when materials are received or issued. The flow charts for stock control in Fig. 16–14 illustrate how the computer analyzes the input by sorting the information,

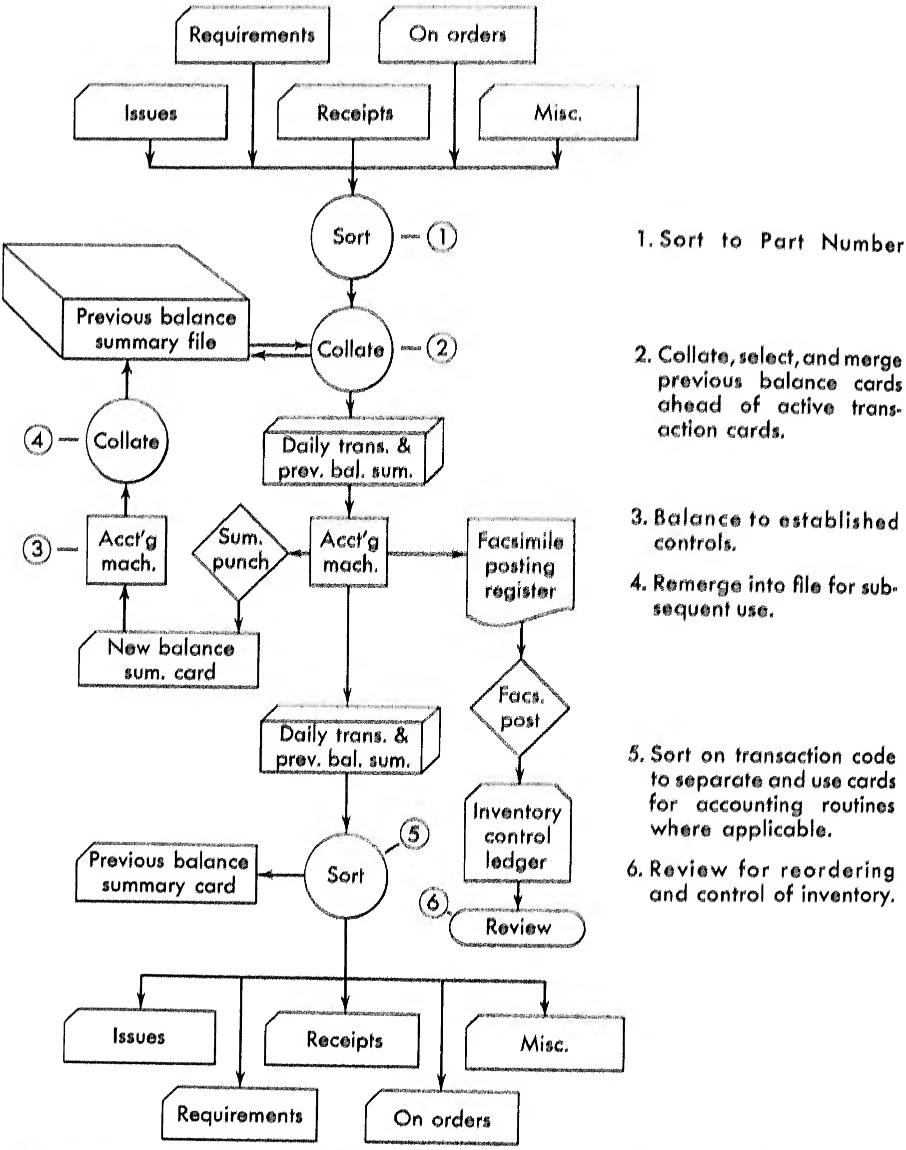

Figure 16–14. ***Inventory control flow chart.*** (Reprinted by permission from "IBM Accounting, Manufacturing Control," International Business Machines Corporation, 1955)

merging data, calculating, and printing the new balance figures. The computer is a very useful tool for inventory planning through studies of demand distributions, queues of orders, effectiveness of delivery dates, etc. Several inventory policies can be evaluated by simulating history for each suggested policy, so that within a fairly short period of time, it is possible to compare the effectiveness of the various policies and select the most promising one.

Machine loading

For this planning function to be effective, it is necessary to have an up-to-date record of available machine capacity. Figure 16–15 illustrates schematically how this information is used by the computer to produce the final job cards.

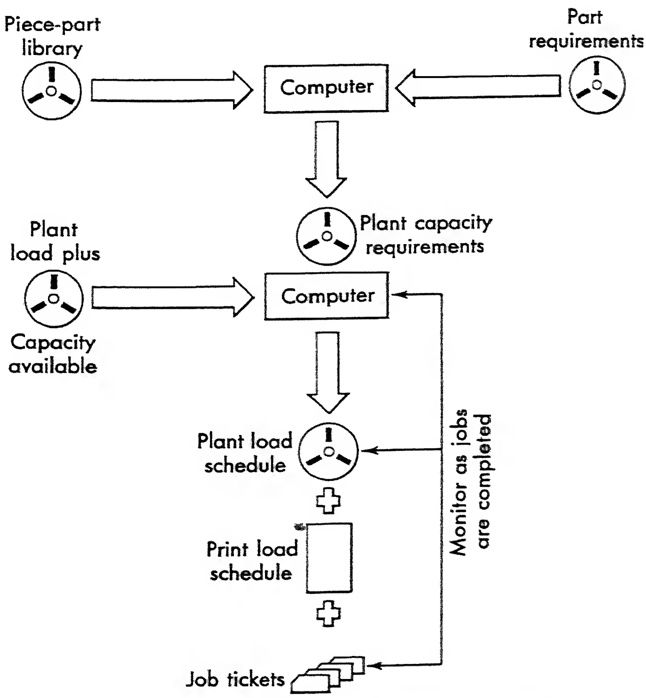

Figure 16–15. ***Outline of machine loading.*** (Contributed by W. J. Kease)

An interesting application of machine loading was reported in the textile industry.[5] Different looms weave different types of cloth. There are versatile machines that are capable of weaving both fine and coarse cloth, and while they are naturally best employed on the highest quality material, it is more economical to use them on producing coarse material rather than having them remain idle.

[5] Ellis, P. V.: Electronic computers and the production engineer (J. Institution of Production Engineers, February, 1959).

In allocating the jobs to available machines, the first step is to classify the capacity into types by reference to the number of shuttle movements of the machines. The loading of orders is also expressed by the number of shuttle movements required to perform a job, so that available capacity is readily obtained by subtracting the loaded orders from given machine capacities. Cards are punched for available capacities of each loom group and for each type of cloth, giving details about the cloth and quantities and delivery dates. The cards are then sorted so that each group card is immediately followed by the stock cards for each of the cloths woven on that group; in allocating the jobs, the computer first selects those with the highest profit margins. Finally the computer prints

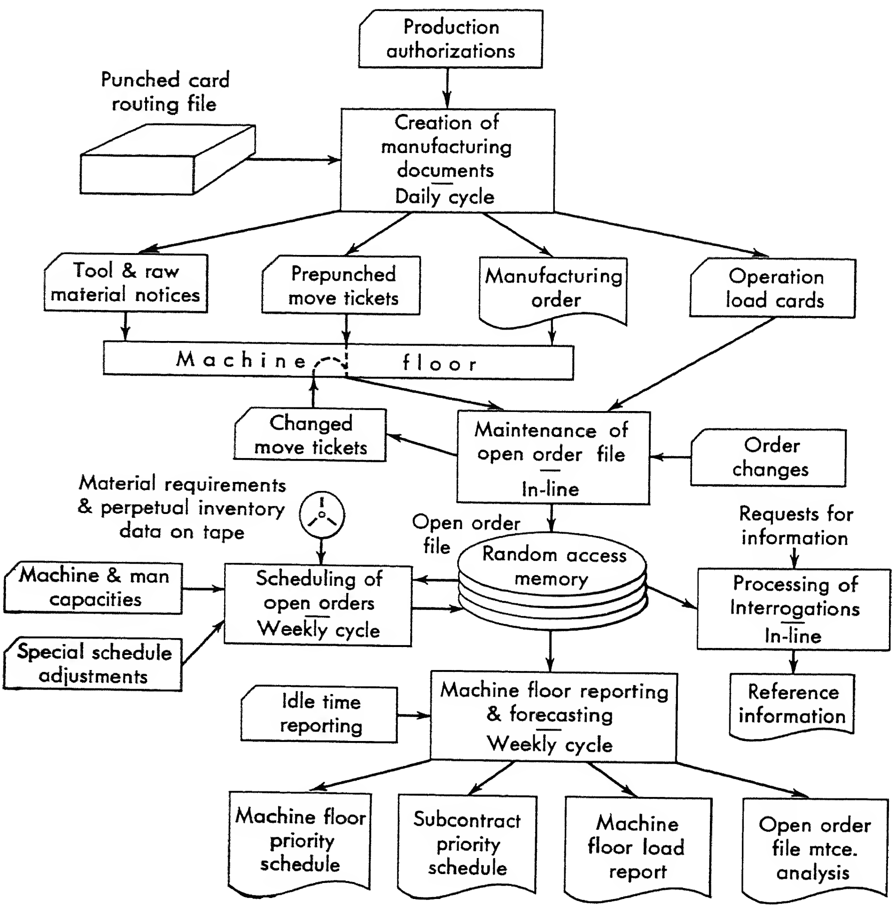

Figure 16–16. ***General flow of information in job production control.*** (From W. R. Elmerdorf, "A Proposed Data-Processing System for Control of a Job-Shop Machine Floor," ASME, paper 58-PROD-3)

lists of exhausted machine capacity for any given production period so that managerial decision can be made about expansion of capacities, subcontracting, etc.

Control of job production

In large job shops products may differ widely in design, types of materials and machines required, skills, and inspection procedures. In many cases scheduling tends to be very inaccurate, since estimating operation times in job production can be based only on limited past experience and is therefore prone to errors. For this reason control must be made as effective as possible so that schedules can be continuously amended and machine loading reviewed with the aim of reducing idle time. A flow chart for flow information in a job shop is shown in Fig. 16–16.

Summary

The electronic computer is capable of handling and manipulating data quickly and reliably, and therefore it can be of invaluable assistance in production control when scheduling and inventory problems are highly dynamic in character. The information is fed into the computer in coded form on a suitable medium (punched cards, punched tapes, or magnetic tapes), and the computer carries out a sequence of predetermined simple operations such as addition or classification of data by comparison with set criteria. This sequence is depicted by program sheets based on a flow operation chart that precisely defines the functions of the computer in the control procedure. The computer is a management tool, very efficient but quite unintelligent; it can be usefully integrated in a control system but it cannot replace it.

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17

INVENTORY CONTROL

Inventory or stock control is an integral part of inventory management, which in turn consists of several functions:

Effective running of the stores: including problems of layout, utilization of storage space, storing media (shelves, bins, etc.), receiving, and issuing procedures.

Technological responsibility for the state of merchandise: methods of storing, maintenance procedures, studies of deterioration and obsolescence.

Stock control systems: stock taking, stock level records, ordering policies, and procedures for stock replenishment. The purpose of stock control is to ensure that while stock levels are not excessive, demand can almost constantly be met and the number of times the store runs out of stock is limited within specified and acceptable bounds.

Of these three functions, stock control deserves special attention here in connection with production planning and control. But before we proceed with describing some stock control systems, perhaps we should ask: What is the purpose of holding stocks in the first place? There are six major reasons:

1. *To create a buffer between input and output*, so that the outgoing flow can be as little dependent on the input characteristics as possible: Consumption of materials either may be continuous or may consist of a series of instantaneous withdrawals of batches from the stores, depending on the type of manufacture, on the process, and sometimes on the stock control system in operation. In chemical and allied industries, where flow of materials is an integral part of the process, supply from the stores is often continuous, while in firms manufacturing engineering products and consumer goods, the assembly line is often fed by small batches of parts, designed to last for a preplanned production period. The supply of materials or components to the stores is, however, almost invariably carried out in batches, so that the stock level increases instantaneously by periodical orders for replenishment (as in Fig. 6–21). The basic difference between the flow characteristics into and from the store compels the firm to hold inventories, which are designed to meet the needs of the production departments until new stock arrives.

2. *To ensure against delays in deliveries:* A batch of incoming materials is intended to last a certain period of time. When an order for fresh stock is made, supply is normally not immediately available, but some time elapses before it arrives. This replenishment period (or "lead time," as it is often called) between the time of placing the order and the time of stock arrival is often subject to some variations, since it depends very much on how heavy the production or delivery schedule is at the vendor's end, on the queue for orders that he has to meet, and on the priority system operating in his organization. A manufacturing enterprise must therefore hold some reserve stocks to allow production operations to continue, if delay in procurement occurs.

3. *To allow for a possible increase in output* if so required: Changes in the manufacturing program may occur because of variations in market requirements, seasonal or stochastic. While production rate cannot always be rigidly geared to demand for the product in the market (and this, incidentally, is why stocks of finished products must also be held), this rate may have to be stepped up or reduced from time to time, and if increased production should be allowed to proceed without interference, reserve stocks of materials must be held (see also Chapter 13).

4. *To take advantage of quantity discounts:* Materials and components may be cheaper when purchased in larger quantities, owing to larger discounts and lower transportation costs. Furthermore, paper work and inspection of incoming goods are often simplified when larger quantities are ordered. On the other hand, capital is tied up in dormant goods, more store space and more handling and maintenance of the goods in the store are required, and greater losses due to deterioration and obsolescence are normally expected.

5. *To ensure against scarcity of materials in the market:* Sometimes there are wild fluctuations in the output of certain materials and in the demand for them, so that materials may become scarce and difficult to get. A reserve stock held by the firm will ensure that production operations are not affected by sporadic scarcities in the market. During periods of rapid economic expansion, or during periods of emergency, the availability or scarcity of materials would very often become a prime mover in stock control policy.

6. *To utilize to advantage price fluctuations:* Price fluctuations of materials may have a marked effect on the procurement policy of a company. If these fluctuations are to be used to some advantage, materials have to be purchased in adequate quantities when prices are low. Industries depending on raw materials, especially basic raw materials, have to pay a great deal of attention to market prices, and some firms feel that they are so dependent on price fluctuations that major policy decisions and the firm's resources become fully geared to the activities of the purchasing department.

The first and basic reason suggested above for holding inventories stems from the fact that withdrawals from the store differ in pattern from the procurement

mechanism. All the other five causes enumerated are of a different character altogether, but they have one thing in common: uncertainty. Because of uncertainties in rates of demand, lead times, price fluctuations and their effects on discounts, reserve stocks must be held in excess of basic inventories. Naturally, the bigger the uncertainties, the higher the reserve or safety stocks that must be carried, and the more costly it is for the firm in question. This is why uncertainties have to be studied and the various possible alternative policies analyzed before the best one can be specified. Needless to say, such an analysis must try to bring in all the relevant parameters in order to obtain as complete and unbiased a picture as possible.

Another point that is often overlooked in connection with the quantitative effect of uncertainties is that the system is usually dynamic. Changes in the market and in the economy as a whole are likely to alter the balance between the various parameters, to increase the effect of some and almost eliminate others; in short, to create a virtually new situation that calls for a new inventory policy. Far too often we find in industry stilted inventory control procedures and replenishment policies that were perhaps suitable to prevailing conditions in the remote past but which have since lost all signs of validity or relevance. Particularly when the stores have to deal with a large number of items, people are usually very reluctant to introduce changes in inventory control systems or treat some items on a different footing than others. It is far easier to maintain a smooth system with as few irregularities as possible, a system whose very backbone is routine, consisting of repeatable actions common to all items, than to keep on changing procedures and even policies.

But inventory control cannot be immunized from the effects of forces acting on the system, and an ultraconservative approach to inventory control often leads to excessive stocks held by the company or sometimes to frequent run-outs of stocks for certain items. Inventory management must therefore constantly evaluate the situation in order to be able to decide when a changeover in policy becomes desirable.

Effect of Demand on Inventories

To illustrate the dynamics of inventory management, let us examine the effects of demand fluctuations on the desirable level of stocks. Suppose a firm is manufacturing a product in four stages:

Making components from raw materials
Making minor assemblies from components
Making major assemblies from components and minor assemblies
Assemblying the final product

Clearly, the stores must hold stocks of items at all stages, as well as stocks of raw materials. If the demand for the final product is, say, 100 units per period, the output at each stage should also be 100 units per period, and raw materials should likewise be ordered for 100 units per period. A reserve or safety stock

should also be held at all stages. Let us consider two policies, one which calls for a reserve stock of 20 per cent and the other for 50 per cent of the output per period. The desirable stock levels according to these policies are shown in Table 17–1.

Table 17–1

Stage	*Demand per Period*	*Reserve Stock*		*Total Stock Available at End of Period*	
		Policy I	*Policy II*	*Policy I*	*Policy II*
0. Finished products	100	20	50	120	150
1. Major assemblies	100	20	50	120	150
2. Minor assemblies	100	20	50	120	150
3. Components	100	20	50	120	150
4. Materials	100	20	50	120	150

If demand for the final product suddenly increases by 20 per cent, i.e., demand at stage 0 is 120, the reserve stock by policy II should now be 60 units. At stage 1 we have to produce 120 units for store 0 and 10 more units to increase the reserve stock at stage 0; hence 130 units are needed, which involve a reserve stock of 65. At stage 2 we now need 130 to supply stage 1, plus the extra 15 units to build the reserve stock of stage 1, and so on. The new figures are shown in Table 17–2.

Table 17–2

Effect of a Demand Increase by 20 Per Cent

Stage	*Policy I*			*Policy II*		
	Demand	*Reserve*	*Total Required at End of 1st Period*	*Demand*	*Reserve*	*Total Required at End of 1st Period*
0	120	24	144	120	60	180
1	124	25	149	130	65	195
2	129	26	155	145	73	218
3	135	27	162	168	84	252
4	142	28	170	202	101	303

A boost of only 20 per cent in the demand for the final product causes a 100 per cent increase in the demand for raw materials in the second policy, but only by 40 per cent when the first policy is adopted. What happens when there is a sudden decline of 20 per cent in demand? Again, we see from Table 17–3 that the effect of a change in demand is augmented as we proceed down the various stages, but that effect in policy II is far larger than in policy I.

Table 17–3

Effect of a Demand Decline by 20 Per Cent

Stage	*Policy I*			*Policy II*		
	Demand	*Reserve*	*Total Required at End of 1st Period*	*Demand*	*Reserve*	*Total Required at End of 1st Period*
0	80	16	96	80	40	120
1	76	15	91	70	35	105
2	71	14	85	55	28	83
3	65	13	78	33	17	50
4	58	12	70	0	0	0

These effects may be expressed in a more general form as follows: If the output per period is Q and if the policy for reserve stocks specifies that a proportion r should be held, then rQ is the safety stock for finished products at the final stores. If there is a sudden impulse x in demand of the product, so that $Q(1 + x)$ are now needed, the reserve stock should now be $rQ(1 + x)$. This would lead to the following results,

Stage 1. Output required:

$$Q(1 + x) \text{ for the final stage}$$
$$+ [rQ(1 + x) - rQ] \text{ for the safety stock at the final stage}$$
$$= Q[1 + x(1 + r)]$$

Safety stock required:

$$rQ[1 + x(1 + r)]$$

Stage 2. Output required:

$$\underset{\text{(for output at stage 1)}}{Q[1 + x(1 + r)]} + \underset{\text{(for safety stock at stage 1)}}{\{rQ[1 + x(1 + r)] - rQ\}}$$
$$= Q[1 + x(1 + r)^2]$$

Safety stock required:

$$rQ[1 + x(1 + r)^2]$$

Stage 3. Output required:

$$Q[1 + x(1 + r)^2] + rQ[1 + x(1 + r)^2] - rQ = Q[1 + x(1 + r)^3]$$

Safety stock required:

$$rQ[1 + x(1 + r)^3]$$

and so on. It can be shown that the output Q_n required at the nth stage would be given by

$$\frac{Q_n}{Q} = 1 + x(1 + r)^n \tag{17-1}$$

from which it is evident that Q_n will increase with the policy r, the impulse x, and above all, with the number of steps n. However, x and, to all intents and purposes, n are dictated by outside circumstances, whereas r is an expression of our inventory policy. Obviously, the smaller the reserve stocks, the smaller the effects of fluctuations caused by this chain reaction along the production line. Had we been able to dispose of reserve stocks altogether (i.e., $r = 0$), there would be no amplification of the impulse through the stages, as then

$$\frac{Q_n}{Q} = 1 + x$$

Equation 17–1 may sometimes give us some useful indications as to how liberal our policy may be allowed to become:

$$r = \sqrt[n]{\frac{1}{x}\left(\frac{Q_n}{Q} - 1\right)} - 1$$

Example

If fluctuations in demand of 10 per cent may be expected, and if it is undesirable for Q_n/Q to exceed 1.2, what policy r may be allowed when the line consists of four production stages?

Solution

Substituting $n = 4$, $x = 0.1$, $Q_n/Q - 1 = 0.2$, we have

$$r = \sqrt[4]{\frac{0.2}{0.1}} - 1 = 0.19 \simeq 0.20$$

i.e., reserve stocks should not be more than 20 per cent of the output volumes.

The figures given in Tables 17–2 and 17–3 indicate the immediate effects of a demand impulse. If in the subsequent period the demand for finished products remains at its new level, the output at stage 1 may be matched with the level required at stage 0, and outputs at other stages should also be adjusted so that the amplitude of the wave that has been generated is considerably reduced with each period, until eventually in the fifth period (Table 17–4) the amount of materials required (stage 4) is the same as the new demand figure. As the orders for output at the various stages are successively adjusted, the safety stock requirements must be adjusted accordingly, and this secondary effect is also generated across the line, before final equilibrium is attained. We have not pursued here the effects of discrepancies between demand forecasts and actual demand, as these were discussed in Chapter 6, but evidently they will contribute to fluctuations along the line.

Table 17–4

Effect of a Demand Impulse on Requirements in Successive Periods

Stage	*Before Demand Impulse*		*Period* 1		2		3		4		5	
	a	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*
0	100	50	120	60	120	60	120	60	120	60	120	60
1	100	50	130	65	120	60	120	60	120	60	120	60
2	100	50	145	73	130	65	120	60	120	60	120	60
3	100	50	168	84	145	73	130	65	120	60	120	60
4	100	50	202	101	168	84	145	73	130	65	120	60

Col. a = demand; col. b = reserve.

The somewhat simplified model given in Table 17–4 illustrates how amplitudes of fluctuations at the market end of the line are amplified as the waves are propagated across the production line, but the model does not account for the fact that if the production rate is suddenly increased, reserve stocks of materials, components and assemblies have to be used until they can be replenished (this, after all, is one of the main reasons for holding safety stocks). Thus, in increasing our production rate, we must supply not only the next stage up the line and the need to raise the level of reserve stock in that stage, but we must also replenish the amount taken from the safety stock when line output is not adequate. This means that the effect at the other end of the line will be even more pronounced than suggested by the accompanying tables, depending on the lead times required for ordering materials and for initiating changes in the production rate.

Take, for example, a case with three stages, as described in Table 17–5. During period 1 the demand for finished products is 100, and the output at each stage is matched to this figure, while safety stocks are 50 units at each stage. During period 2 the demand rises to 110 units, but output of finished products is only 100, so that 10 units from the safety stock are withdrawn, leaving it with 40 units. Suppose this increased demand is expected to continue during period 3; the output of stage 0 should be

$$110 \text{ (to supply the line)} + 0.5 \times 110 \text{ (new safety level)} - 40 \text{ (present level)} = 125$$

Table 17–5

Effect of Demand Impulse When Policy Requires Safety Stock to be 50% of Momentary Output for the Line

Stage	*Period Number* 1 *a*	1 *b*	2 *a*	2 *b*	3 *a*	3 *b*	*End of 3* *a*	*End of 3* *b*
Market demand	100		110		110		110	
0. Final product	100	50	100	40	125	40	110	55
1. Assemblies	100	50	100	50	150 (163)	25	110	65
2. Components	100	50	100	50	150 (258)	* (−13)	110	40
3. Materials	100	50	100	50	495→	* (−108)	110	385

Column *a*: actual outputs for the period. In brackets: required, but unattainable output levels.

Column *b*: safety stocks.

*: Safety stock run-out.

Arrows show flow of materials, components, etc.

Table 17–6

Effect of Demand Impulse When Policy Requires Safety Stock to be 50% of Eventual Output for the Line

Stage	*Period Number* 1 *a*	1 *b*	2 *a*	2 *b*	3 *a*	3 *b*	*End of 3* *a*	*End of 3* *b*	4 *a*	4 *b*	5 *a*	5 *b*
Market demand	100		110		110		110		110		110	
0. Final product	100	50	100	40	125	40	110	55	110	55	110	55
1. Assemblies	100	50	100	50	140	25	110	55	110	55	110	55
2. Components	100	50	100	50	150 (155)	10	110	50	115	50	110	55
3. Materials	100	50	100	50	170	* (−5)	110	55	115	55	110	55

Column *a*: actual outputs for the period. In brackets: required, but unattainable output levels.

Column *b*: safety stocks.

*: Safety stock run-out.

Arrows show flow of materials, components, etc.

But in order to produce 125 units, we must have 125 assemblies, and these we can get as follows: 100 from output of stage 1 + 25 from the safety stock of stage 1, leaving now only 25 in stock. The output required now for assemblies is:

$$125 \text{ (for the line)} + 0.5 \times 125 \text{ (safety)} - 25 \text{ (present level)} = 163.$$

But with the policy for safety stock adopted so far, it is impossible to produce 163 units at stage 1, the maximum possible output being 150 (100 from stage 2 + 50 from the safety stock), so that an outstanding demand for 13 units from the safety stock at stage 2 cannot be met. Similarly, the output required at stage 2 is

$$163 + 0.5 \times 163 - (-13) = 258$$

Again, only 150 can be produced and a deficiency of 108 units in the safety stock at stage 3 is recorded, and so on. For safety we have specified here stocks being 50 per cent of the portion of the output that is required for replenishment of the safety stock of that stage. Even so, it is remarkable how a 10 per cent impulse in demand of finished products can (one might say "artifically") create a demand for 495 units for materials after only three stages.

Restrictions on output increase

There are two main restrictions on increasing the output at various stages: The first, as already indicated above, is caused by the level of safety stocks. In the example given in Table 17–5, this would limit the output to 150 at each stage during period 3. The second restriction lies in the limited capacity of the machines, in the limited available labor, and in the limited flexibility of the

technological and administrative arrangements that can be made at a short notice.

Let us assume for a moment that in our example, the only limitation to increasing the output is the amount of materials available at the input to each stage; in other words, the amount that can be drawn from the safety stock. Orders for the amounts shown in parentheses in Table 17–5 have been issued with the full knowledge that they cannot be fulfilled, and this naturally introduces an atmosphere of strain and uncertainty into the shop. But what does actually happen in period 3?

Stages 1, 2, and 3 produce 150 units (the maximum possible amount), and at the end of the period 110 units are transferred along the line. The balance goes into the safety stocks and the picture is not really so bleak as we have been led to believe at the beginning of the period. In fact we have now 65 units as safety stock at stage 1, whereas only 55 are required. This means that although the quota of 163 was not met, the production order for the next period will have to specify a quantity smaller than 110, while stage 0 (where the quota was met) will have to produce 110! This is a typical state of affairs when safety stock policy:

(i) Specifies a certain percentage of the momentary stage output, rather than of the ultimate stage output;
(ii) Tries to restore immediately the safety stocks to their desirable level.

Too often, unfortunately, these features are characteristic of decentralized production control, where each stage is responsible for its own output and reserve stocks, and where decisions are taken on the basis of output figures at the next stage in the line, rather than on the end requirements. One can only sympathize with the frustrated stock controller of materials at stage 3, who is faced with unruly fluctuations in requirements. At the beginning of period 3 he puts in an order for 495 units, 110 of which are sent along the line and 385 left as reserve. The safety stock has, therefore, 50 at the beginning of the period, all of which are withdrawn. A further demand for 108 cannot be met, and then the stock level surges to 385—and all this in one period! Furthermore, after increasing fivefold the order for materials from period 2 to 3, there will now pass several periods in which no orders for fresh materials will be made.

If, however, outputs and safety stocks are computed with the *eventual* required output in mind, the amount ordered at stage 0 would still be 125, but at stage 1:

$$110 \text{ (for stage 0)} + 55 \text{ (safety)} - 25 \text{ (present safety level)} = 140$$

Similarly, at stage 2, 155 would be needed, 170 at stage 3, etc., the increase being 15 units per stage (10 for satisfaction of the demand, 5 for putting up the safety stock), as shown in Table 17–6. The safety stock is now depleted at stage 3, so that stage 2 can produce 150 instead of 155 units. This is not so serious, as shown in the next column, which gives the position at the end of period 3. Apart from the safety stock at stage 3 (which is only 5 units too low), all is well. In period 4,

therefore, we have to produce 115 at stage 3 and order 115 for materials at stage 4, and in this way we succeed in smoothing out the effects of the demand impulse by the end of period 4.

Fluctuations in distribution channels

Distribution channels in marketing present a similar background, each channel being analogous to one of the stages in the line referred to above, and changes in demand at the retailers' end may cause serious fluctuations in orders from the plant. Very often, however, the plant cannot immediately switch over to new outputs in every period, and the change is usually gradual and within certain limits (e.g., production may be adjusted from period to period by ± 20 per cent but cannot be increased fivefold as in the example in Table 17–5), and this often helps to damp these fluctuations.

Two interesting examples of the effect of a demand impulse by 10 per cent at the retail end are shown in Figs. 17–1 and 17–2. The processing of the orders, including mailing, receiving, and accounting, may often involve a lag time of about one month from retailers' to distributors' orders. While the retailer in this case orders 10 per cent more, the distributor has to order 15 per cent more (Fig. 17–1), and by the time the effect is felt at the factory warehouse, the order is increased by 28 per cent. The inventory level at the factory warehouse first falls by 13 per cent in order to comply with the orders. The factory output, now delayed by six weeks, reaches a peak of $+40$ per cent above the initial level, while the increase in demand is still only 10 per cent up. Figure 17–2 shows a similar state of affairs when the demand fluctuates seasonally by ± 10 per cent. The effects on the stock-level fluctuations at the factory warehouse and on the factory production output are remarkable.

To sum up, combinations of uncertainties in market demand and replenishment times seem to have a marked effect on the requirements at the source, and for this reason inventories of goods at the beginning of a distribution pipe line, or inventories of raw materials, tend to be rather high. Indeed, the Department of Commerce has estimated that more than 50 per cent of the total inventories in the United States are held by manufacturers, and this represents a colossal sum of money. The study of inventory levels and the characteristics of chain reactions of the type mentioned above seems, therefore, to be well worth while.

Stock Control Systems

Briefly, stock control systems constitute the framework of laid down procedures, according to which quantity control is exercised. This control follows the familiar pattern of any control system, namely:

- Find out how much we have (stock records, stock taking).
- Compare with how much we should have (as specified after study of stock-level fluctuations, rates of demand, etc.).
- Take steps to close the gap between the two (stock replenishment, re-evaluation of the desirable average and safety stock levels).

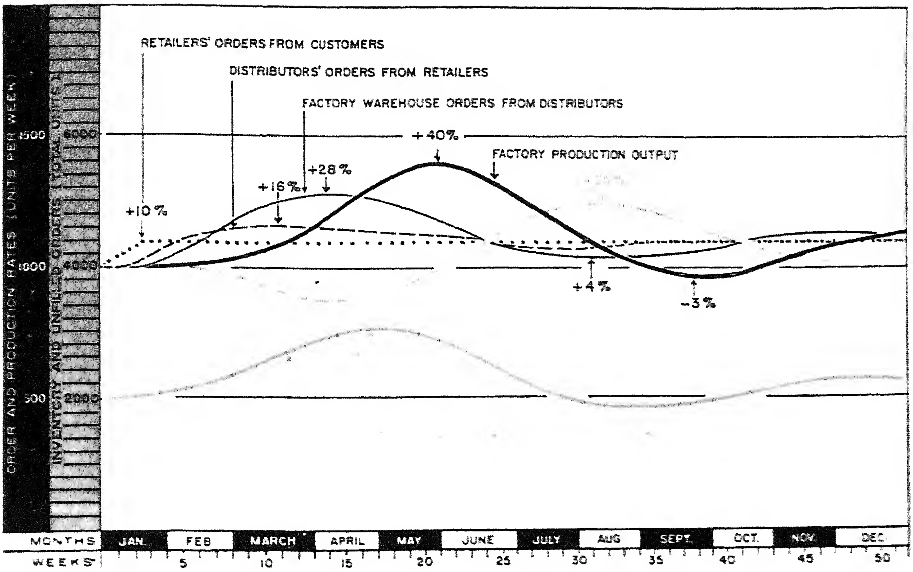

Figure 17–1. ***Response of production-distribution system to a sudden 10% increase in retail sales.*** (From "Industrial Dynamics, A Major Breakthrough for Decision Makers," by Jay W. Forrester, Harvard Business Review, July-August 1958, reproduced courtesy Harvard Business Review)

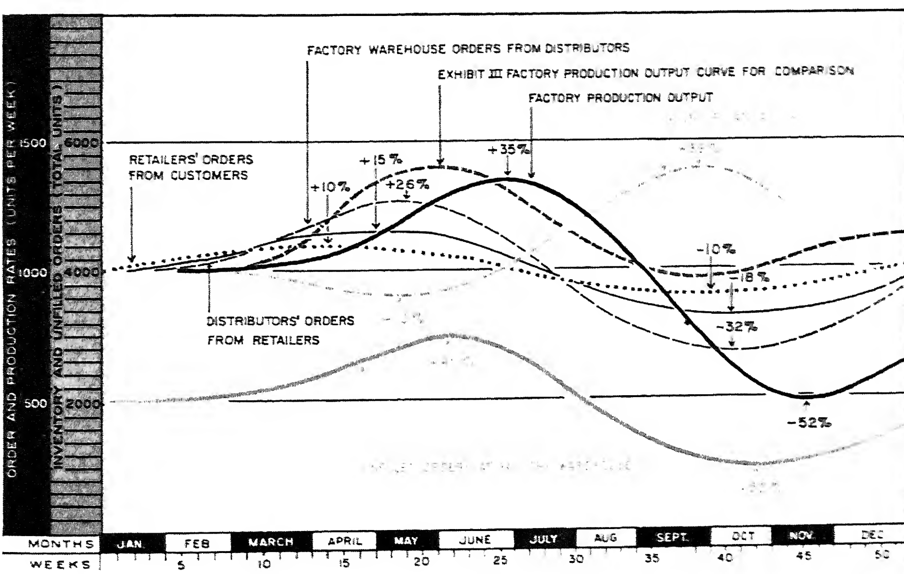

Figure 17–2. ***Response of production-distribution system to a sudden 10% annual rise and fall in retail sales.*** (From "Industrial Dynamics, A Major Breakthrough for Decision Makers," by Jay W. Forrester, Harvard Business Review, July-August 1958, reproduced courtesy Harvard Business Review)

The study of the discrepancies that occur between the planned and the actual stocks is essential if the control system is to be effective. Indeed, the effectiveness of any system or procedure is determined by its success or failure to realize the aims for which it has been adopted in the first place. If the nature of these discrepancies is known and understood, management can be in a position to decide whether it is possible and worth while to change the system or modify it in order to attain a higher degree of correlation.

We have discussed in the preceding section some of the vertical effects of demand fluctuations on stock levels, "vertical" in the sense that stages follow in a manufacturing sequence and the dependence of stages on each other lies in the feeder flow of materials or components that form the line. There are, however, horizontal effects as well. A store of different components, for instance, may be fed by the same shop in such a way that no two items can be supplied simultaneously. The machines in the shop produce one item at a time and are set to produce another item when the quota or batch of the currently produced one is fulfilled. All the items supplied by this shop are not vertically dependent on each other (they may even be intended for different products) and coexist on the same horizontal footing, so to speak. Now, when the demand for one item changes, the quantity ordered for that item from the shop and thereby the production schedule are affected. This means that delivery for another item may be delayed or put forward, and hence its stock level will assume a pattern of change different from that expected.

Horizontal effects also occur when materials or components are ordered from an outside source of supply. It is sometimes desirable to exploit favorable terms of sale, which involve a common delivery date for different items, and this date may not coincide with the ones specified had each item been ordered separately. It is probably true to say that the larger the store and the larger the number of items held in it, the more prominent these horizontal effects are likely to be.

Ordering procedures

The main features of a stock control system are defined by the ordering procedure, which is designed to answer the following two simple questions: How much to order? When to order? Some of the more common stock control systems will now be described.

The two-bin system (sometimes called the min-max system)

This is perhaps the oldest and the most commonly used system. The stock is divided into two bins: the first one is intended for satisfying current demand, the second for satisfying demand during the replenishment period. Thus the second bin comes into use only after the first bin has been completely depleted, and when that happens, an order for stock replenishment is put in. When the ordered batch arrives, the level of the second bin is restored to its original high value and the balance is put in the first bin, from which current demand is now supplied again.

This division into bins either may take the form of physical division in the store, with the storeskeeper giving the ordering signal as soon as he has to start using the second bin, or the two bins may be segregated only on the stock card, so that in the stores the two quantities are kept together. There is, however, a fundamental difference between the two methods. When the stock is physically divided into two bins, the ordering procedure is "automatic," so to speak. The authority to order replenishment is not dependent on stock taking, stock balance calculations, consumption rate fluctuations, trends, etc. The system is automatic in the sense that it can dispense with all this paper work and rely solely on the stock level to trigger off the order, as soon as this level reaches a predetermined value.

When the stock is divided into two bins on paper only (either on the same stock card or by the use of two stock cards, one for each bin), the order for replenishment must be initiated at the control office, and this necessitates accurate and up-to-date records, hence an effective reporting procedure, involving a fair amount of paper work, minimum time lag for information from the store to be transferred into the records, and periodical stock takings to compare the records with physical stocks. Although valuable information is gained by this method on the current stock level and demand characteristics, the system loses some of its "automaticity."

The main advantage of the two-bin system lies in its simplicity and reliability. It is comparatively cheap to operate and easy to explain to new stock control personnel. One of its disadvantages, however, is the absence of adequate data on stock levels and consumption rates in the simpler forms of the system. This hampers to a certain extent our ability to re-evaluate batch sizes for orders, which become particularly difficult to control in the case of slow-moving stock. Also, when different items have to be ordered from the same source, in order to reduce transportation costs or to take advantage of credit or discount terms, it is necessary to order several items simultaneously, even when only one reaches the reorder point (an example of such a system is given later), while the basic idea of the two-bin system is that products are independent of each other in the replenishment procedure.

True enough, all these shortcomings in the system can be alleviated by the introduction of suitable modifications such as segregation between slow-, medium-, and fast-moving items with a different amount of paper work and consumption rate analysis for each group or setting up groups of goods that have to be ordered simultaneously from the same vendors, with proper cross checks of stock levels. Most of these modifications work very well in practice, but more often than not they rob the two-bin system of its basic advantage: simplicity of performance and minimum paper work.

The ordering cycle system

Whereas in the two-bin system the reorder mechanism is linked to the stock level of each individual item, the ordering cycle system is based on periodic

reordering of all items. With every cycle the stock of each item is brought up to its level, which is dependent on the length of the cycle, the replenishment period, and the consumption rate. When the replenishment period and demand are not subject to variations, the reorder quantity obviously increases with the cycle time, so that short cycles are required if rapid turnover of stock is desirable. The main advantage of this system over the two-bin system is that all orders for replenishment are issued at the same time. The ordering mechanism is regular and not subject to sporadic arrivals of warning signals from the store, and the personnel in the control office can perhaps better plan its operations and utilize its time more efficiently. Usually, however, more stock is held when this system is adopted than with the two-bin system. Among the variations of the ordering cycle system are the following:

1. *All items one cycle;* i.e., all the items are replenished every cycle. This is a particularly useful method when the number of items is not too large and when the differences in demand are not so marked as to create large contrasts in the rate of stock turnover. In some cases, when the different goods have similar demand characteristics, replenishment times, etc., one reorder quantity may be adopted for all items, with readjustments for individual items made once every few cycles. This method greatly simplifies the ordering procedure.
2. *Multicycles;* i.e., the items are divided into groups and each group has its own ordering cycle, independent of the other groups. The groups are formed either by selecting goods that have to be ordered from the same vendor or by taking items whose demand characteristics are very close. The multicycle system is adopted when the stores have to deal with a large number of items. Whereas with "all items one cycle" the average stock level tends to increase with the number of items, this failing can be remedied by splitting the store into groups and assigning an appropriate cycle time to each.

Combinations of the two-bin and the ordering cycle systems

There are several stock control systems in industry that try to combine certain features of the two-bin and of the cycle systems, in order to enjoy the advantages of both. These combinations vary in character, depending on how much is really borrowed from the two basic systems. Two examples are discussed below.

Example 1

The store is divided into groups and all items of each group are ordered simultaneously. When any member of the group reaches the reorder point (i.e., its second bin comes into use), all items in the group are automatically ordered. The danger of this method is that slow moving items will be ordered again and again, and some measure of control is required to prevent their being reordered when a certain level of stock is attained. This can be done by reviewing the stock cards of all members of the group before reordering. Better still, one can introduce a quick visual check in the stores by using a shelf or a rack with a capacity

equal to the maximum desirable amount of stock. If the shelf is not full, the item may be reordered when the other members of the group are ordered. If there is an overspill, the item is deleted from the group order.

This method constitutes in fact a three-bin system with the following usage and ordering rules: Do not use bin 3 before bin 2 is depleted and do not use bin 2 before bin 1 is depleted. Do not order an item as long as its bin 1 is used. If a reorder signal is given by another item, include the item when bin 2 is in use. As soon as bin 2 is depleted, give the reorder signal for the group (but exclude from the order all items that have bin 1 in use). In this way, excessive accumulation of stocks is avoided, while some of the main features of the basic systems are retained, namely, the principle of the two-bin system by which reordering is triggered off by stock levels, and the simultaneous reordering characteristic of the ordering cycle. It may be said, however, that the method is basically a two-bin system with an ordering cycle modification.

Example 2

The store is divided into groups and each group has its own ordering cycle. In order to ensure, however, that we shall not run out of stock before the time for reorder for the whole group has come, reordering individual items by the two-bin system is introduced. Hence, if the first bin of any member of the group is depleted, an order for this item alone is automatically sent out. This method allows flexibility in determining the length of the reorder cycle. When the cycle is short, most orders are made by the cycle system; when the cycle is made longer, more and more orders are sent out by the two-bin system. Here, too, it is necessary to prevent excessive stock build-up, and this may be achieved in the manner described above. This system is basically an ordering cycle system with a two-bin system modification, but the two-bin bias increases as the cycle length for any specific group increases.

Reorder Quantity

A typical example for variation of stock level with linear demand was shown in Fig. 6–21, and as pointed out in Chapter 10, the optimal batch size for stock replenishment is a special case in the theory for optimal batch sizes for production. Clearly, if we order too often, the cost per item may increase, owing to the cost involved in preparing and issuing a large number of orders. If we keep this number down and order too much each time, the carrying costs due to a high average level of stock would be excessive.

Since the stock increases instantaneously when the batch arrives, the rate of production is not included in the consideration of the optimal batch, or

$$a_p = 0$$

and for minimum costs per unit the batch size is

$$Q_m = \sqrt{\frac{s}{K}} \tag{17–2}$$

where s is the cost incurred in processing an order and K is the carrying costs factor:

$$K = \frac{I + 2B}{2a} \tag{17–3}$$

where a is now the consumption rate, I the interest charges per piece per unit time, and B the average carrying costs in stores per unit time. It should perhaps be noted here that consumption often takes the form of short instantaneous spurts, as shown in the example of Table 17–7 and Fig. 17–3. As long as these spurts are comparatively small in amplitude, an approximately continuous demand pattern may be assumed, and the average rate of demand may be regarded as a for application in formulae.

Table 17–7

Stock Record Card in the Two-Bin System (an example)

Item: ROTA Spindles — *Order Quantity*: 1,200
Spec. No.: 0375B6 — *Reorder Point:* 800
Safety Stock: 300

Date		*In*	*Out*	*Balance*	*On Order*
May	31	1,200		1,200	0
June	7		100	1,100	0
	10		250	850	0
	20		100	750	1,200
July	1		150	600	1,200
	6		225	375	1,200
	10	1,200		1,575	0
	16		320	1,255	0
	26		300	955	0
Aug.	8		55	900	0
	10		350	550	1,200
	15		100	450	1,200
	20		150	300	1,200
Sept.	5		100	200	1,200
	10	1,200		1,400	0
	22		150	1,250	0
	30		250	1,000	0
Oct.	10		325	675	1,200
	18		125	550	1,200

Average demand in the first period, $\frac{825 \text{ units}}{40 \text{ days}} \simeq 21/\text{day}$.

Average demand in the second period $\simeq 23/\text{day}$.

Reorder range

A discussion on criteria selection for optimal batch determination is included in Chapter 10, and those remarks will generally apply here. The term *production range*, which was introduced in Chapter 10, may be substituted by *reorder range*. This range covers those batch sizes for which the total costs per unit will not exceed a predetermined value (conveniently defined by the parameter p),

and its upper and lower limits may be computed as those of the production range (except that now, again, $a_p = 0$). The reorder range is an important concept in the formulation of an effective stock control system, as we shall see later on.

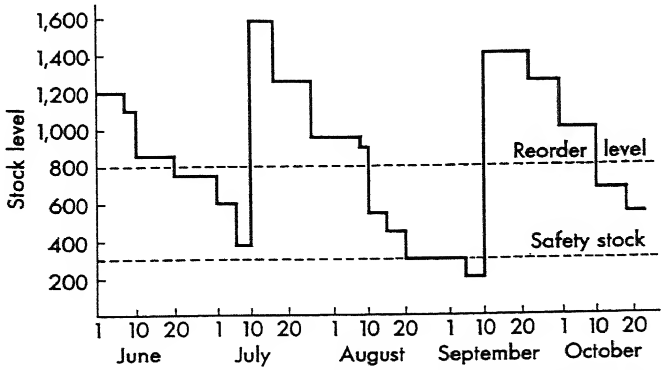

Figure 17–3. ***Random instantaneous withdrawals from stock.*** (*See* Table 7)

It should be noted here that the ratio u, introduced in Chapter 10, has a specific meaning in stock control. Previously u was defined as

$$u = \frac{cQm}{s}$$

where c is the constant cost per unit; hence the numerator denotes the value of the order when the minimum-cost batch size is specified, and u is the ratio of the order value to the cost of placing and dealing with an order. While the value per unit denoted on the order is c, the actual costs incurred per unit are higher, owing to the additional costs of placing orders and carrying stocks:

$$Y = c + \frac{s}{Q} + KQ$$

so that the actual costs per order of Q_m units are

$$\left(YQ\right)_m = cQ_m\left(1 + \frac{2}{u}\right)$$

Some case studies in industry have shown that sometimes a comparatively large percentage of the total orders issued is for small sums of money, so that the ratio of the order value to the cost of issuing the order is comparatively small. The question arises whether it would not be wise to depart from the policy of specifying Q_m for each item, but instead stipulate larger quantities per order so that the number of orders for these particular items would be reduced. The maximum amount that can be allowed is obviously the upper limit of the reorder range,

since by definition it is undesirable to specify batch sizes outside this range; in this connection it is useful to bear in mind the following brief remarks:

1. Items that are comparatively cheap may often enjoy a higher allowance p than expensive items, since larger deviations from the optimum in cheap items may first seem ominous percentage-wise (compared with their cost), but when compared with the stock value of all items, this increase may be negligible. In cases where the cost per item varies widely, it is often worth while to analyze critically the policy laid down for p, and especially when it is a uniform policy to all items, it should be viewed suspiciously.

2. The increase of batch quantities of the cheaper items results in a reduction of the total number of orders placed. This number reduces drastically even further when cheaper items are put on a reorder cycle system, with several items included in one order from one vendor instead of a separate order per item. This is often accompanied by a change in the cost of placing and dealing with an order. The total cost of placing, say N orders per annum, may sometimes be expressed as $k_1 + k_2N$, so that the cost per order is $s = (k_1/N) + k_2$. This implies that by reducing the total number of orders N, the cost per order increases and the ratio u for all the items therefore decreases. We know, however, that as u decreases, the function of costs per unit becomes more sensitive to variations in batch sizes, and if p remains unchanged, the reorder range contracts. We must, then, do the following:

   (i) Recalculate the optimal batch sizes and the reorder ranges.
   (ii) Check whether the reorder quantities selected for the cheap items, which caused the reduction in the total number of orders, still comply with the new reorder ranges.

3. Increase in the ordered batch sizes may result in quantity discounts, hence in reduction of the costs c. This may often offset any reduction in the value of u, caused by reducing the number of annual orders, and the analysis of this effect may proceed on the lines suggested in Chapter 11, where the question of price breaks was discussed.

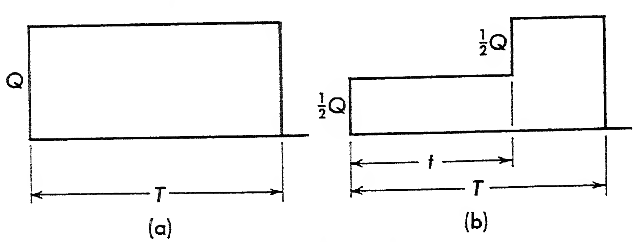

Figure 17-4. ***Comparing the alternatives when Q units are required for an assembly line. (a) One order for Q units T days ahead of time. (b) Two orders for $\frac{1}{2}Q$ units each.***

Effect of splitting an order

There are cases involving instantaneous demand in which the number of orders per period has to be analyzed. Take as an example the supply of parts to an assembly line engaged on batch production. The parts are supplied to the stores, say, at the beginning of a period (Fig. 17–4), and the whole batch is issued to the line at the end of the period. Now, we know that it would be far cheaper to supply the batch to the stores not at the beginning of the period, but as near as possible to the end, and save in this way the carrying costs for the period. But sometimes this is impossible because of scheduling and other commitments. Supposing we are faced with the following problem: Under what circumstances is it worth our while to split the quantity Q if half of it can be supplied at a later date? At present we have one order for quantity Q, involving a cost of

$$y_1 = \underbrace{s}_{\substack{\text{cost of order,}\\ \text{setting up, etc.}}} + \underbrace{QCT}_{\substack{\text{carrying}\\ \text{costs}}}$$

where C is the cost of holding one unit per day. If by the new arrangement the second batch is delivered after t days, the cost would be

$$y_2 = 2s + \tfrac{1}{2}QCT + \tfrac{1}{2}QC(T - t)$$

The splitting of the batch is worth while when

$$y_1 > y_2$$

or $y_1 - y_2 = -s + \frac{1}{2}QCT - \frac{1}{2}QC(T - t) > 0$

or when
$$t > \frac{2s}{QC}$$

If orders for delivery beyond this value of t cannot be entertained, it would obviously not pay to have two orders.

Reorder Procedure

The reorder procedure is evidently dependent on the stock control system in use, and we have seen that even modifications to any system, such as special measures to prevent excessive piling of stock, may have a marked effect on the methods adopted. Let us now examine in some more detail this procedure in the two common stock control systems.

Reorder point in the two-bin system

If the consumption were constant and not subject to any fluctuations, and if the replenishment period were fixed and reliable, the stock could be allowed to dwindle to zero and the new stock should be planned to come to the stores only at that point (Fig. 17–5). If the lead time between the reorder point and

replenishment point is t_0, the stock level at the reorder point is Q_0, and the consumption rate is a; then

$$Q_0 = at_0 \tag{17-4}$$

and Q_0 is the amount of stock in the second bin.

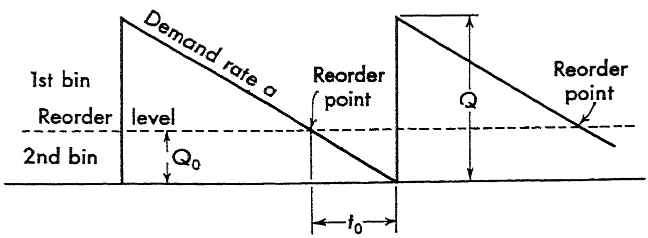

Figure 17–5. ***Reorder point in the two-bin system.***

If, however, the demand forecast is a, with a possible maximum rate of $a(1 + \beta)$, the reorder point may have to be planned in such a manner as to allow the new stock to arrive when the old stock reduces to zero at the maximum consumption rate. From Fig. 17–6 the reorder point is

$$Q_0 = at = a(1 + \beta)t_0$$

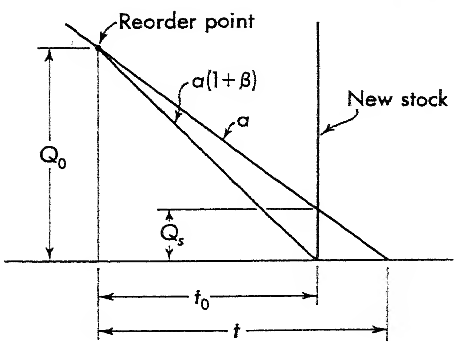

Figure 17–6. ***Placing the reorder point when a higher rate of demand is expected.***

Running out of stock is avoided in this way. If in actual case the demand is only a, the residual stock at the point of replenishment is

$$Q_s = Q_0 \frac{t - t_0}{t} = Q_0 \left(1 - \frac{t_0}{t}\right)$$

or

$$Q_s = \frac{\beta}{1 + \beta} Q_0 \tag{17-5}$$

This safety stock is caused by our being unable to ascertain the demand rate and the replenishment time accurately. When the normal rate a and the normal

replenishment time t_0 occur, the safety stock has to be carried over to the next cycle. The question is: What safety stock should be aimed at? This problem (following an analysis of the loss function due to uncertainty) is discussed in the next chapter.

Reorder procedure in the cycle system

Different items in the group may have different consumption rates (Fig. 17–7), but if we want to prevent stock piling or too early depletion of stock, the length of the consumption cycle must be the same. Hence,

$$\frac{Q_1}{a_1} = \frac{Q_2}{a_2} = \frac{Q_3}{a_3} = \cdots \tag{17–6}$$

which is the condition that we had in multiproduct batch scheduling. We may therefore use the methods given in Chapter 14 to compute the length of the reordering cycle and reorder batch sizes for each product. A reorder range may be defined (in the same way as the production range) for each product, and similarly, an economic reorder range may be defined.

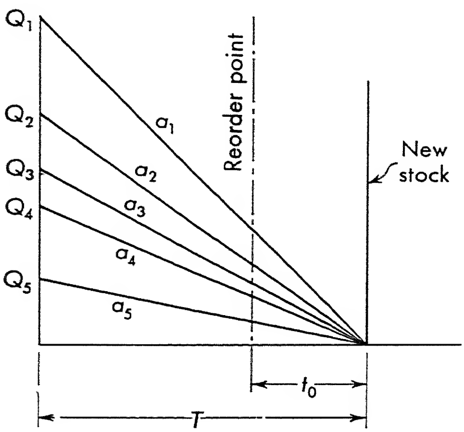

Figure 17–7. ***Reordering five items by the cycle system: the consumption period must be the same for all items; replenishment times are also equal in this example.***

In the solution for the multiproduct production schedule, the computed batch sizes had to pass two tests: (1) the p test, to ensure that the quantity is within the defined range, and (2) the cycle test, to ensure that the required schedule may be matched by the capacity of the plant. In the case of stock control the solution has to pass only the p test. If it does, the solution is acceptable. If it does not, we may do either of two things:

1. Take out from the group those items that do not conform to the p test; in other words, the division of the stores into groups is mainly guided by the principle of having products of similar characteristics, so that each group is ordered every cycle and all the products are included in the order.

2. Introduce shorter and longer cycles in addition to the basic reordering cycle. In this way products whose respective computed batch sizes are higher than the upper limits of the reorder ranges could be ordered more frequently by dividing each batch into two or more sub-batches. When the time for reordering of the group comes, these products are ordered alongside the others but only at reduced quantities, and during the cycle period another date for reordering is fixed just for these products. The length of this short cycle depends on the number of sub-batches in the computed batch size. If the batch size is divided into two (and in many cases this is adequate), the short cycle will be half the basic reordering cycle.

On the other hand, products whose computed batch sizes are below the respective lower limits of the reorder ranges could be ordered every second cycle, so that the ordered quantities (which are brought in this way within the range limits) would satisfy the demand for two cycles. The long cycle would therefore have a duration of two ordinary cycles. To summarize, the group is divided into three subgroups A, B, and C, and the reordering procedure is as follows:

Short cycle: order subgroup A

Basic cycle: order subgroups A, B

Long cycle: order subgroups A, B, C

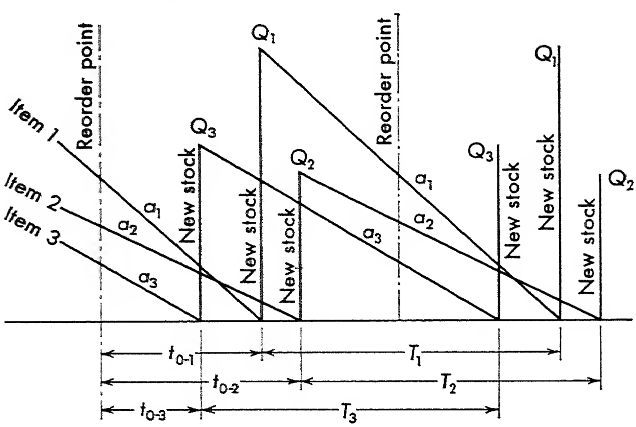

Figure 17–8. ***Reordering cycle when replenishment periods are not equal but consumption periods are still the same*** $(T_1 = T_2 = T_3)$.

Another complication arising from the reordering cycle is that the replenishment time should be the same for all products if the new stocks are to arrive at the same time, as in Fig. 17–7. This would sometimes be the case when the whole group is supplied by the same vendor. Very often, however, replenishment

times differ, so that the individual stock level curves are displaced in time, as shown in Fig. 17–8. The consumption periods remain equal: $T_1 = T_2 = \cdots$. At the reorder point

$$Q_{01} = a_1 t_{01}; \qquad Q_{02} = a_2 t_{02}; \qquad \text{etc.}$$

and these quantities may be used for control purposes in order to check and adjust time displacements.

Effect of Uncertainty

It was remarked above that when all the factors governing the behavior of the system could be accurately determined, there would be no need to plan for a safety stock at all. The purpose of the safety stock is to guard against uncertainty; first the uncertainty caused by variations in the rate of consumption, and secondly the uncertainty in assessing the replenishment time. At a certain point on the consumption curve a decision regarding replenishment has to be made. This is a typical situation in the two-bin system. A prediction regarding the consumption trend has to be made on the basis of past data and any available information on possible changes in demand in the comparatively near future. This "prediction point," as it may be called, should not be selected too far down in the consumption curve, lest not enough time is left for replenishment of stock.

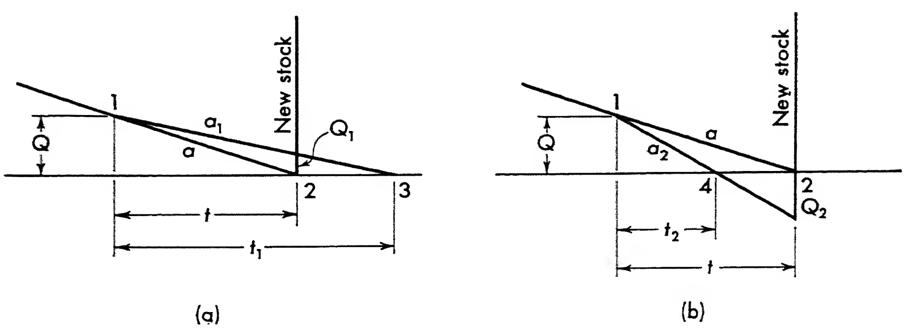

Figure 17–9. ***Two possible outcomes of uncertainty in continuous demand.***

(a) Rate of demand has been overestimated.
Result: Stock residual Q_1 at end of cycle.

(b) Rate of demand has been underestimated.
Result: Demand for Q_2 units cannot be met.

What is the effect, or the cost, of uncertainty? Figure 17–9a describes a situation in which the rate of consumption is overestimated. At point 1, the prediction point, the consumption rate a is predicted, so that the stock is expected

to deplete at point 2. The actual rate, however, is smaller, $[a_1 = a(1 - \beta)]$, and the stock is really depleted at point 3 (t days after the prediction point), so that a residual stock Q_1 has to be carried to the next cycle. Since

$$Q = at = a_1t_1$$

the residual stock is

$$Q_1 = Q\frac{t_1 - t}{t_1} = Q\left(1 - \frac{t}{t_1}\right) = \beta Q$$

If the cost of carrying a unit in stock is c_1 per day, the cost of carrying Q_1 units for the next cycle, whose length is T, is

$$y_1 = \beta Q c_1 T \tag{17–7}$$

Suppose now that the rate of consumption is underestimated (Fig. 17–9b) and that the actual rate is $a_2 = a(1 + \beta)$, where a is the forecast rate. The stock is actually depleted at point 4 (t_2 days after the prediction point), but since provision is made for the new stock to arrive at point 2, we are not able to satisfy demand for a period $t - t_2$. This means that in the case of continuous demand, a stock of Q_2 cannot be supplied. From Fig. 17–9b:

$$Q = at = a_2t_2$$

and

$$Q_2 = Q\frac{t - t_2}{t_2} = Q\left(\frac{t}{t_2} - 1\right) = \beta Q$$

There is a penalty attached to being unable to satisfy demand, but we should perhaps distinguish between two situations, insufficient supply to customers and insufficient supply to the assembly line.

Situation a: Insufficient supply to customers

When we supply to outside customers, we may try in most cases to assign a flat penalty rate for each unit that we are unable to supply. This penalty includes the loss of profit for sales that are not concluded and the loss of good will of the customers. The first loss is easily established, provided we know that the customer has decided not to wait until new stock arrives, but goes and buys elsewhere. The loss of good will is not that easily determined quantitatively. One customer may perhaps be only slightly disappointed at not getting proper service; another may be utterly disgusted and leave for good, or perhaps even take his orders for other products elsewhere.

There is also the factor of accumulated displeasure. A customer may be prepared to put up with bad service once or twice, but no more. It is evident that thorough and painstaking customer surveys may have to be carried out in order

to determine these factors. If the penalty for not meeting demand is denoted by c'_2 per each unit that cannot be supplied, the cost is

$$y''_2 = \beta Q c'_2 \tag{17-8}$$

Situation b: Insufficient supply to assembly line

When the supply to an assembly line must be continuous but we suddenly run out of stock, the assembly may be held up. Men, machines, and parts may be kept idle until supply is resumed. In some such cases the penalty may be expressed as c''_2 per unit for each day that demand cannot be met. Under these circumstances the penalty is

$$\begin{aligned} y''_2 &= \tfrac{1}{2}Q_2 c''_2(t - t_2) \\ &= \frac{1}{2}\beta Q c''_2 t\left(1 - \frac{t_2}{t}\right) \\ &= \frac{1}{2}\frac{\beta^2}{1+\beta}\, Q c''_2 t \end{aligned} \tag{17-9}$$

The above expressions lead to the following obvious conclusions: First, the higher the prediction point on the stock depletion curve, the higher is the cost of uncertainty. Secondly, this cost increases with the actual deviation from the forecasted value of the consumption rate.

Comparison of replenishment policies

In many cases the predicted consumption rate is expressed in the form of an average expected value and an assessment of variations that may occur. Suppose we forecast a consumption rate of $a(1 \pm \beta)$, when the rate is expected to fall between the two extreme limits within a certain known confidence level, say, 99 per cent. This means that only in 1 per cent of the cases would the rate of consumption be expected to fall out of this range. When 1 per cent is considered to be too high, wider limits at a higher confidence level should be chosen. What replenishment policy should be adopted?

Figure 17–10 shows this situation as viewed from the prediction point. The upper limit of the consumption rate is $a_2 = a(1 + \beta)$; the lower limit, $a_1 = a(1 - \beta)$. Let us examine three replenishment policies: optimistic, realistic, and pessimistic.

The "optimistic" policy (I)

According to this policy we play safe, i.e., we plan for a situation where the maximum consumption rate occurs, and since we do not want to run out of stock, replenishment is planned to take place in t_2 days' time. The average consumption

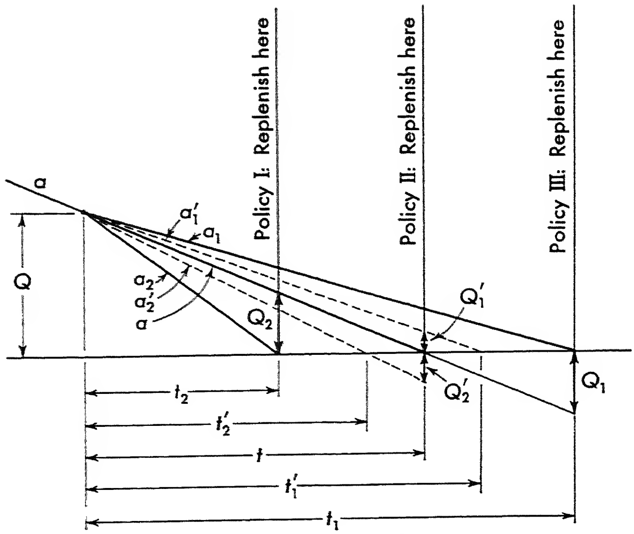

Figure 17–10. ***Analysis of three replenishment policies.***

rate expected is a, so that on the average a stock Q_2 is expected to remain unconsumed when the new batch arrives:

$$\frac{Q_2}{Q} = \frac{t - t_2}{t} = \frac{a_2 - a}{a_2} = \frac{\beta}{1 + \beta}$$

The cost of this policy, when c_1 is the cost of carrying one unit in stock per day, is

$$y_{\mathrm{I}} = Q_2 c_1 T = \frac{\beta}{1 + \beta} Q c_1 T \tag{17–10}$$

The "realistic" policy (II)

Here we plan to replenish after t days, i.e., we take the expected average consumption rate as a guiding criterion. If the distribution of expected rates is symmetrical about the mean value a, the chances that the rate will be above or below the mean are 50:50. When the rate is below a, it will on the average be

$$a'_1 = a(1 - \beta_0)$$

where $\beta_0 < \beta$, so that Q'_1 units have to be carried in stock (and $Q'_1 = \beta_0 Q$), the cost being

$$\tfrac{1}{2}\beta_0 Q c_1 T$$

The factor $\frac{1}{2}$ is introduced because consumption is expected to be below the

average for 50 per cent of the time. When the rate exceeds a, it will on the average be

$$a'_2 = a(1 + \beta_0)$$

and $Q'_2 = \beta_0 Q$ cannot be supplied. The cost of this occurrence in situation a (i.e., flat penalty rate per unit) is

$$\tfrac{1}{2}\beta_0 Q c'_2$$

and in situation b (i.e., penalty cost is per unit per day),

$$\tfrac{1}{4}\beta_0 Q c''_2(t - t'_2)$$

Hence, the total cost of this policy is expected to be:

For situation a: $$y_{\mathrm{II}a} = \tfrac{1}{2}\beta_0 Q(c_1 T + c'_2) \tag{17–11}$$

For situation b: $$y_{\mathrm{II}b} = \tfrac{1}{2}\beta_0 Q[c_1 T + \tfrac{1}{2}c''_2(t - t'_2)] \tag{17–12}$$

The "pessimistic" policy (III)

Here we take a dim view of life and plan for replenishment at the lowest level, i.e., in t_1 days' time. This policy definitely leads to running out of stock, and within the confidence limits, no stock has to be carried to the next cycle. The average expected consumption is a and on the average the demand that cannot be met is

$$Q_1 = Q\,\frac{t_1 - t}{t} = \frac{\beta}{1 - \beta}\,Q$$

and the cost of this policy is

For situation a: $$y_{\mathrm{III}a} = \frac{\beta}{1 - \beta}\,Qc'_2 \tag{17–13}$$

For situation b: $$\begin{aligned} y_{\mathrm{III}b} &= \frac{1}{2}\,\frac{\beta}{1 - \beta}\,Qc''_2(t_1 - t) \\ &= \frac{1}{2}\left(\frac{\beta}{1 - \beta}\right)^2 Qc''_2 t \end{aligned} \tag{17–14}$$

Which of these policies is preferable? It seems that it pays to be optimistic rather than pessimistic, as normally policy I is cheaper.

For situation a: $$\frac{y_{\mathrm{I}}}{y_{\mathrm{III}a}} = \underbrace{\frac{c_1 T}{c'_2}}_{\substack{<1 \\ \text{(usually)}}}\;\underbrace{\frac{1 - \beta}{1 + \beta}}_{<1} < 1$$

For situation b:
$$\frac{y_{\mathrm{I}}}{y_{\mathrm{III}b}} = \frac{c_1 T}{\frac{1}{2}c''_2 t}\,\frac{1}{\beta}\,\frac{(1-\beta)^2}{1+\beta}$$
$$= \underbrace{\frac{c_1 T(1+\beta)}{\frac{1}{2}c''_2 t\beta}}_{\substack{<1\\ \text{(usually)}}}\;\underbrace{\frac{(1-\beta)^2}{(1+\beta)^2}}_{<1} < 1$$

Similarly, the realistic policy is preferable to the pessimistic one.

For situation a:
$$\frac{y_{\mathrm{II}a}}{y_{\mathrm{III}a}} = \frac{\beta_0}{\beta}(1-\beta)\frac{1}{2}\left(1+\frac{c_1 T}{c'_2}\right) < 1$$

because
$$\frac{\beta_0}{\beta} < 1$$
$$1-\beta < 1$$
$$\frac{1}{2}\left(1+\frac{c_1 T}{c'_2}\right) < 1$$

For situation b:
$$\frac{y_{\mathrm{II}b}}{y_{\mathrm{III}b}} = \frac{\beta_0(1-\beta)^2}{\beta^2}\,\frac{c_1 T + \frac{1}{2}c''_2(t-t'_2)}{c''_2 t}$$
$$= \frac{\beta_0}{\beta}\,\frac{(1-\beta)^2}{1+\beta}\cdot\left[\frac{1+\beta}{\beta}\,\frac{c_1 T}{c''_2 t} + \frac{1}{2}\,\frac{1+\beta}{\beta}\,\frac{t-t'_2}{t}\right] < 1$$

because
$$\frac{\beta_0}{\beta} < 1$$
$$\frac{(1-\beta)^2}{1+\beta} < 1$$
$$\frac{1+\beta}{\beta}\,\frac{c_1 T}{c''_2 t} < \frac{1}{2}$$
$$\frac{1}{2}\,\frac{1+\beta}{\beta}\,\frac{t-t'_2}{t} = \frac{1}{2}\,\frac{1+\beta}{\beta}\,\frac{\beta_0}{1+\beta_0} = \frac{1}{2}\,\frac{\beta_0+\beta\beta_0}{\beta+\beta\beta_0} < \frac{1}{2}$$

Comparison of policies I and II is inconclusive.

Situation a:
$$\frac{y_{\mathrm{II}a}}{y_{\mathrm{I}}} = \frac{\frac{1}{2}\beta_0(c_1 T + c'_2)}{[\beta/(1+\beta)]c_1 T}$$
$$= \underbrace{(1+\beta)}_{>1}\,\underbrace{\frac{\beta_0}{\beta}}_{<1}\,\underbrace{\frac{1}{2}\left(1+\frac{c'_2}{c_1 T}\right)}_{>1}$$

Situation b: $$\frac{y_{\mathrm{II}b}}{y_{\mathrm{I}}} = \frac{\frac{1}{2}\beta_0[c_1T + \frac{1}{2}c''_2(t - t'_2)]}{[\beta/(1+\beta)]c_1T}$$

$$= (1+\beta)\frac{\beta_0}{\beta}\frac{1}{2}\left(1 + \frac{1}{2}\frac{c''_2t}{c_1T}\frac{\beta_0}{1+\beta_0}\right)$$

$$= \underbrace{(1+\beta)}_{>1}\underbrace{\frac{\beta_0}{\beta}\frac{1}{2}}_{<1}\left(1 + \frac{1}{2}\underbrace{\frac{c''_2t}{c_1T}\frac{\beta}{1+\beta}}_{>1}\underbrace{\frac{\beta_0+\beta\beta_0}{\beta+\beta\beta_0}}_{<1}\right)$$

The value of c'_2/c'_1, c''_2t/c_1T is important here. If these ratios have a high enough value to offset the fraction β_0/β, policy I would be cheaper. In marginal cases the distribution of expected consumption rates may also affect the issue.

Example[1]

For situation a the cost ratio $c'_2/c_1T = 10.0$, and the consumption rate is forecast with tolerances of ± 20 per cent. Compare the costs of policies I, II, and III as indicated in Fig. 17–10, when the distribution of expected consumption rates is (1) normal, (2) rectangular.

Solution

We have $\beta = 0.2$. Comparing I and III:

$$\frac{y_{\mathrm{I}}}{y_{\mathrm{III}}} = \frac{c_1T}{c'_2}\frac{1-\beta}{1+\beta} = \frac{1}{10}\frac{0.8}{1.2} = 0.067$$

Comparing II and III:

$$\frac{y_{\mathrm{II}}}{y_{\mathrm{III}}} = \frac{\beta_0}{\beta}(1-\beta)\frac{1}{2}\left(1 + \frac{c_1T}{c'_2}\right)$$

$$= \frac{\beta_0}{\beta}\frac{0.8}{2}1.1 = 0.44\frac{\beta_0}{\beta}$$

1. When the distribution is normal, the average value of all the consumption rates above a is given by (as shown in the next chapter)

$$\beta_0 = -\frac{\varphi_\infty - \varphi_0}{\Phi_\infty - \Phi_0}$$

Substitute: $\varphi_\infty = 0$; $\Phi_\infty = 1.0$; $\Phi_0 = 0.50$

From the Appendix table: $\varphi_0 = 0.399 \simeq 0.40$

$$\therefore \qquad \beta_0 = \frac{0.40}{0.50} = 0.8$$

[1] This example may be deleted in first reading.

and
$$\frac{\beta_0}{\beta} = \frac{0.8}{3.0} = 0.27$$

(*Note:* β is taken as 3.0 for $\pm 3\sigma$ confidence limits; i.e., when in 99.7 per cent of the cases the consumption rate is expected to fall within the forecast range.)

Therefore
$$\frac{y_{\text{II}}}{y_{\text{III}}} = 0.44 \times 0.27 = 0.119$$

2. When the distribution is rectangular, the average value of the consumption rates above a is $\frac{1}{2}\beta$, or[2]
$$\frac{\beta_0}{\beta} = \frac{1}{2}$$

$\therefore$
$$\frac{y_{\text{II}}}{y_{\text{III}}} = 0.44 \times 0.5 = 0.220$$

Comparing I and II:

1.
$$\frac{y_{\text{I}}}{y_{\text{II}}} = \frac{y_{\text{I}}/y_{\text{III}}}{y_{\text{II}}/y_{\text{III}}} = \frac{0.067}{0.119} = 0.56$$

2.
$$\frac{y_{\text{I}}}{y_{\text{II}}} = \frac{0.067}{0.220} = 0.65$$

Hence, policy I is by far the best to choose in this case.

This analysis merely compares three specific policies; it does not tell us which is the optimum policy, which might lie between I and II. The problem of determining the optimal reorder quantity in the case of uncertainty is discussed in the next chapter.

Summary

When the mechanism of supply differs from that of demand, a buffer stock is required. It should include a safety stock to guard against any adverse effects of uncertainties attached to the rate of supply, to the rate of demand, to lag time in ordering, to price fluctuations of materials, etc. Briefly, the inventory problem is: How much to order and when to order, so as to optimize the effectiveness of the store? If supply and demand are linked together in several stages, the output of one constituting the input to another (as in a production line or in channels of distribution), i.e., any demand variation for finished goods at the end of the line, triggers off a chain reaction, and the effect is amplified as the wave travels along the line. These effects can be damped by selection of an appropriate percentage of safety stock, by proper directives in the stock control system as to how to react in cases of fluctuations (often effective in a centralized control system), and (obviously) by reduction of the number of links in the chain.

[2] This is the highest value this fraction can attain with a uni-modal symmetrical distribution.

Stock control systems are the tools through which the inventory policies are exercised, and some of the common systems in use (two-bin, reorder cycle, combinations of the two) and their reorder procedures are described in this chapter. The optimal reorder batch size and some preliminary considerations of the effects of uncertainties are also analyzed.

References

Forrester, Jay W.: Industrial dynamics, a major breakthrough for decision makers (Harvard Business Review, July–August, 1958).

Magee, J. F.: *Production Planning and Inventory Control* (McGraw-Hill Book Co., Inc., 1958).

Whitin, T. M.: *The Theory of Inventory Management* (Princeton University Press, 1957).

Problems

1. Show from Eq. 17–1 that

$$\frac{Q_n}{Q} > 1 + x$$

and comment on the conclusions that may be drawn from this relation.

2. It was shown through Eq. 17–1 that the smaller the safety stock, the smaller the effects along the production line, but from Tables 17–4 and 17–5 it would seem that with larger safety stocks, the effects can be damped. How do you explain this apparent contradiction?

3. With an initial line output of Q per period and a demand impulse denoted by x, find an expression for the required output at the nth stage with safety stock policy r under situations described by Tables 17–5 and 17–6.

4. A production line has three stages, apart from the final assembly center, and the inventory policy requires that safety stock be 50 per cent of the eventual output at each stage, as suggested in Table 17–6. The initial line output is 100 units per period, the subsequent changes in demand being as follows:

Period	*Units*
1	100
2–3	110
4	100
5	110
6	90
7–10	100

If the maximum output capacity at each stage cannot be raised above 125 units per period, construct a table and then plot the fluctuations in demand, outputs, and safety stocks at each stage. Comment on the safety stock policy, assuming that fluctuations of demand for the final product range between 90 and 110 units per period.

5. Summarize the effects of a demand impulse of the end product on the production line and indicate in what ways these effects can be reduced. When this chain reaction phenomenon recurs owing to several stages in the distribution channels, how can management control the situation?

6. An investigation in a firm showed that a considerable number of items were ordered 25 per cent in excess of the minimum-cost batch size. The financial manager said it was a disastrous situation, as the cost per unit increased thereby by 25 per cent and the firm could not face the severe competition in the market. The store superintendent claimed that, since the stock control was based on the two-bin system, it was virtually impossible to reduce the size of the ordered batches without drastic changes in the control system. He suggested that these changes would cost far more than the anticipated saving. Furthermore he thought the estimate given by the financial manager about the excessive costs per unit was exaggerated. What comments can you offer?

7. Demand from a general store for a certain item fluctuated between 4,000 and 8,000 units per week, and data for accumulated demand in the past is summarized in the following table:

Demand above 4,000 units per week	100% of the time
4,500	90%
5,000	79%
5,500	64%
6,000	50%
6,500	22%
7,000	8%
7,500	3%
8,000	0%

The cost per unit may be adequately expressed by

$$Y = 0.80 + 360\,Q^{-1} + 10^{-5}Q \quad \text{dollars}$$

where Q is the ordered batch size, which has so far been 6,000 units per week. Since there have been so many complaints that the store is running out of stock far too often, management has decided to review the situation. What batch size would you recommend if the following restrictions are to be satisfied:

(i) The cost per unit should not exceed the minimum possible costs by more than 0.5 per cent.

(ii) Level of satisfaction should not be below 90 per cent (i.e., in 90 per cent of the cases demand can be met).

(iii) The maximum capacity of the store is 7,500 units.

8. A stock control system has the following features: There are ten products in the store and, by present practice, three months' supply of each product is ordered when the quantity on hand of that product reaches the reorder point. The demand is fairly uniform, with no seasonal fluctuations, so that

four orders per year are required for each product. Data are given in the accompanying table.

Data on the Products

Product No.	*Annual Usage*	*Order Size*	*Average Inventory**
1	\$ 2,000	\$ 500	\$ 300
2	\$ 3,000	\$ 750	\$ 450
3	\$ 5,000	\$ 1,250	\$ 750
4	\$ 6,000	\$ 1,500	\$ 900
5	\$20,000	\$ 5,000	\$ 3,000
6	\$28,000	\$ 7,000	\$ 4,200
7	\$30,000	\$ 7,500	\$ 4,500
8	\$45,000	\$11,250	\$ 6,750
9	\$48,000	\$12,000	\$ 7,200
10	\$80,000	\$20,000	\$12,000

* Average inventory is based on the policy that, at replenishment point, a safety stock of 10 per cent of the order quantity is held.

The cost of one order is \$5.00 and interest charges are 12 per cent.

(i) Is this a two-bin or a reordering cycle system? Why?
(ii) What recommendations for changes in this system would you make?
(iii) If management decides to have a reordering cycle system, explain how the theory of multiproduct scheduling for maximum return could be used for this control system.

9. A store has 10,000 items, and replenishment is based on the two-bin system aiming at minimum costs per unit. On the average, each item is ordered five times a year, and the total cost of placing orders amounts to \$50,000 a year. We may assume that the cost per order is the same for all items. Furthermore the history of the firm in recent years suggests that the total annual costs of placing orders follows approximately the linear expression $18{,}000 + 0.8N$ dollars, where N is the total number of orders per annum. A breakdown of the orders by value of goods is given in the accompanying table.

Class 1:	each of 20% of the orders is for less than \$10 worth of merchandise
Class 2:	each of 38% of the orders is for \$10 ÷ 20 worth of merchandise
Class 3:	each of 14% of the orders is for \$20 ÷ 50 worth of merchandise
Class 4:	each of 10% of the orders is for \$50 ÷ 100 worth of merchandise
Class 5:	each of 10% of the orders is for \$100 ÷ 500 worth of merchandise
Class 6:	the remainder of the orders is for more than \$500 each

Comment on the reorder system used at present and suggest in what ways it can be improved. What further data would you require and how would you use the data to determine what modifications should be recommended?

10. A repair garage, specializing in a limited number of makes, maintains a spare-parts store, which is designed to supply the immediate requirements of the various departments. Repairs are generally classified in three categories:

Class 1: repairs amounting to less than 2 hours work
Class 2: repairs amounting to between 2 and 12 hours work
Class 3: repairs amounting to more than 12 hours

It is the policy of the garage management to finish each job as soon as possible so that the automobile owner does not have to wait too long, and this policy was formulated as follows:

Automobiles belonging to class 1: should be ready within 4 hours of their entry to the garage
class 2: within 2 days
class 3: usually within 1 to 5 days

As orders for these spares normally require a lag time of about 24 hours, it has been often found necessary to send somebody on a special errand to get parts when classes 1 and 2 repairs are involved. Explain in detail what data you would look for in order to analyze the situation, and suggest a suitable stock control system.

11. If the annual consumption of a commodity is A and the cost of placing an order is

$$s = kn^{-\frac{1}{2}}$$

where n stands for the number of orders per year, and k is a constant, show that the optimal reorder batch for minimum costs per piece is given by

$$Q = \left(\frac{k^2}{4K^2A}\right)^{\frac{1}{3}}$$

12. If the allowable increase in costs per unit is given as

$$\xi = \frac{Y}{Y_m}$$

find an expression for q as a function of u and ξ.

13. A stock of Q components must be available at a certain date for the assembly line. The available procurement period is T days before the dead line. There are three alternatives for building up this stock:

(i) Getting the whole batch of Q components into the store T days before the dead line

(ii) Getting half the batch at $-T$ days and half the batch at $-\frac{1}{2}T$ days.

(iii) Splitting the batch into three and getting each lot at $-T$ days, $-\frac{2}{3}T$ days, and $-\frac{1}{3}T$ days.

Find the cheapest method by comparing the costs incurred by each method, when the total carrying costs per piece in the store are C per day.

14. In the preceding problem, the vendor is prepared to deliver half the order at $-T$ days and split the rest into equal quantities, spread evenly during the period $-\frac{1}{2}T$ to 0.

Find the cheapest method for the stores by splitting the batch into n lots, and draw characteristic curves to which the purchasing department can refer on future similar occasions.

15. A certain component is used for an assembly line and is ordered so that Q units are held in stock T days before assembly begins. The safety stock Q_0 is 20 per cent of Q, and it was specified by the chief production engineer that on no account should the stock during the period $-T$ to 0 be below Q_0.

    (i) After reviewing this situation, an industrial engineer has suggested that if only Q_0 were received at $-T$ and the balance at a later date, he could achieve a substantial saving in costs, in spite of the additional setup costs involved. He claims that this saving amounts to 60 per cent of the present carrying costs. Can you substantiate this statement?

    (ii) The industrial engineer has been told that his plan is not practical because it is impossible to get the second lot at the time he has specified. However, the management thinks highly of his scheme and should like him to state a limiting situation at which the second lot does not lead to a loss, as well as plot a curve showing the saving as a function of time between the limiting situation and the best one. Can you comply with this request?

16. A stock Q is required for the assembly line in T days' time. Two schemes are suggested: (a) order and receive Q immediately so that Q units will be held in stock for T days and then issued to the shop; (b) order the quantity Q in n equal batches to be received every T/n days, the first batch to be received immediately; each order incurs a cost s.

    (i) When is scheme (a) less costly?

    (ii) For what value of n does the changeover from scheme (a) to scheme (b) yield the maximum saving? If this optimal n does not happen to be a whole number, how would you decide which of the two neighboring whole numbers to select?

18[1]

SOME FURTHER CONSIDERATIONS OF QUANTITY CONTROL

It was pointed out in the last chapter that quantity control of inventories requires detailed study in view of the many uncertainties involved. There may be random variations in the consumption rate and in the time required for replenishment, and combinations of these factors alone may cause serious deviations of the stock at the point of replenishment from the anticipated level. The ordered batch may also be subjected to variations in size: One quantity may be ordered and another one actually received, the discrepancy being due either to mistakes or disturbances at the vendor's depot or to rejection of products by quality inspection.

In the midst of these uncertainties, we want to try to outline a policy for controlling the level of stocks; we want to specify how much to order and when to order; and we want our policy to yield optimum results as judged by the criterion through which we measure the effectiveness of our performance. Richard Bellman summarized the inventory problem very aptly:

> We possess various quantities of different items for which there is a demand of stochastic nature from time to time. Since there is a penalty of some type attached to not being able to satisfy this demand, at various stages additional quantities of these items are ordered, at costs dependent upon types and quantities ordered, the times of ordering, the rate of delivery desired, and other factors as well. The problem is to determine the ordering policies which are optimal with respect to pre-assigned measures of efficiency.[2]

In quantity control of raw materials, components, and semifinished products, we normally wish to minimize the cost of holding inventories. In stock control of finished products for sale, we often select maximization of profit as our yardstick.

First, we have to study the distribution of variables that affect the inventory control system and express in a probabilistic form our expectations of the behavior of the system. We can then try to build an inventory model and proceed

[1] This chapter may be omitted on first reading.

[2] Management Science, October, 1958, p. 139.

to find the value of the variables at the point of optimum. This model need not always be a mathematical one. The number of variables is sometimes too large and the distribution too awkward to be easily expressed in simple algebraic forms, so that simulation methods may become preferable.

In the absence of a general all-purpose inventory model, we shall first analyze two models: one for continuous demand and the other for instantaneous demand when the form of the demand distribution is not specified. We shall then proceed to examine the special case of stocking perishable goods for a normal distribution and some aspects of apportioning problems.

Mathematical Comments on Notations and Derivations

Before we proceed with the analysis of the loss function, it would be useful at this stage to introduce several notations. If $f(D)$ is the probability density function of demand, then the probability that a quantity up to Q will be required is

$$\int_0^Q f(D)\,dD = F \tag{18–1}$$

The total cumulative probability of demand is

$$\int_0^\infty f(D)\,dD = 1 \tag{18–2}$$

Therefore the probability of demand for quantity D when $D > Q$ is

$$\int_Q^\infty f(D)\,dD = \int_0^\infty f(D)\,dD - \int_0^Q f(D)\,dD = 1 - F \tag{18–3}$$

Another function we shall encounter is

$$\int_Q^\infty \frac{f(D)}{D}\,dD = F_1 \tag{18–4}$$

An expression describing the first moment of the distribution function is

$$\int_0^Q Df(D)\,dD = F_2 \tag{18–5}$$

The mean value $\bar{D}$ of the distribution is given by the definition

$$\int_0^\infty Df(D)\,dD = \bar{D} \tag{18–6}$$

Hence

$$\int_Q^\infty Df(D)\,dD = \bar{D} - F_2 \tag{18–7}$$

The following derivations are given without proof, which may be sought in suitable texts on calculus:

$$\frac{d}{db}\int_a^b f(x)\,dx = f(b) \tag{18–8}$$

$$\frac{d}{db}\int_b^a f(x)\,dx = -\frac{d}{db}\int_a^b f(x)\,dx = -f(b) \tag{18–9}$$

$$\frac{d}{db}\int_a^b xf(x)\,dx = bf(b) \tag{18–10}$$

$$\frac{d}{db}\int_a^b bf(x)\,dx = \int_a^b f(x)\,dx + bf(b) \tag{18–11}$$

$$\frac{d}{db}\int_a^b b^2f(x)\,dx = 2b\int_a^b f(x)\,dx + b^2f(b) \tag{18–12}$$

The Cost of Uncertainty for Continuous Demand

Suppose we have continuous demand and we start with a stock Q (see Fig. 18–1). It is planned to replenish the stock after a period T. The demand is D, with a probability density function $f(D)$. When $D < Q$, a stock $Q - D$ is left

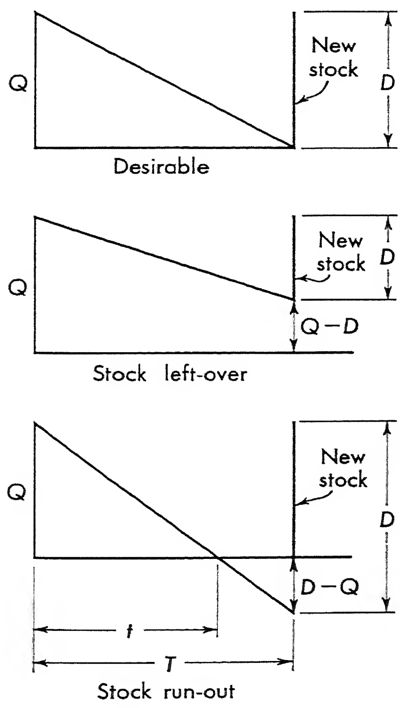

Figure 18–1. ***Stock patterns in continuous demand.***

over to be carried during the next cycle at a cost of c_1 per unit per day. The average stock during the cycle is $Q - \frac{1}{2}D$; hence the carrying cost for the case $D \leqslant Q$ is

$$\int_0^Q c_1(Q - \tfrac{1}{2}D)\,Tf(D)\,dD$$

Situation a

When $D > Q$, the number of units that cannot be supplied is $D - Q$, and the penalty for each unit is c'_2. The average stock during the time t is $\frac{1}{2}Q$. The cost for $D > Q$ is

$$\int_Q^\infty \left[c_1 \frac{Q}{2} t + c'_2(D - Q)\right] f(D)\, dD$$

or

$$\int_Q^\infty \left[c_1 \frac{Q^2}{2D} T + c'_2(D - Q)\right] f(D)\, dD$$

when $t/T = Q/D$ is substituted. The total cost function is, therefore,

$$C = c_1 T \int_0^Q (Q - \tfrac{1}{2}D) f(D)\, dD + c_1 T \int_Q^\infty \frac{Q^2}{2D} f(D)\, dD$$

$$+ c'_2 \int_Q^\infty (D - Q) f(D)\, dD \tag{18–13}$$

or

$$\frac{C}{c'_2} = \epsilon_1 \int_0^Q Qf(D)\, dD - \frac{1}{2}\epsilon_1 \int_0^Q Df(D)\, dD + \frac{1}{2}\epsilon_1 \int_Q^\infty \frac{Q^2}{D} f(D)\, dD$$

$$+ \int_Q^\infty Df(D)\, dD - \int_Q^\infty Qf(D)\, dD \tag{18–14}$$

where

$$\epsilon_1 = \frac{c_1 T}{c'_2}$$

The quantity that should be ordered to minimize costs corresponds to $dC/dQ = 0$. Hence

$$\epsilon_1 \underbrace{\int_0^Q f(D)\, dD}_{=F} + \epsilon_1 Qf(Q) - \frac{1}{2}\epsilon_1 Qf(Q) + \epsilon_1 Q \underbrace{\int_Q^\infty \frac{f(D)}{D}\, dD}_{=F_1} - \frac{1}{2}\epsilon_1 Q^2 \frac{f(Q)}{Q}$$

$$- Qf(Q) - \underbrace{\int_Q^\infty f(D)\, dD}_{=1-F} + Qf(Q) = 0$$

$$\therefore \qquad \epsilon_1 F + \epsilon_1 Q F_1 - (1 - F) = 0$$

$$\therefore \qquad Q = \frac{1 - F - \epsilon_1 F}{\epsilon_1 F_1} \tag{18–15}$$

Situation b

If the penalty per unit is c''_2 per unit per day, the cost incurred when $D > Q$ is

$$\int_Q^\infty \left[c_1 \frac{Q}{2} t + c''_2 \frac{D - Q}{2} (T - t)\right] f(D)\, dD$$

or

$$\int_Q^\infty \left[c_1 \frac{Q^2}{2D} T + c''_2 \frac{(D - Q)^2}{2D} T\right] f(D)\, dD$$

The total cost function in this situation is, therefore,

$$C = c_1 T \int_0^Q \left(Q - \frac{1}{2} D\right) f(D)\, dD + c_1 T \int_Q^\infty \frac{Q^2}{2D} f(D)\, dD + c''_2 T \int_Q^\infty \frac{(D-Q)^2}{2D} f(D)\, dD \tag{18–16}$$

This cost is a minimum when $dC/dQ = 0$. If now

$$\epsilon_2 = \frac{c_1}{c''_2}$$

we get for $dC/dQ = 0$:

$$\epsilon_2 \underbrace{\int_0^Q f(D)\, dD}_{=F} + \epsilon_2 Q f(Q) - \frac{1}{2}\epsilon_2 Q f(Q) + \epsilon_2 Q \underbrace{\int_Q^\infty \frac{f(D)}{D}\, dD}_{=F_1} - \frac{1}{2}\epsilon_2 Q f(Q)$$
$$- \frac{1}{2} Q f(Q) - \underbrace{\int_Q^\infty f(D)\, dD}_{=1-F} + Q f(Q) + Q \underbrace{\int_Q^\infty \frac{f(D)}{D}\, dD}_{=F_1}$$
$$- \frac{1}{2} Q f(Q) = 0$$

$$\therefore \qquad \epsilon_2 F + \epsilon_2 Q F_1 - (1 - F) + Q F_1 = 0$$

or

$$Q = \frac{1 - F - \epsilon_2 F}{F_1 (1 + \epsilon_2)} \tag{18–17}$$

Profit maximization as a criterion for reorder quantity

The foregoing discussion of the loss function was detached from the criterion of profit. This is a common situation in stores of a manufacturing enterprise where the main consideration is satisfaction of the demand by the production centers. It costs to keep goods in the stores and (figuratively speaking) it costs not to keep goods in the stores, when we are in some way penalized for failing to meet demand. The question that we have dealt with was: What costs more and how can the total cost function be minimized? Under such circumstances, profit is not really involved because it is virtually impossible in most cases to attach a profit tag to each component that moves toward the assembly line. The question of profit or loss becomes a real issue for the product that has to be sold to customers. If the product is sold during the cycle, a certain profit is realized (say, z per unit), but we still have to bear the carrying costs and the penalty costs when we run out of stock. The total profit is

$$Z = \underbrace{\int_0^Q z D f(D)\, dD}_{\substack{\text{Profit for } D \\ \text{units when} \\ D<Q}} + \underbrace{\int_Q^\infty z Q f(D)\, dD}_{\substack{\text{Profit for } Q \\ \text{units when} \\ D>Q}} - \underbrace{C}_{\substack{\text{Cost of} \\ \text{uncertainty} \\ \text{(see above)}}} \tag{18–18}$$

For maximum profit:

$$\frac{dZ}{dQ} = zQf(Q) + z\Big[\underbrace{\int_Q^\infty f(D)\,dD}_{=1-F} - Qf(Q)\Big] - \frac{dC}{dQ} = 0$$

$$\therefore \qquad \frac{dC}{dQ} = z(1 - F)$$

or, for situation *a*,

$$\frac{d(C/c'_2)}{dQ} = \zeta(1 - F)$$

and for situation *b*,

$$\frac{d(C/c''_2T)}{dQ} = \zeta(1 - F) \qquad (18\text{–}19)$$

where, for situation *a*,

$$\zeta = \frac{z}{c'_2}$$

and for situation *b*,

$$\zeta = \frac{z}{c''_2T} \qquad (18\text{–}20)$$

By substituting dC/dQ from the foregoing analysis, we get for situation *a*:

$$\epsilon_1F + \epsilon_1QF_1 - (1 - F) = \zeta(1 - F)$$

$$\therefore \qquad Q = \frac{(1 - F)(1 + \zeta) - \epsilon_1F}{\epsilon_1F_1} \qquad (18\text{–}21)$$

This is a similar expression to Eq. 18–15, which may be considered as a special case of Eq. 18–21 when $\zeta = 0$. It is interesting to note that the reorder quantity in this case is higher than the one obtained by Eq. 18–15; also that the criterion of minimum costs is not synonymous with that of maximum profit.

Under conditions of situation *b*, where the penalty is expressed in the form of c''_2 per unit per day, the appropriate value of dC/dQ has to be substituted, yielding

$$\epsilon_2F + \epsilon_2QF_1 - (1 - F) + QF_1 = \zeta(1 - F)$$

or

$$Q = \frac{(1 - F)(1 + \zeta) - \epsilon_2F}{F_1(1 + \epsilon_2)} \qquad (18\text{–}22)$$

To summarize, the reorder quantity for continuous demand under the conditions described above is

$$Q = \frac{(1 - F)(1 + \zeta) - AF}{BF_1} \qquad (18\text{–}23)$$

and the coefficients A, B are given in the accompanying table.

Criterion		*Situation a**	*Situation b**
Minimize cost due to uncertainty	$\zeta = 0$	$A = B = \epsilon_1$	$A = \epsilon_2$
Maximize profit in case of uncertainty	$\zeta > 0$		$B = 1 + \epsilon_2$

* Situation a: flat penalty cost per unit that cannot be supplied.
Situation b: penalty cost for stock run-out is per unit per day.

Combinations of situations a and b

In some situations in practice the penalty for not meeting demand may not be adequately described by either definitions a or b. When supply stops, immediate loss may be incurred owing to stoppage of machines or assembly lines, but some accumulative loss may also result for every day of stoppage. The cost per unit that we fail to supply may then be expressed as

$$c_2 = c'_2 + \tfrac{1}{2}c''_2 t'' \tag{18–24}$$

Where t'' stands for the number of days in which demand cannot be satisfied. The total expected profit is given by Eq. 18–18, and for maximum profit by Eq. 18–19:

$$\frac{dC}{dQ} = z(1 - F)$$

where

$$C = c_1 T \int_0^Q Qf(D)\,dD - \frac{1}{2}c_1 T \int_0^Q Df(D)\,dD + \frac{1}{2}c_1 T \int_Q^\infty \frac{Q^2}{D} f(D)\,dD$$
$$+ \int_Q^\infty c_2(D - Q)f(D)\,dD$$

After dividing by $c'_2 + c''_2 T$, we get

$$\frac{d[C/(c'_2 + c''_2 T)]}{dQ} = \epsilon F + \epsilon Q F_1 - \frac{\epsilon}{\epsilon_1}(1 - F) - \frac{\epsilon}{\epsilon_2}(1 - F) + \frac{\epsilon}{\epsilon_2} Q F_1$$
$$= \epsilon F + QF_1\epsilon\left(1 + \frac{1}{\epsilon_2}\right) - (1 - F)\left(\frac{\epsilon}{\epsilon_1} + \frac{\epsilon}{\epsilon_2}\right)$$

where

$$\left.\begin{aligned} \epsilon &= \frac{c_1 T}{c'_2 + c''_2 T} \\ \epsilon_1 &= \frac{c_1 T}{c'_2} \\ \epsilon_2 &= \frac{c_1}{c''_2} \end{aligned}\right\} \tag{18–25}$$

so that $(\epsilon/\epsilon_1) + (\epsilon/\epsilon_2) = 1$.

If we now denote

$$\zeta = \frac{z}{c'_2 + c''_2 T} \tag{18-26}$$

then

$$\epsilon F + QF_1\epsilon\left(1 + \frac{1}{\epsilon_2}\right) - (1 - F) = \zeta(1 - F)$$

or

$$Q = \frac{(1 - F)(1 + \zeta) - \epsilon F}{\epsilon F_1[1 + (1/\epsilon_2)]} \tag{18-27}$$

This is the general solution, incorporating situations *a* and *b* and the criteria for minimization of costs (in which case $\zeta = 0$) or maximization of profit. If situation *a* applies, we put $\epsilon_2 \to \infty$, then $\epsilon = \epsilon_1$ and expressions 18–15 and 18–21 are obtained. If situation *b* applies, we have $\epsilon_1 \to \infty$, so that $\epsilon = \epsilon_2$ and expressions 18–17 and 18–22 are obtained.

Cost of Uncertainty for Instantaneous Demand

In discrete or instantaneous demand a quantity D is withdrawn from the store at one time, as illustrated in Fig. 18–2. We assume here that the cycle

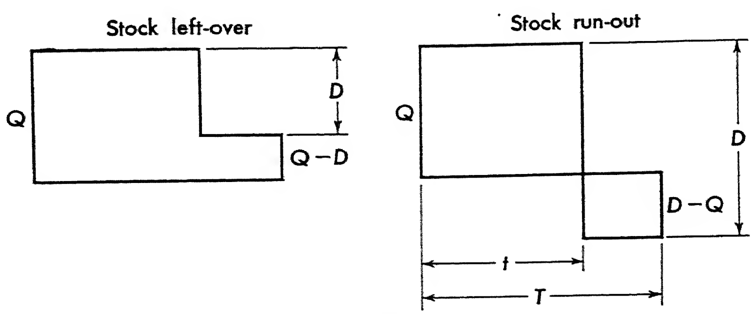

Figure 18–2. ***Instantaneous demand.***

begins with a batch Q and that stock is replenished every T days. The probability density function for demand is $f(D)$ and the quantity D is withdrawn only once during the cycle, after t days from the beginning of the cycle, t being a random variable.

Cost function

The cost function for situations *a* and *b* is given below.

Situation a

For $Q > D$, carrying Q units in stock for first t days:

$$\int_0^Q Qc_1 t f(D)\, dD$$

Carrying the residual:

$$\int_0^Q (Q - D)c_1(T - t)f(D)\,dD$$

For $Q < D$, carrying Q units in stock for first t days:

$$\int_Q^\infty Qc_1tf(D)\,dD$$

Penalty for not supplying $D - Q$ units:

$$\int_Q^\infty (D - Q)c'_2f(D)\,dD$$

The total cost is, therefore,

$$C = Qc_1t\underbrace{\left[\int_0^Q f(D)\,dD + \int_Q^\infty f(D)\,dD\right]}_{=1} + c_1(T - t)\left[\int_0^Q Qf(D)\,dD - \int_0^Q Df(D)\,dD\right] + c'_2\left[\int_Q^\infty Df(D)\,dD - \int_Q^\infty Qf(D)\,dD\right] \tag{18-28}$$

$$\frac{dC}{dQ} = c_1t + c_1(T - t)\Big[F + Qf(Q) - Qf(Q)\Big] + c'_2\Big[-Qf(Q) - (1 - F) + Qf(Q)\Big] = 0$$

or

$$c_1t + Fc_1(T - t) - c'_2(1 - F) = 0$$

Substitute $\epsilon_1 = c_1T/c'_2$ and $\tau = t/T$:

$$\epsilon_1\tau + \epsilon_1F(1 - \tau) - (1 - F) = 0$$

or

$$F = \frac{1 - \epsilon_1\tau}{1 - \epsilon_1\tau + \epsilon_1} \tag{18-29}$$

Situation b

$$C = Qc_1t + c_1(T - t)\left[\int_0^Q Qf(D)\,dD - \int_0^Q Df(D)\,dD\right] + c''_2(T - t)\left[\int_Q^\infty Df(D)\,dD - \int_Q^\infty Qf(D)\,dD\right] \tag{18-30}$$

For $dC/dQ = 0$:

$$\epsilon_2\tau + (1 - \tau)[\epsilon_2F - (1 - F)] = 0$$

where now $\epsilon_2 = c_1/c''_2$. Therefore

$$F = \frac{1 - (1 + \epsilon_2)\tau}{1 - (1 + \epsilon_2)\tau + \epsilon_2} \tag{18-31}$$

Criterion of profit

If the criterion of profit is adopted in the case of discrete demand, the total profit is given by Eq. 18–18, which is maximum when

$$\frac{dC}{dQ} = z(1 - F)$$

Situation a

Substitute $$\frac{d(C/c'_2)}{dQ} = \epsilon_1\tau + \epsilon_1 F(1 - \tau) - (1 - F)$$

and $$\frac{z}{c'_2} = \zeta$$

Hence $$F = \frac{1 + \zeta - \epsilon_1\tau}{1 + \zeta - \epsilon_1\tau + \epsilon_1} \tag{18–32}$$

which includes Eq. 18–29 as a special case when $\zeta = 0$.

Situation b

Substitute

$$\frac{d(C/c''_2T)}{dQ} = \epsilon_2\tau + (1 - \tau)\Big[\epsilon_2 F - (1 - F)\Big]$$

Hence $$F = \frac{1 + \zeta - \tau(1 + \epsilon_2)}{1 + \zeta - \tau(1 + \epsilon_2) + \epsilon_2} \tag{18–33}$$

In the general case, when the penalty for not meeting demand is expressed in the form $c_2 = c'_2 + \frac{1}{2}c''_2t''$ per unit, the solution for the optimal batch size can be shown to be at

$$F = \frac{1 + \zeta - \epsilon\tau[1 + (1/\epsilon_2)]}{1 + \zeta - \epsilon\tau[1 + (1/\epsilon_2)] + \epsilon} \tag{18–34}$$

where the definitions of ϵ, ϵ_2 are given by Eq. 18–25. As in the case of continuous demand, the general solution includes as special cases the optimal batches for the minimum-cost and the maximum-profit criteria; namely, the solutions given by Eqs. 18–29, 18–31, 18–32, and 18–33.

Stocking of Perishables

Stocking and replenishment of perishables, when the reordering cycle is used, confront us with some special problems. At the beginning of a period the store is stocked to a certain level, and during this period consumption takes place. If we run out of stock before the end of the period, any subsequent demand in that period can obviously not be satisfied. If we stock too much, we run the risk of having a residue in stock at the end of the period. This residue may in some situations be a total loss; for instance, in such cases as newspapers, journals, foodstuffs, certain chemicals, pharmaceutical goods, and photographic materials.

These products are *perishables*. They carry their full value throughout the cycle but become virtually worthless beyond a certain dead line. Some products lose a substantial part of, but not all, their value when they are carried over the dead line. These may be termed *semiperishables*. Certain kinds of books, style goods, consumer goods, such as household appliances and automobiles, sometimes spare parts—all these belong to the semiperishable class.

Take the case of an automobile: It will fetch its full price as long as newer models are not introduced into the market, but after they appear, it becomes "an old model" and has to be sold at a reduced price. The life of these semiperishables is often longer than the inventory cycle, so that for several cycles the carryover from cycle to cycle involves only carrying charges, as in the cases discussed in previous sections. After a certain number of inventory cycles, the product is relegated to a lower price level, and the difference between the original price and the lower price constitutes the loss due to overstocking.

Example 1

A typical example in the stocking of perishables is that of newspapers and journals. A newspaper can be sold for the full price printed on it for one day only. On the day after, its news has become stale, and although it carries perhaps many features and articles that maintain their value for several days to come, the paper as an article for sale has become valueless.

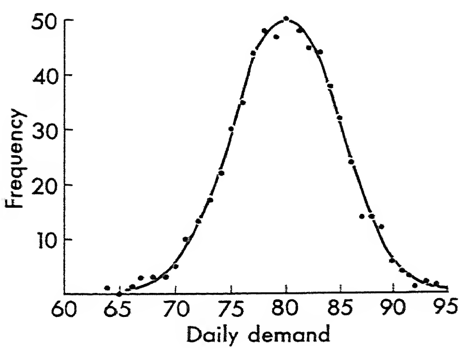

Figure 18–3. ***Demand frequency for newspapers.***

Suppose that a news agent had a full record of the demand (= actual sales + demand that he could not meet) of a certain paper, as shown in Table 18–1; the frequency of demand could be plotted as in Fig. 18–3. The distribution in this case is quite close to the normal one. The average demand was found to be 80 papers a day with a maximum recorded daily demand of 95 and a minimum of 64 papers. Suppose the news agent sells each paper for 10 cents and has to pay 7 cents for each that he orders. Assuming that he cannot return unsold papers (whether this is a wise policy from the publisher's point of view is another matter),

the news agent makes 3 cents on every paper that he sells, he misses an opportunity to gain 3 cents on every paper that he cannot supply, and loses 7 cents on every paper he is unable to sell. How many papers, then, should he order, assuming no periodical demand fluctuations occur? He decided to take the average demand as his order quantity (namely, order 80 papers per day), and in this way (see Table 18–2) he

sold 48,242 papers, yielding a profit of $1,447.26;
was unable to supply 1,215 papers, losing a potential gain of $36.45;
had 1,198 papers left over, losing $83.86;

Table 18–1

Demand Record for a Daily Newspaper

No. of Papers	*Frequency of Demand*	*Total Number Required*
64	1	64
65	0	0
66	1	66
67	3	201
68	3	204
69	3	207
70	5	350
71	10	710
72	13	936
73	17	1,241
74	22	1,628
75	30	2,250
76	35	2,660
77	44	3,388
78	48	3,744
79	47	3,713
80	50	4,000
81	48	3,888
82	45	3,690
83	44	3,652
84	38	3,192
85	32	2,720
86	24	2,064
87	14	1,218
88	14	1,232
89	12	1,068
90	6	540
91	4	364
92	1	92
93	2	186
94	1	94
95	1	96
Total	618	49,457

$$\text{Average demand} = \frac{49{,}457}{618} = 80.03$$

his total gain being $1,363.40. Had he decided to stock 76 papers a day, he would have

sold 46,617 papers, yielding a profit of $1,398.51;
been unable to supply 2,840 papers, losing a potential gain of $85.20;
had 351 papers left over, losing $24.57;

thereby increased his earnings to $1,373.94. Admittedly, the increase is only by about 1 per cent, but the example illustrates how the news agent could affect the total profit by adjusting his order quantity level.

His considerations have so far not included any penalty for turning away customers, whose wishes cannot be met. This point, naturally, requires some further study to answer the question: Does the fact that not all customers can be satisfied all the time affect the pattern of demand? If it does not, the demand distribution is not expected to change owing to readjustment of the order quantity and no penalty for dissatisfied customers need be considered, but if it does have an adverse effect on demand, this aspect should not be overlooked. In some industries a thorough analysis to establish this penalty quantitatively may be called for.

Example 2

A slightly different situation would arise when our news agent is compensated by the publisher for unsold papers. For instance, the publisher could charge the news agent 7 cents for each copy sold but only 4 cents for each copy that the news agent returned. In this way the publisher may hope to encourage the news agent to order more papers and thereby increase circulation. The problem, then, belongs to the class of semiperishables, since the news agent does not lose all the value of the paper at the end of the selling period. Admittedly, this is a special and perhaps a comparatively simple case of semiperishables, since the goods are not offered to the public and the leftovers are therefore not likely to affect the pattern of demand. The reader will not fail to notice that the problems of the news agent and those of the publisher are different. The news agent would naturally welcome any reduction in the price he pays for unsold papers, since in this way he can afford to order more without taking too high a risk. The publisher is also interested in larger orders, but only up to a point. If he does not charge the news agent anything for unsold papers, the latter would tend to put in orders equivalent to the maximum possible demand (perhaps even higher than that), and the publisher would find that he got back large numbers of papers, of which he could make very little use. Unless it is definitely proved that abundance of the papers on the news stands does stimulate demand, the publisher will start losing if he has too lenient a policy toward the news agent. The optimal policy of the publisher would also belong to the class of these problems.

Throughout this discussion, the underlying assumption has been that we know, or the news agent knows, the distribution of demand, and having assembled our figures, we strive to define the best course of action that ought to be taken. We

Table 18–2

Profits and Losses for Two Policies of Stocking Newspapers

Demand (No. of papers)	*Frequency of demand*	*Policy I: Order 80 Papers* No. sold	*Policy I* No. not supplied	*Policy I* Leftovers	*Policy II: Order 76 Papers* No. sold	*Policy II* No. not supplied	*Policy II* Leftovers
64	1	64 × 1 = 64	0	(80 − 64) × 1 = 16	64	0	12
66	1	66 × 1 = 66	0	(80 − 66) × 1 = 14	66	0	10
67	3	67 × 3 = 201	0	(80 − 67) × 3 = 39	201	0	27
68	3	68 × 3 = 204	0	36	204	0	24
69	3	69 × 3 = 207	0	33	207	0	21
70	5	350	0	50	350	0	30
71	10	710	0	90	710	0	50
72	13	936	0	104	936	0	52
73	17	1,241	0	119	1,241	0	51
74	22	1,628	0	132	1,628	0	44
75	30	2,250	0	150	2,250	0	30
76	35	2,660	0	140	2,660	0	0
77	44	3,388	0	132	3,344	44	0
78	48	3,744	0	96	3,648	96	0
79	47	3,713	0	47	3,572	141	0
80	50	4,000	0	0	3,800	200	0
81	48	3,840	1 × 48 = 48	0	3,648	240	0
82	45	3,600	2 × 45 = 90	0	3,420	270	0
83	44	3,520	3 × 44 = 132	0	3,344	308	0
84	38	3,040	4 × 38 = 152	0	2,888	304	0
85	32	2,560	160	0	2,432	288	0
86	24	1,920	144	0	1,824	240	0
87	14	1,120	98	0	1,064	154	0
88	14	1,120	112	0	1,064	168	0
89	12	960	108	0	912	156	0
90	6	480	60	0	456	84	0
91	4	320	44	0	304	60	0
92	1	80	12	0	76	16	0
93	2	160	26	0	152	34	0
94	1	80	14	0	76	18	0
95	1	80	15	0	76	19	0
		Total 48,242	1,215	1,198	46,617	2,840	351

also assume that the pattern of demand in the future is not liable to change. In fact, we take the past demand distribution and suggest by inference that it represents the distribution in the future. Although in many cases (especially on a short-term basis) this assumption is adequate, its limitations should always be borne in mind.

Example 3

There are cases, however, that call for an even higher level of forecasting and evaluation, especially when direct data collection is impossible. Suppose we have to build a theater and a decision regarding its seating capacity has to be made. We can construct our break-even charts to determine the minimum economic capacity that need be contemplated, but that still does not tell us very much about the size that should finally be adopted. The seating capacity in this case is our "order quantity." Seats that remain vacant are analogous to left-over goods that cannot be sold. On the other hand, if the theater is full, the management is obliged to turn away potential customers, and each customer represents a demand for one unit of commodity that cannot be supplied. The management of the theater collects a profit for every seat or ticket sold but suffers a loss (due to upkeep, maintenance, and overhead) for each seat that remains vacant. This is, therefore, an inventory problem belonging to the class of perishable goods, with the same seats constituting the fresh stock as each performance starts.

Problems dealing with the size of stores, reservoirs, or containers are also essentially of this kind; some deal with perishables only, others have to incorporate the question of additional space required for left-overs or semiperishables. In all these cases, however, we find ourselves in a vicious circle: We need the theater or store to find the demand distribution, but we need the demand distribution to determine the required capacity. Sometimes it is possible to suggest an interim solution for the capacity and modify the building at a later stage when enough data have been collected and analyzed, but in most cases it is necessary to infer from existing stores the form of expected demand distribution, on which determination of size has to be based.

The desirable capacity of modes of transportation, such as trains, trucks, buses, or ships, or the capacity of each of a number of storehouses, compartments in a silo, etc., is also allied to this problem, but apart from the optimal size of each unit, it is necessary in most of these problems to determine the number of units required. A typical industrial problem belonging to this class would be: determining the capacity of each machine and the number of machines required to cater to a given distribution of loads. This problem is somewhat more complicated than the one discussed above and is beyond the scope of this work.

Optimal Order Quantity for Normal Demand Distribution

For the purpose of our analysis we may start by looking upon perishables and semiperishables as belonging to the same category. For any leftovers at the end of the period, there is a penalty or a loss c_1 per unit. In the case of perishables,

c_1 would be equivalent to the full value of the product, including storage costs. In the case of semiperishables, c_1 would be only that part of the full value which is lost when the product is transferred to the next cycle. Thus perishables and semiperishables differ in the numerical value attached to the penalty c_1.

There is, however, a fundamental difference between the two groups. In the case of perishables, the leftovers are simply scrapped and the stock level at the beginning of the period is equal to the reorder quantity. In the case of semiperishables, the stock level at the beginning of the period consists of the reorder quantity and the residue from the preceding period. The old stock also affects the sales of fresh stock during the new cycle, and thereby the profit. If, for example, an appliance of an older model is sold to a customer at a reduced price, it may be done at a risk of not selling him a new model, which would have fetched the full profit. Thus, while it is probably true to say that the consumption of old stock is partially an addition to the normal demand (being inspired by the attractive low prices), the actual sales of new stock may be reduced because of competition from old stock. Reduction in sales of new stock results in a larger residue at the end of the cycle, i.e., in a larger amount transferred as old stock to the next cycle. Such a situation calls for an analysis of the quantitative effect of old stock on profit and on reorder quantities.

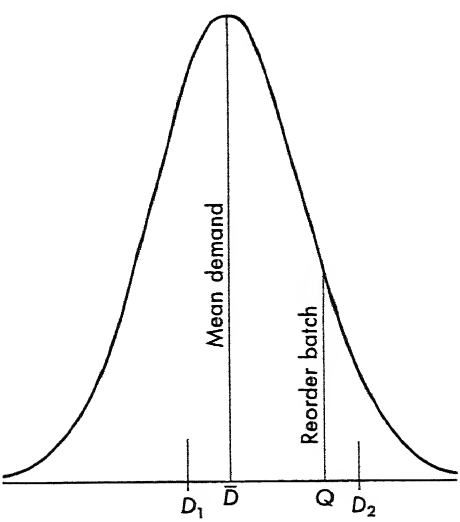

Figure 18–4. ***Demand following the normal distribution.***

$\bar{D}$ = Mean demand

Q = Reorder batch

D_1 = Mean demand value that can be satisfied

D_2 = Mean demand value that cannot be satisfied

Minimizing loss

Suppose that the distribution density function for demand $f(D)$ is known, the mean demand being $\bar{D}$. If we plan to reorder Q (see Fig. 18–4), the probability that demand will be below Q is

$$\int_0^Q f(D)\, dD = F$$

and the probability that demand cannot be satisfied ($D > Q$) is

$$\int_Q^{\infty} f(D)\, dD = 1 - F$$

If the average demand for $D < Q$ is D_1, the number of units expected to remain in stock at the end of the period is $(Q - D_1)F$. When c_1 is the loss per unsold unit, the expected loss for $(Q - D_1)$ units is

$$F(Q - D_1)c_1$$

If the average demand for $D > Q$ is D_2, the number of units that cannot be supplied is expected to be $(1 - F)(D_2 - Q)$, and if the loss per each unit that cannot be supplied is c_2, the expected loss for $D > Q$ is

$$(1 - F)(D_2 - Q)c_2$$

Hence, the total loss:

$$C = F(Q - D_1)c_1 + (1 - F)(D_2 - Q)c_2 \tag{18–35}$$

where D_1, D_2 can be evaluated (when the probability density function is known) by:

$$D_1 \int_0^Q f(D)\, dD = \int_0^Q Df(D)\, dD \tag{18–36}$$

and

$$D_2 \int_Q^{\infty} f(D)\, dD = \int_Q^{\infty} Df(D)\, dD \tag{18–37}$$

Let us take, for example, the case when demand may be described by the normal curve, so that

$$f(D) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{D-\bar{D}}{\sigma}\right)^2} \tag{18–38}$$

where σ is the standard deviation. Case studies in industry have shown that the normal distribution can often be taken as an adequate approximation to describe the demand pattern. When we denote

$$\frac{D - \bar{D}}{\sigma} = t$$

then

$$f(D) = \frac{1}{\sigma}\varphi(t)$$

where

$$\varphi(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} \tag{18–39}$$

and the cumulative probability to have a value $t \leqslant t_1$ is

$$\Phi_1 = \int_{-\infty}^{t_1} \varphi\, dt = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t_1} e^{-\frac{1}{2}t^2} dt \tag{18–40}$$

φ and Φ are extensively tabulated in books on statistics (see also Appendix).

Although theoretically the normal distribution function extends from $-\infty$ to $+\infty$ (thus including some absurd demand values), it is still a very useful model to use, since within the range of $\pm 3\sigma$ from the mean value (i.e., $t = \pm 3$), we have 99.7 per cent of the total area under the curve. Provided $\bar{D} > 3\sigma$, the negative portion of the curve is so small that it may be neglected. We may therefore use the values of Φ as derived from tables (e.g., the Appendix) and not worry too much about the fact that Φ is the cumulative probability from $-\infty$, whereas we are actually interested in the cumulative probability from 0 (the demand being always positive):

$$F \simeq \Phi = \int_{-\infty}^{t_Q} \varphi \, dt$$

where

$$t_Q = \frac{Q - \bar{D}}{\sigma}$$

Hence the loss is

$$C = \Phi(Q - D_1)c_1 + (1 - \Phi)(D_2 - Q)c_2 \tag{18-41}$$

In order to find D_1, D_2, suppose we have two ordinates on the normal curve and we have to find the average value between the two limits (see Fig. 18-5).

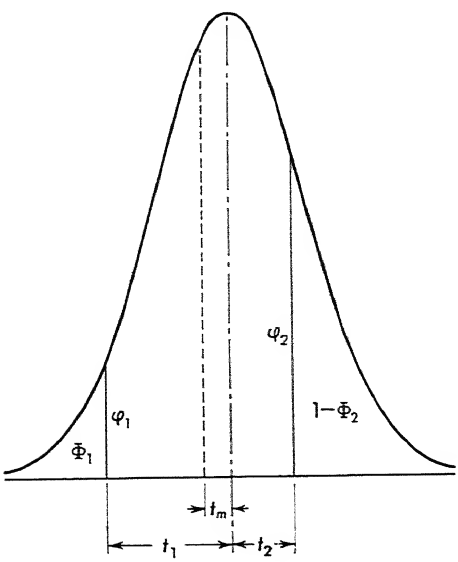

Figure 18-5. ***A standardized normal distribution.***

This average, call it t_m, is related to the abscissa of the center of gravity of the area between 1 and 2:

$$t_m \int_{t_1}^{t_2} \varphi \, dt = \int_{t_1}^{t_2} t\varphi \, dt$$

or

$$t_m \frac{1}{\sqrt{2\pi}} \int_{t_1}^{t_2} e^{-\frac{1}{2}t^2} \, dt = \frac{1}{\sqrt{2\pi}} \int_{t_1}^{t_2} t e^{-\frac{1}{2}t^2} \, dt$$

By definition $\quad \dfrac{1}{\sqrt{2\pi}}\displaystyle\int_{t_1}^{t_2} e^{-\frac{1}{2}t^2}\,dt = \dfrac{1}{\sqrt{2\pi}}\left[\int_{-\infty}^{t_2} e^{-\frac{1}{2}t^2}\,dt - \int_{-\infty}^{t_1} e^{-\frac{1}{2}t^2}\,dt\right]$

$$= \Phi_2 - \Phi_1$$

$$\therefore \qquad t_m(\Phi_2 - \Phi_1) = \frac{1}{\sqrt{2\pi}}\int_{t_1}^{t_2} te^{-\frac{1}{2}t^2}\,dt$$

$$= \frac{1}{\sqrt{2\pi}}\int_{t_1}^{t_2} e^{-\frac{1}{2}t^2}d(\tfrac{1}{2}t^2) = -\frac{1}{\sqrt{2\pi}}\left[e^{-\frac{1}{2}t_2^2} - e^{-\frac{1}{2}t_1^2}\right]$$

$$= -(\varphi_2 - \varphi_1)$$

or

$$t_m = -\frac{\varphi_2 - \varphi_1}{\Phi_2 - \Phi_1} \tag{18–42}$$

To find the average for $D < Q$, we have to look at the position of the normal curve from $-\infty$ to Q; i.e., $\varphi_1 = 0$, $\Phi_1 = 0$, or

$$t_{m_1} = \frac{D_1 - \bar{D}}{\sigma} = -\frac{\varphi}{\Phi}$$

$$\therefore \qquad D_1 = \bar{D} - \frac{\varphi}{\Phi}\sigma \tag{18–43}$$

Similarly, for $D > Q$, we have the two limits at Q and $+\infty$, so that $\varphi_2 = 0$, $\Phi_2 = 1$, and

$$t_{m_2} = \frac{D_2 - \bar{D}}{\sigma} = \frac{\varphi}{1 - \Phi}$$

or

$$D_2 = \bar{D} + \frac{\varphi}{1 - \Phi}\sigma \tag{18–44}$$

By substituting Eqs. 18–43 and 18–44 into the loss function, Eq. 18–41, we get

$$C = \Phi\left(Q - \bar{D} + \frac{\varphi}{\Phi}\sigma\right)c_1 + (1 - \Phi)\left(\bar{D} + \frac{\varphi}{1 - \Phi}\sigma - Q\right)c_2 \tag{18–45}$$

If now we denote $(Q - \bar{D})/\sigma = t$, then (18–46)

$$\frac{C}{\sigma} = (\Phi t + \varphi)c_1 + [\varphi - t(1 - \Phi)]c_2$$

$$= -c_2 t + (\varphi + \Phi t)(c_1 + c_2)$$

We define a function

$$E = \frac{C}{\sigma c_2} \tag{18–47}$$

Hence
$$E = -t + (\varphi + \Phi t)(1 + \epsilon) \tag{18–48}$$

where
$$\epsilon = \frac{c_1}{c_2} \tag{18–49}$$

where ϵ is the cost ratio for the two types of losses that we may expect, and its numerical value will naturally affect the loss function E. For the special case $\epsilon = 0$ (i.e., $c_1 = 0$, meaning that we do not lose anything by having surplus at the end of the period; for instance, in some cases where we have the option to return unsold goods to the vendor without loss to ourselves):

$$E_0 = \varphi - t(1 - \Phi) \tag{18–48a}$$

and in the special case $\epsilon = 1$ (i.e., $c_1 = c_2$, meaning that the loss per unit remaining in stock is the same as the loss per unit that we are unable to supply):

$$E_1 = -t + 2(\varphi + \Phi t) \tag{18–48b}$$

In cases where $0 < \epsilon < 1$, the loss function E would be between E_0 and E_1. Figure 18–6 gives values for E in terms of t with ϵ as a parameter. There are, however, cases where the loss per unsold unit is higher than the loss per unsupplied one, i.e., $\epsilon > 1$.

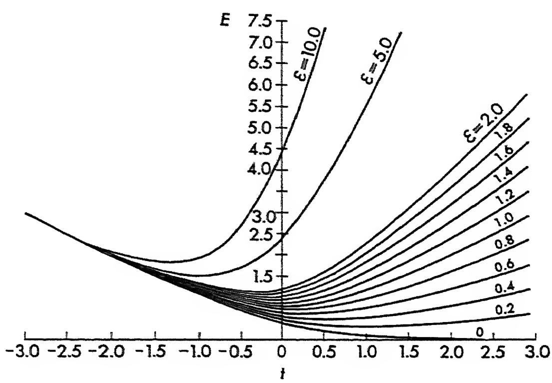

Figure 18–6. ***The cost function E.*** (From S. Eilon, "Inventory Control: A Problem in Stocking Perishable Goods," The Production Engineer, April 1960)

What Q should be planned in order to reduce the expected loss function to a minimum? C is minimum when E is minimum; i.e., when

$$\frac{dE}{dt} = -1 + (1 + \epsilon)\left(\frac{d\varphi}{dt} + t\frac{d\Phi}{dt} + \Phi\right) = 0$$

But, by definition,

$$\frac{d\Phi}{dt} = \varphi$$

and
$$\frac{d\varphi}{dt} = \frac{d}{dt}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}t^2} = -t\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}t^2} = -t\varphi$$

$\therefore$
$$\frac{dE}{dt} = -1 + (1+\epsilon)(-t\varphi + t\varphi + \Phi) = 0$$

or
$$\Phi = \frac{1}{1+\epsilon} \tag{18-50}$$

The solution is shown in Fig. 18–7. This is, in fact, a special case in instantaneous demand. A quantity D is required during the cycle, and since no carrying or storage costs are charged as a function of the time the goods stay in stores, we may regard the consumption as a single withdrawal at the beginning of the cycle. Expression 18–50 is therefore obtainable from Eq. 18–29 when $\tau = 0$.

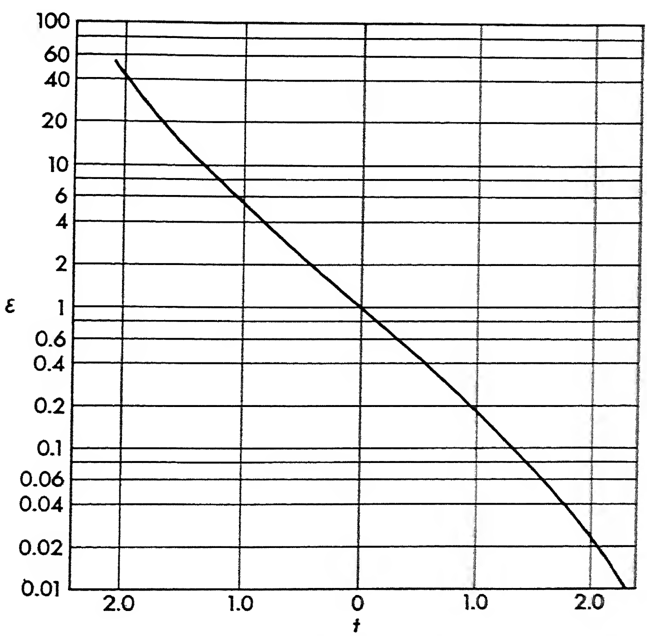

Figure 18–7. ***Optimum solution for stocking of perishable goods.*** (From S. Eilon, "Inventory Control: A Problem in Stocking Perishable Goods," The Production Engineer, April 1960)

Maximizing profit

Similarly, it can be shown that if maximum profit is the desirable objective in stocking of perishables, the optimal reorder quantity is given by

$$\Phi = \frac{1 + \zeta}{1 + \zeta + \epsilon} \tag{18–50a}$$

where $\zeta = z/c_2$ and z is the difference between the sales price and cost of acquisition per unit (the reader can derive this solution as an exercise). The optimal solution, Eq. 18–50, is, therefore, a special case of Eq. 18–50a, obtained when $\zeta = 0$. Figure 18–7 may still be used for this case, since

$$\Phi = \frac{1}{1 + [\epsilon/(1 + \zeta)]}$$

except that ϵ is replaced by $\epsilon/(1 + \zeta)$

Example

The probable demand for a product is given by a normal distribution with a mean expected demand of 1,000 units during the period and a standard deviation of 80 units. If the loss per unsold unit is \$1.04 and the loss per unit that cannot be supplied is \$8.00, what quantity should be ordered to minimize loss and what is the cost for this case?

Solution

$$c_1 = \$1.04$$

$$c_2 = \$8.00$$

$$\therefore \qquad \epsilon = \frac{1.04}{8.00} = 0.13$$

The optimal policy is at

$$\Phi = \frac{1}{1 + \epsilon} = \frac{1}{1.13} = 0.885$$

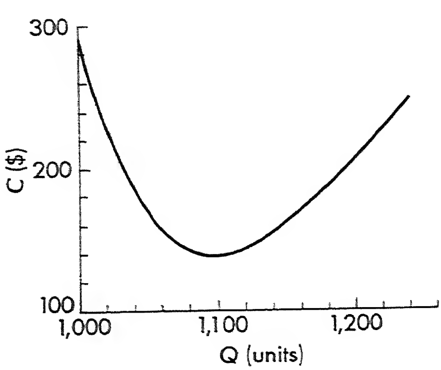

Figure 18–8. ***An example to illustrate how the cost function C varies with the reorder batch Q.***

This value of Φ corresponds to (from tables, see Appendix):

$$t = 1.2$$

But
$$t = \frac{Q - \bar{D}}{\sigma}$$

$$\therefore \qquad Q = \bar{D} + t\sigma = 1{,}000 + 1.2 \times 80 = 1.096 \simeq 1{,}100 \text{ units}$$

The cost of this policy is

$$C = \sigma[-c_2t + (\dot{\varphi} + \Phi t)(c_1 + c_2)]$$

From the Appendix table, $\varphi = 0.194$.

$$\therefore \qquad C = 80[-8.00 \times 1.2 + (0.194 + 0.885 \times 1.2)(1.04 + 8.00)]$$
$$= \$140$$

This cost increases quite appreciably as deviations from the optimal policy increase (see Fig. 18–8).

Determining the level of satisfaction

The main difficulty in practice is to assign a value to c_2. As mentioned above, c_2 consists of:

1. The loss of profit per unit that we are unable to supply, which (provided we know for sure that the customer is not prepared to wait until new stock arrives) is easy to establish.
2. The loss of good will, which as explained later, is not that easy to express quantitatively.

In order to avoid the issue, management sometimes states the average level of satisfaction that should be aimed at. The level of satisfaction specifies the percentage of the times in which demand can be met. For instance, if the policy requires that the level of satisfaction be 99 per cent, then only in 1 per cent of the cases are we prepared to put up with the situation in which demand exceeds the stock. The issue of value assignment is, in fact, not avoided in this way, since the decision to specify a certain level of satisfaction automatically puts a price tag on c_2. In many cases this is a more convenient way for management to express its desirable policy, but at the same time it is useful to know just how much this policy costs and whether the cost function is steep in the specified region, in which case the policy may intelligently be reconsidered.

Example

For the data given in the previous example, suppose that instead of specifying c_2, management rules that we should aim to meet demand 98 per cent of the time. Find the c_2 to which this policy is equivalent, what it costs to implement it, and how much should be ordered.

Solution

Here $\Phi = 0.980$. Corresponding to $t = 2.05$ (from Appendix table) and $\varphi = 0.048$, from

$$\Phi = \frac{1}{1+\epsilon}$$

we have $$\epsilon = \frac{1}{\Phi} - 1 = \frac{1}{0.98} - 1 = 0.0204$$

and $$c_2 = \frac{c_1}{\epsilon} = \frac{1.04}{0.0204} = \$50.98 \simeq \$\$51.0$$

The cost is

$$\begin{aligned} C &= \sigma[-c_2 t + (\varphi + \Phi t)(c_1 + c_2)] \\ &= 80[-51 \times 2.05 + (0.048 + 0.98 \times 2.05)(1\ .04 + 51.0)] \\ &= \$200 \end{aligned}$$

And the amount that should be ordered is

$$Q = \bar{D} + t\sigma = 1{,}000 + 2.05 \times 80 = 1{,}164 \simeq 1{,}160 \text{ units}$$

The reorder quantity is naturally somewhat larger than in the preceding example because the penalty for not supplying is higher. It is interesting to note that the cost of maintaining the described policy and the reorder quantities are quite sensitive functions when the level of satisfying demand is high (as illustrated in Fig. 18–9, where the cost C and the reorder quantity Q are given for various values of ϵ). When σ is comparatively high, the recorder quantity becomes even more sensitive to changes in policy.

Apportioning Problems

There are numerous situations in industry and trade where a certain commodity is ordered or produced and then divided into portions, these portions being required for further operations in the manufacturing process or for marketing as a finished product. Packaging problems fall within this category: The commodity, when in the form of powder, for instance, is weighed and packed in packets, cartons, or sacks; when it is in liquid form, it is poured into bottles or containers, often by volume specifications. These packets, cartons, or bottles are supplied to the market as the final product, and they have to meet predetermined specifications defining weight, volume, or any other property (cutting paper or cloth, for instance, would be governed by dimensional specifications).

Variability

Apportioning operations normally involve a certain variability of the quantity allotted to each unit. As we often find that reduction or practical elimination of

this variability is rather costly, we have to put up with some deviations from the desirable specified quantity. If the number of units of the final product is known and the penalty for not meeting the specifications is given, how much of this commodity should be ordered or made for each production cycle? We must strive to determine the optimal setting of the process that would yield such an average quantity per product unit that the over-all cost or waste will be minimized. The problem becomes more complex when either or a combination of the following situations occur.

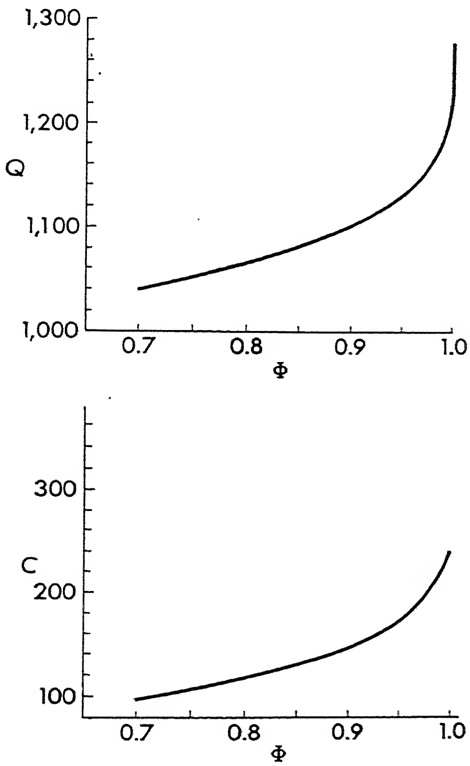

Figure 18–9. ***An example to illustrate the effect of level of satisfaction Φ on the reorder batch Q and the cost function C.***

Legal restrictions

Some specifications are laid down by law; for instance, minimum weights of packaged or bottled foodstuffs. Since the penalty for underweight units is high, the manufacturer tends to increase the average amount per unit as a safety measure. On the other hand, he is reluctant to increase the average too much, both because material is thereby wasted and because it may reduce potential demand for a second purchase.

Measurement limitations

In many cases the specifications for ordering the commodity and for marketing the final product differ in the property selected for measurement purposes. In rolling, extrusion, and similar processes, the inlet may be measured by weight, whereas the outlet (or final product) may be defined by dimensions of width or length. In such cases, variations of specific gravity during the process (as in rolling or extrusion) or gage (as in production of paper or metal sheets) have to be considered.

Deviations from specified packaged weights

As we have pointed out, there is an inherent variability in the actual apportioning operation. If, for instance, we have to produce 1 pound packages of a commodity, we find that the packages vary in weight, and numerous case studies conducted in industry suggest that the distribution of weights resembles the normal curve.

If the apportioning process is so adjusted as to yield a mean package weight of 1 pound, half the packages are expected to be overweight and half are expected to be underweight. The underweight packages do not meet the requirements. They may have to be sold at a lower price, or perhaps be opened in order to be repackaged again (for instance, when marketing packages below 1-pound weight is forbidden, or undesirable from the point of view of maintaining the quality of the product, so that marketing underweight packages should be avoided). In some cases underweight packages may simply have to be scrapped. In any event, a loss due to underweight is involved. The overweight packages are marketable, but again a loss is incurred by putting too much material in each packet. The question arises: How can the total loss be minimized? We can reduce the loss by using one or combinations of the following three methods.

1. We may try to reduce the variability of the process, either by change of equipment or by periodical checks through machine setting, maintenance, and repair (this problem is further discussed in Chapter 19). In this way the tails of the distribution curve are pruned down, and a higher precentage of the production volume can be brought within prespecified tolerances (see Fig. 19–9).

2. An appropriate inspection mechanism could be installed, which would let through only those products that are within the desirable tolerances. This would ensure against the penalties for underweights.

3. For a given process variability, the loss may be reduced by process adjustment, i.e., by changing the mean, until an optimal proportion between oversized and undersized products is attained. Deviations from this optimal setting would involve an increase in the loss function, either due to excessive surplus material input (when the mean is too high) or to too many rejects of undersized products.

Process adjustment

We shall now proceed to analyze mathematically the question of process adjustment. Suppose the mean of the distribution is $\bar{x}$, while the amount specified for each product is x_0, and suppose the process is adjusted in such a way that the probability of obtaining an underweight product is 0.15 per cent; there would be an average excess of material of $\bar{x} - x_0$ in each package. If N packages or portions are made, the total amount of material required to produce N units is $N\bar{x}$, so that the loss ratio is

$$S_0 = \frac{N(\bar{x} - x_0)}{N\bar{x}} = 1 - \frac{x_0}{\bar{x}} \tag{18–51a}$$

The probability of 0.15 per cent for underweight units corresponds to

$$\bar{x} - x_0 = 3\sigma$$

or

$$\frac{x_0}{\bar{x}} = 1 - \frac{3\sigma}{\bar{x}} = 1 - \frac{3}{\gamma}$$

where $\gamma = \bar{x}/\sigma$ is the inverse of the coefficient of variation, which gives us some measure of the dispersion of the distribution. Hence

$$S_0 = \frac{3}{\gamma} \tag{18–51b}$$

If the distribution is shifted to the right so that the mean weight per unit increases, the chances of obtaining underweight units are further reduced, but the amount of excess material used is increased and thus the loss ratio evidently increases. If the distribution is shifted to the left so that the probability for obtaining underweight packages is Φ, the loss would be

Amount of loss due to underweights $= N\Phi x_1$
where x_1 is the average amount lost per packet.

Amount of loss due to overweights $= N(1 - \Phi)(x_2 - x_0)$
where x_2 is the average weight of the overweight packages.

If underweight products are a total loss and have to be scrapped, the total amount of material lost is $N[\Phi x_1 + (1 - \Phi)(x_2 - x_0)]$, while the total amount of material input is $N\bar{x}$. The scrap ratio is

$$S = \frac{N[\Phi x_1 + (1 - \Phi)(x_2 - x_0)]}{N\bar{x}} = \Phi \frac{x_1}{\bar{x}} + (1 - \Phi) \frac{x_2 - x_0}{\bar{x}} \tag{18–52}$$

If

$$t_{x_1} = \frac{x_1 - \bar{x}}{\sigma}$$

$$t_{x_2} = \frac{x_2 - \bar{x}}{\sigma}$$

then, by Eq. 18–42,

$$t_{x_1} = -\frac{\varphi}{\Phi}$$

$$t_{x_2} = \frac{\varphi}{1 - \Phi}$$

or

$$\left.\begin{aligned} \frac{x_1}{\bar{x}} &= 1 - \frac{\varphi}{\Phi}\frac{1}{\gamma} \\ \frac{x_2}{\bar{x}} &= 1 + \frac{\varphi}{1 - \Phi}\frac{1}{\gamma} \end{aligned}\right\} \qquad (18\text{–}53)$$

Define

$$\left.\begin{aligned} t &= \frac{x_0 - \bar{x}}{\sigma} \\ \text{or} \quad \frac{x_0}{\bar{x}} &= 1 + \frac{t}{\gamma} \end{aligned}\right\} \qquad (18\text{–}54)$$

By substituting these expressions into the scrap ratio function, we get

$$\begin{aligned} S &= \Phi\left(1 - \frac{\varphi}{\Phi}\frac{1}{\gamma}\right) + (1 - \Phi)\left(\frac{\varphi}{1 - \Phi}\frac{1}{\gamma} - \frac{t}{\gamma}\right) \\ &= \Phi - (1 - \Phi)\frac{t}{\gamma} \end{aligned} \qquad (18\text{–}55)$$

How can this function be minimized by selection of $\bar{x}$ (obtained through an appropriate adjustment of this process), when the requirement x_0 is given? The answer to this question is obtained by

$$\frac{dS}{dt} = 0$$

From Eq. 18–54 we get

$$t = m - \gamma \qquad (18\text{–}56)$$

where

$$m = \frac{x_0}{\sigma} \quad \text{is known} \qquad (18\text{–}57)$$

$$\therefore \qquad \frac{t}{\gamma} = \frac{m}{\gamma} - 1$$

By substituting this expression into Eq. 18–55:

$$\begin{aligned} S &= \Phi - (1 - \Phi)\left(\frac{m}{\gamma} - 1\right) \\ &= 1 - (1 - \Phi)\frac{m}{\gamma} \end{aligned} \qquad (18\text{–}58)$$

If changes in $\bar{x}$ do not cause any changes in σ, the ratio m is constant for any given situation. An example showing the scrap ratio function is given in Fig. 18–10. The function becomes a minimum when

$$\frac{dS}{d\gamma} = \frac{m}{\gamma^2}(1 - \Phi) + \frac{m}{\gamma}\frac{d\Phi}{d\gamma} = 0$$

But

$$\frac{d\gamma}{dt} = -1$$

from Eq. 18–56, and

$$\frac{d\Phi}{dt} = \varphi$$

$$\frac{d\Phi}{d\gamma} = -\varphi$$

and

$$\frac{m}{\gamma^2}(1 - \Phi) - \frac{m}{\gamma}\varphi = 0$$

or

$$\gamma = \frac{1 - \Phi}{\varphi} \tag{18–59}$$

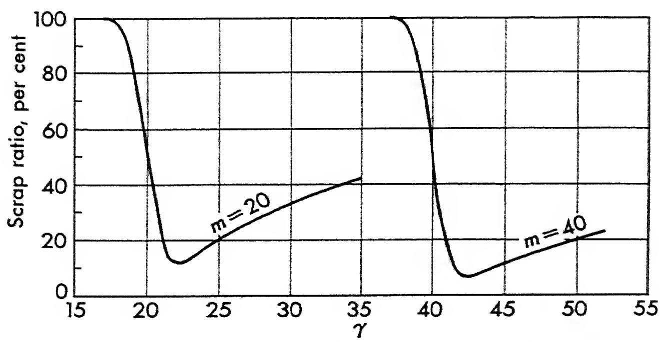

Figure 18–10. ***The scrap function (for γ=constant).***

The minimum value of the scrap ratio function is obtained by substituting the optimal γ into the scrap ratio function, yielding

$$S_m = 1 - m\varphi \tag{18–60}$$

Example

A liquid is bottled and sealed in containers that should not hold less than 100 grams of the material. The unsealing of the containers may adversely affect the properties of the liquid, so that underweight containers have to be thrown away as scrap. A study of the operation showed that the standard deviation of the distribution of weights was 8 grams and the mean weight was set at 120 grams. The underweight products were sifted from the finished product stores

and discarded through 100 per cent inspection. Disregarding the loss due to containers, and also the labor associated with the underweight products, find:

1. The scrap ratio for the above situation.
2. The optimal setting that would reduce the scrap ratio to a minimum.
3. What saving may be expected at the optimal setting.

Solution

1. Given $x_0 = 100$; $\bar{x} = 120$; $\sigma = 8$. Therefore

$$t = \frac{x_0 - \bar{x}}{\sigma} = \frac{100 - 120}{8} = -2.5$$

and

$$\gamma = \frac{\bar{x}}{\sigma} = \frac{120}{8} = 15$$

For $t = -2.5$, from the Appendix table,

$$\Phi = 0.006$$

$$1 - \Phi = 0.994$$

$$\varphi = 0.0175$$

$$\therefore \qquad S = \Phi + (1 - \Phi)\frac{t}{\gamma} = 0.006 + 0.994\frac{2.5}{15} = 0.172, \quad \text{or } 17.2\%$$

2. We know that

$$m = \frac{x_0}{\sigma} = \frac{100}{8} = 12.5$$

The optimal setting can be found through solution of Eq. 18-59 by trial and error:

First trial. First assume that the answer corresponds to $t = -2.0$, or

$$\gamma = m - t = 12.5 + 2.0 = 14.5$$

For $t = -2.0$ we find from the Appendix table:

$$1 - \Phi = 0.977$$

$$\varphi = 0.0540$$

Checking for γ by formula 18-59,

$$\frac{1 - \Phi}{\varphi} = \frac{0.977}{0.054} = 18.1 \neq \text{our assumption of } 14.5$$

Second trial. Assume $t = -1.8$ or $\gamma = 12.5 + 1.8 = 14.3$. Now

$$1 - \Phi = 0.964$$

$$\varphi = 0.0790$$

Check:
$$\frac{1 - \Phi}{\varphi} = \frac{0.986}{0.0355} = 12.2 < 14.3$$

The answer must lie between $t = -2.0$ and -1.8

Third trial. Assume $t = -1.9$ or $\gamma = 12.5 + 1.9 = 14.4$. Now

$$1 - \Phi = 0.971$$

$$\varphi = 0.0656$$

Check:
$$\frac{1 - \Phi}{\varphi} = \frac{0.971}{0.0656} = 14.8 > 14.4$$

but the difference is now fairly small, and a small change in t (in the second decimal place) will hardly affect the last assumption, $\gamma = 14.4$, and the value of $1 - \Phi = 0.971$. We can, of course, find t quite accurately from

$$\varphi = \frac{1 - \Phi}{\gamma} = \frac{0.971}{14.4} = 0.0675$$

which corresponds (see Appendix table) to $t = -1.88$ or $\gamma = 14.38$, the optimal mean setting being

$$\bar{x} = \gamma\sigma = 14.38 \times 8 = 115.0 \text{ grams}$$

but obviously the accuracy provided by $\gamma = 14.4$ is quite adequate.

3. The minimum scrap ratio is

$$S_m = 1 - m\varphi = 1 - 12.5 \times 0.0675 = 0.156, \quad \text{or } 15.6\%$$

compared with 17.2 per cent resulting from the present setting.

The cost function when the penalty for underweights is different from that for overweights

The foregoing analysis is related to the scrap ratio function, when material in underweight products has to be scrapped and the cost per unit weight to the plant is the same as the cost of excess material in overweight products. In many cases, however, the cost for each category is different. Material in rejected products may be reclaimed for reapportioning and the loss may be only partial, owing to additional labor, machine time, and packaging that have to be reinvested in the material. On the other hand, in some cases the procedure or cost of inspection may be such that it would not enable reclaiming or even stopping the

rejects from going through, and the penalty for these may be rather high. In such cases it is useful to analyze the cost function:

$$C = \underbrace{N\Phi x_1 c_1}_{\substack{\text{cost of} \\ \text{underweights}}} + \underbrace{N(1-\Phi)(x_2 - x_0)c_2}_{\substack{\text{cost of} \\ \text{overweights}}} \tag{18-61}$$

where C is the total cost, c_1 the cost per unit amount of underweight, c_2 the cost of unit amount of surplus material. By substituting

$$\frac{x_1}{\sigma} = \gamma - \frac{\varphi}{\Phi}$$

$$\frac{x_2}{\sigma} = \gamma + \frac{\varphi}{1-\Phi}$$

$$\frac{x_0}{\sigma} = \gamma + t$$

$$\epsilon = \frac{c_1}{c_2}$$

and considering the nondimensional cost function

$$E = \frac{C}{Nc_2\sigma} \tag{18-62}$$

we get

$$\begin{aligned} E &= \gamma\epsilon\Phi - t(1-\Phi) + (1-\epsilon)\varphi \\ &= (m-t)\epsilon\Phi - t(1-\Phi) + (1-\epsilon)\varphi \end{aligned} \tag{18-63}$$

For given values of m and ϵ, the cost function E can be plotted in terms of t, and this function will indicate:

1. What value of t should be selected in order to minimize the cost.
2. How sensitive the function is to changes in t (i.e., how "flat" the curve is in the vicinity of the point of minimum), so that an appropriate policy may be outlined to show just at what deviation from the optimal t it is desirable to readjust the process.

As might have been expected, the parameter ϵ greatly affects the cost function and hence our considerations when optimal solutions are sought. For the extreme case $\epsilon = 0$ (i.e., when the penalty for underweights is nil), we get

$$E_0 = \varphi - t(1-\Phi) \tag{18-63a}$$

Another special case is $\epsilon = 1$ (i.e., the two cost rates are the same), for which

$$\begin{aligned} E_1 &= \gamma\Phi - t(1-\Phi) \\ &= m\Phi - t \end{aligned} \tag{18-63b}$$

This expression corresponds to Eq. 18–58; thus we see that the criterion of scrap ratio (i.e., ratio of absolute scrap to material input) is a special case of Eq. 18–63, in which the criterion of absolute cost is used.

The cost function becomes a minimum when

$$\frac{dE}{dt} = 0$$

The situation is defined by the requirement $m = x_0/\sigma$, which is constant; hence

$$\frac{dE}{dt} = -\epsilon\Phi + \epsilon(m - t)\varphi - (1 - \Phi) + t\varphi - (1 - \epsilon)t\varphi = 0$$

or

$$\frac{1 - \Phi + \epsilon\Phi}{\epsilon\varphi} = m \tag{18–64}$$

Solutions to this equation are given in Fig. 18–11, from which the optimal setting t can be found for known values of m and ϵ. Alternatively, the equation may be solved by the trial-and-error method, similar to the solution illustrated in the preceding example.

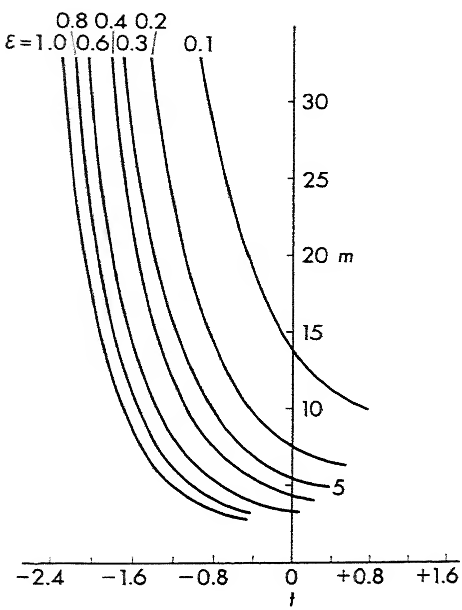

Figure 18–11. ***Optimal solutions for determining the mean in apportioning problems (when σ = constant).***

For the two special cases mentioned above when $\epsilon = 0$, we get $t \to \infty$ (i.e., we should avoid the production of overweight products, since there is a penalty for overweights, but no loss due to underweights), when $\epsilon = 1$, we get $\varphi = 1/m$

(as m is known for any given situation, we can easily find φ and then the corresponding t from the Appendix table).

As already pointed out, this situation is based on the assumption that any adjustment of the process does not change the standard deviation of the distribution of weights, volumes, etc., of the products. In other words, it is implied

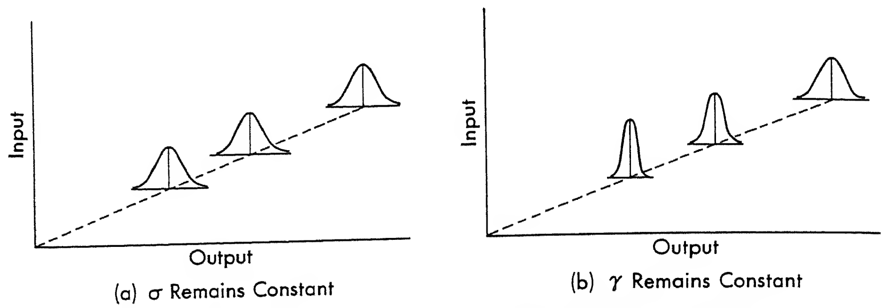

Figure 18–12. ***Effect of input on the distribution of outputs.***

that by selecting a new mean $\bar{x}$ for the distribution, we merely shift its position, as suggested by Fig. 18–12a. In many cases, however, an increase of the mean results in an increase of σ, so that the coefficient of variation remains constant, as suggested by Fig. 18–12b, or

$$\gamma = \text{constant}$$

The cost function may now be defined as

$$G = \frac{C}{c_2 N x_0} \tag{18–65}$$

or

$$G = \frac{E}{x_0/\sigma} = \frac{E}{t + \gamma}$$

By using Eq. 18–63, we get

$$G = \frac{1}{t + \gamma}\left[\gamma\epsilon\Phi - t(1 - \Phi) + (1 - \epsilon)\varphi\right] \tag{18–66}$$

and this cost function is minimum when

$$\frac{dG}{dt} = -\frac{1}{(t+\gamma)^2}\left[\gamma\epsilon\Phi - t(1 - \Phi) + (1 - \epsilon)\varphi\right] + \frac{1}{t+\gamma}\left[(t + \gamma)\epsilon\varphi - (1 - \Phi)\right] = 0$$

or, by substituting $t + \gamma = m$,

$$\epsilon m^2 - \frac{\gamma}{\varphi}(1 - \Phi + \epsilon\Phi) = 1 - \epsilon \tag{18–67}$$

Solutions for some values of ϵ are presented in Fig. 18–13. For the two special cases

$$\epsilon = 0, \text{ we get } t \to \infty$$

$$\epsilon = 1, \text{ we get } \gamma = m^2\varphi$$

Example

Consider the cost function in the preceding example, when the loss of material due to overweight products is 0.5 cents per gram of excess material, while $c_2 = 5$ cents per gram.

1. Show how the cost function depends on the setting t, (a) assuming that σ remains constant; (b) assuming that γ remains constant.
2. Find the cost of the present policy.
3. Determine the optimal policy when
   (a) σ is assumed to be constant.
   (b) γ is assumed to be constant.

Solution

1. We have

$$\epsilon = \frac{c_1}{c_2} = \frac{0.5}{5} = 0.1$$

(a) From Eq. 18–63, we find that

$$E = (12.5 - t)0.1\Phi - t(1 - \Phi) + 0.9\varphi$$

and the cost $C = 40E$ cents per unit.

(b) From Eq. 18–66, we get

$$G = \frac{1}{t + 15}\left[1.5\Phi - t(1 - \Phi) + 0.9\varphi\right]$$

and the cost $C = 500G$ cents per unit.

2. The present policy is at $t = -2.5$, which yields $E = 2.59$, or $C = c_2\sigma \times 2.59 = 1.03$ dollars per unit.

3. (a) From Fig. 18–11 we find that the optimal setting is at $t = -0.1$ or at $\bar{x} = x_0 - t\sigma = 100 + 0.8 = 100.8$ grams, incurring a cost of 40 cents per unit. This is substantially below the cost at the present moment.

(b) From Fig. 18–13, the optimal setting is at $t = -0.05$, but with this new setting, the standard deviation is changed, so that

$$\bar{x} = x_0 - t\sigma$$

but

$$\sigma = \frac{\bar{x}}{\gamma}$$

$\therefore$

$$\bar{x}\left(1 + \frac{t}{\gamma}\right) = x_0$$

or
$$\bar{x} = \frac{x_0}{1 + (t/\gamma)} = \frac{100}{1 - (0.05/15)} = 100.3 \text{ grams}$$

the new standard deviation being

$$\sigma = \frac{\bar{x}}{\gamma} = \frac{100.3}{15} = 6.7 \text{ grams}$$

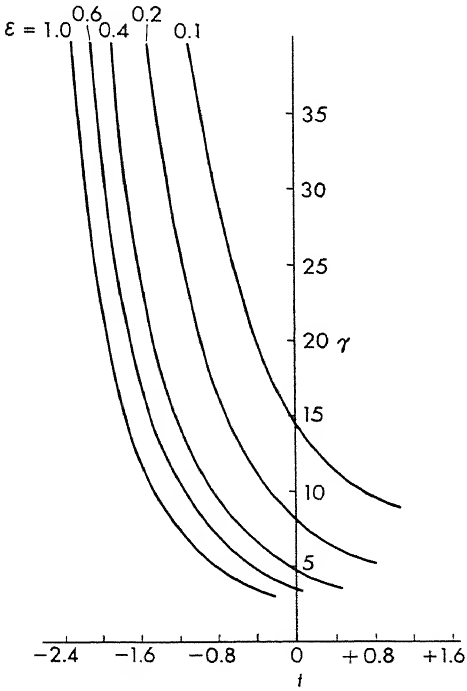

Figure 18–13. ***Optimal solutions for determining the mean in apportioning problems (when the process is governed by γ=constant).***

Apportioning outputs from a known distribution into several identical products[2]

This section deals with subdivision of a quantity, belonging to a known distribution, into smaller quantities. The original quantities are the outcome of a given process, and these form a distribution, as in the case of filling sacks of sugar or barrels of wine. Now, the contents of each of these sacks or barrels have to be divided into smaller containers or bottles. If we can subdivide the contents accurately into a whole number of smaller packages, we shall have no surplus. Of course, in the case of sugar or wine, we could have receptacles larger than sacks or barrels, or we could collect residues from several containers to make additional packages. But in some cases this cannot be done, and the residue of

[2] Acknowledgement is due to the Institution of Mechanical Engineers, London, for permission to quote from the author's papers mentioned in the References and to reproduce Figs. 18–12 to 18–20.

each original quantity incurs a certain loss. A typical example is in the manufacture of metal bars marketed by length. The bars are obtained by a rolling process, where the input to the mills consists of billets of known length and cross-section, and the bars form the output. These bars vary in length, the variability of the process being caused by many factors, such as variations in physical dimensions and chemical and mechanical properties of the input to the process, variations in the cross-section of the bars, variations in the process conditions, including temperature, wear of rollers, lubrication, etc.

If each bar of the bar-length distribution yields only one product, we have one of the inventory problems discussed above, namely:

1. The input to the process may be fixed, so that we have a certain bar-length distribution, of which an optimal bar length for marketing has to be selected. The bar-length distribution is analogous to the distribution of demand, and the optimal bar length to be chosen is analogous to the optimal reorder batch size, as analyzed previously.
2. The final bar length may be fixed by standards or by market requirements, but the distribution of bars at the output may be shifted by process adjustment, and the optimal setting may be specified to reduce the loss function.

In our apportioning problem, the mean of the bar-length distribution is large compared with the required length of the product, and each bar has to be sheared several times, yielding several products and a remainder. This remainder is scrap, and it naturally is smaller than the required product length.

Class distribution

In the bar-length distribution, the shortest bar that should be considered is $x_0 = n\mu$, where μ is the length of the required product; i.e., n products are obtained from the shortest bar. The distribution may be divided into classes, as shown in Fig. 18–15, the width of each being μ. The first class includes all the bars with a length between x_0 and x_1, i.e., between $n\mu$ and $(n + 1)\mu$, but all the bars in this class yield only n units of the product. The bars in the second class are between x_1 and x_2 in length, i.e., between $(n + 1)\mu$ and $(n + 2)\mu$, but each yields the same number of units, namely, $n + 1$; etc.

First, let us assume that the division into classes puts the symmetry line of the distribution in the middle of one class, as in Fig. 18–15; $2k + 1$ classes are thereby obtained, where k is a whole number. In order to cover the practical range of the distribution function (99.7 per cent), we should have

$$(2k + 1)\mu \geqslant 6\sigma$$

or

$$k \geqslant \frac{3}{m} - \frac{1}{2} \tag{18—68}$$

where

$$m = \frac{\mu}{\sigma}$$

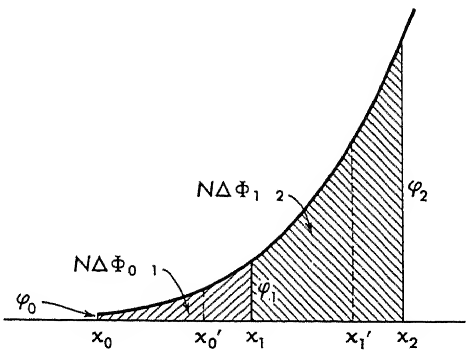

Figure 18–14. ***The first two classes (magnified).***

The scrap resulting from the class 0–1 is (Fig. 18–14)

$$s_{0,1} = N(x'_0 - x_0)(\Phi_1 - \Phi_0) = N(x'_0 - x_0)\Delta\Phi_{0,1}$$

where x'_0 is the average length of the bars in this class, $\Delta\Phi_{0,1}$ is the probability to obtain a bar in this class, and N is the total number of bars in the distribution.

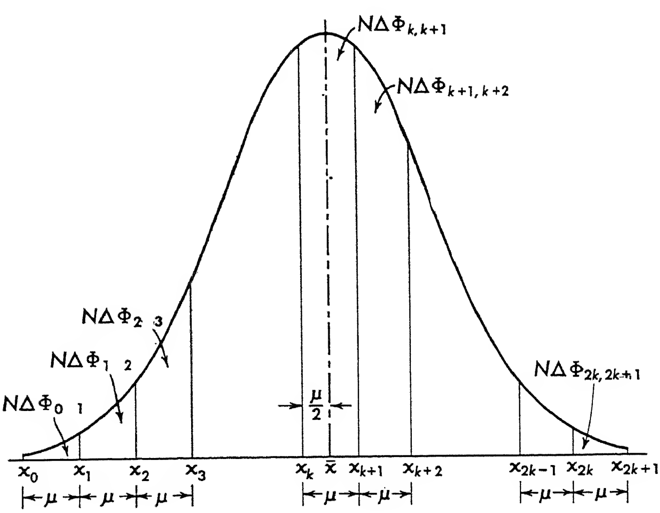

Figure 18–15. ***A symmetrical class-grid.***

Similarly, the scrap for class 1–2 is

$$s_{1,2} = N(x'_1 - x_1)\Delta\Phi_{1,2}$$

and the total scrap is

$$S = \sum_{i=0}^{2k} s_{i,i+1} = N \sum_{i=0}^{2k} (x'_i - x_i)\Delta\Phi_{i,i+1} \tag{18–69}$$

Since by Eq. 18–42,

$$t_{x'_i} = -\frac{\varphi_{i+1} - \varphi_i}{\Delta\Phi_{i,i+1}}$$

and as, by definition,

$$x'_i = \bar{x} + \sigma t_{x'_i}$$

then

$$x'_i = \bar{x} - \frac{\sigma}{\Delta\Phi_{i,i+1}}(\varphi_{i+1} - \varphi_i)$$

When this expression is substituted into Eq. 18–69, the scrap per bar is

$$\frac{S}{N} = \sum_{i=0}^{2k} (\bar{x} - x_i)\Delta\Phi_{i,i+1} - \sum_{i=0}^{2k} \sigma(\varphi_{i+1} - \varphi_i)$$

$$= \bar{x}\sum_{i=0}^{2k} \Delta\Phi_{i,i+1} - \sum_{i=0}^{2k} x_i\,\Delta\Phi_{i,i+1} - \sigma(\varphi_{2k+1} - \varphi_0)$$

Practically,

$$\sum_{i=0}^{2k} \Delta\Phi_{i,i+1} = 1; \quad \varphi_{2k+1} = \varphi_0 = 0$$

and $x_i = (n + i)\mu$. Therefore

$$\frac{S}{N} = \bar{x} - \Big[n\,\Delta\Phi_{0,1} + (n + 1)\,\Delta\Phi_{1,2} + (n + 2)\,\Delta\Phi_{2,3} + \cdots$$
$$+ (n + 2k)\,\Delta\Phi_{2k,2k+1}\Big]\mu$$

$$= \bar{x} - n\mu \sum_{i=0}^{2k} \Delta\Phi_{i,i+1} - \mu\Big[0\,\Delta\Phi_{0,1} + 1\Delta\Phi_{1,2}$$
$$+ 2\Delta\Phi_{2,3} + \cdots + 2k\,\Delta\Phi_{2k,2k+1}\Big]$$

But due to the symmetry of the class grid placed on the distribution,

$$\Delta\Phi_{0,1} = \Delta\Phi_{2k,2k+1}$$
$$\Delta\Phi_{1,2} = \Delta\Phi_{2k-1,2k}$$
$$\vdots$$
$$\Delta\Phi_{k-1,k} = \Delta\Phi_{k+1,\ k+2}$$

and
$$\Delta\Phi_{k,k+1} = 2\Delta\Phi_{k,k+\frac{1}{2}}$$

$$\therefore \quad \frac{S}{N} = \bar{x} - n\mu - \mu \cdot 2k(\underbrace{\Delta\Phi_{0,1} + \Delta\Phi_{1,2} + \cdots + \Delta\Phi_{k-1,k} + \Delta\Phi_{k,k+\frac{1}{2}}}_{=\frac{1}{2}}$$

By substituting $\bar{x} = (n + k + \frac{1}{2})\mu$, the scrap per bar is

$$\frac{S}{N} = \frac{1}{2}\mu \tag{18–70}$$

Similarly, when the class grid is superimposed symmetrically on the distribution, so that the ordinate at the mean coincides with the boundary line between

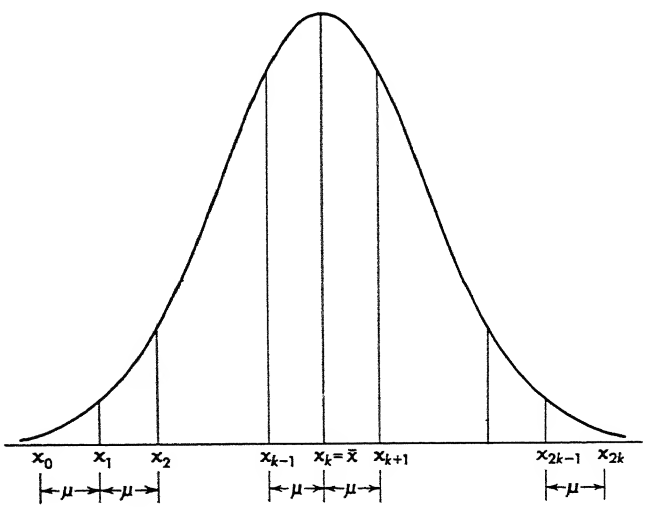

Figure 18–16. ***A symmetrical class-grid.***

two classes (Fig. 18–16), the same value for scrap is obtained. This is an important result, since it implies that neither the variability of the process nor the type of the distribution (as long as it is a symmetrical one), has any effect on the amount of scrap.

When the grid of classes is not symmetrical, i.e., when the ordinate at $\bar{x}$ is not in the middle of a class but at a distance a from the ordinate at $\bar{x}$ (Fig. 18–17), the scrap per bar is again

$$\frac{S}{N} = \bar{x} - \sum_{i=0}^{2k} x_i \, \Delta\Phi_{i,i+1} = \bar{x} - n\mu - \mu \sum_{i=0}^{2k} i \, \Delta\Phi_{i,i+1} \tag{18–71}$$

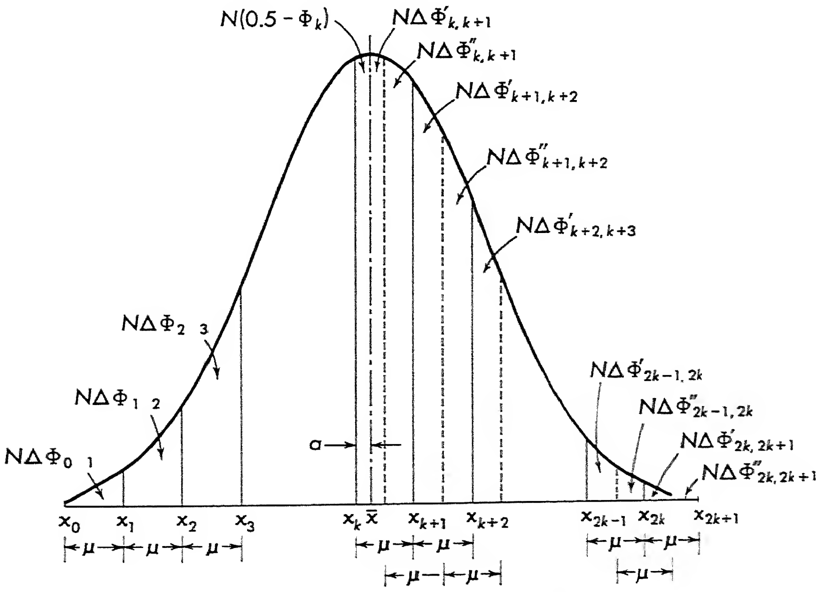

Figure 18–17. ***A nonsymmetrical class-grid.***

If the grid on the left of the symmetry line is reflected on its right, every class on the right is divided into two subclasses so that each $\Delta\Phi = \Delta\Phi' + \Delta\Phi''$. Now, to evaluate the third term in Eq. 18–71,

$$\begin{aligned}\sum_{i=0}^{2k} i \, \Delta\Phi_{i,i+1} &= 0 \, \Delta\Phi_{0,1} + 1 \, \Delta\Phi_{1,2} + 2 \, \Delta\Phi_{2,3} + \cdots + (k-1) \, \Delta\Phi_{k-1,k} \\ &\qquad + k(0.5 - \Phi_k) \quad \text{Below } \bar{x} \\ &+ k(\Delta\Phi'_{k,k+1} + \Delta\Phi''_{k,k+1}) + (k+1)(\Delta\Phi'_{k+1,k+2} + \Delta\Phi''_{k+1,k+2}) + \cdots \\ &\qquad + 2k(\Delta\Phi'_{2k,2k+1} + \Delta\Phi''_{2k,2k+1}) \quad \text{Above } \bar{x}\end{aligned}$$

(where $\Delta\Phi'_{k,k+1}$ is from the ordinate of $\bar{x}$ and not from the ordinate of x_k)

(see Fig. 18–17). But due to the symmetry of the distribution curve,

$$\Delta\Phi_{0,1} = \Delta\Phi''_{2k-1,2k} + \Delta\Phi'_{2k,2k+1}$$

$$\Delta\Phi_{1,2} = \Delta\Phi''_{2k-2,2k-1} + \Delta\Phi'_{2k-1,2k}$$

$$\vdots$$

$$\Delta\Phi_{k-1,k} = \Delta\Phi''_{k,k+1} + \Delta\Phi'_{k+1,k+2}$$

also

$$(0.5 - \Phi_k) = \Delta\Phi'_{k,k+1}$$

and practically

$$\Delta\Phi''_{2k,2k+1} = 0$$

Hence

$$\sum_{i=0}^{2k} i\,\Delta\Phi_{i,i+1} = (2k-1)\,0.5 + \Delta\Phi'_{k,k+1} + \Delta\Phi'_{k+1,k+2} + \cdots + \Delta\Phi'_{2k,2k+1}$$

$$= k - \Delta\Phi''_{k,k+1} - \Delta\Phi''_{k+1,k+2} - \cdots - \Delta\Phi''_{2k,2k+1}$$

By substituting this relation and

$$\bar{x} = (n + k)\mu + a$$

into Eq. 18–71, we get

$$\frac{S}{N\mu} = \frac{1}{2} + \frac{a}{\mu} - \sum_{i=k}^{2k} \Delta\Phi'_{i,i+1}$$

$$= \frac{a}{\mu} + \sum_{i=k}^{2k} \Delta\Phi''_{i,i+1}$$

or

$$\frac{S}{N\mu} = \frac{1}{2} + \frac{m'}{m} - \sum_{i=k}^{2k} \Delta\Phi'_{i,i+1} \tag{18–72}$$

where $m = \mu/\sigma$, $m' = a/\sigma$, m expresses the relation of the product length to the variability of the process, m' indicates the degree of asymmetry of the class grid in relation to the mean of the distribution. The effect of these parameters in the case of a normal distribution is shown in Fig. 18–18. For $m' = 0$, $m' = \frac{1}{2}$ expression 18–70 is obtained, whatever the value of m. If the class grid is not symmetrical, the amount of scrap becomes more and more sensitive to m as m increases above 2.0. Hence the longer the required bar and the smaller the variability of the process, the more important it is to adjust the mean in order

to ensure minimum scrap. The scrap function Eq. 18–72, has the shape of a symmetrical wave, the amplitude of which increases with m, and the minimum

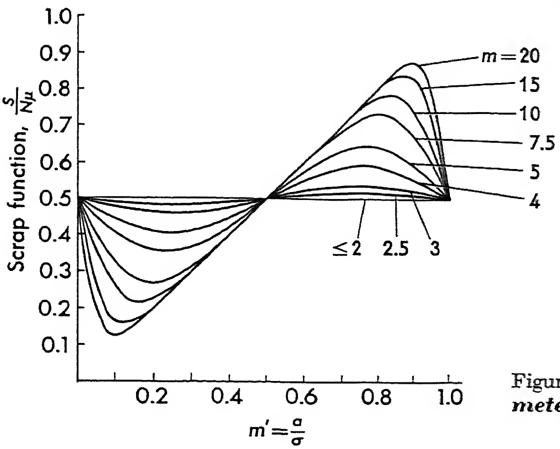

Figure 18–18. ***Effect of the parameters m, m′ on scrap.***

scrap reduces with m, as shown in Fig. 18–19. For $m < 2.0$, it can be shown that practically

$$\sum_{i=k}^{2k} \Delta\Phi'_{i,i+1} = \frac{a}{\mu} = \frac{m'}{m} \tag{18–73}$$

$$\therefore \qquad \frac{S}{N\mu} = \frac{1}{2} \tag{18–74}$$

which is the same as Eq. 18–70.

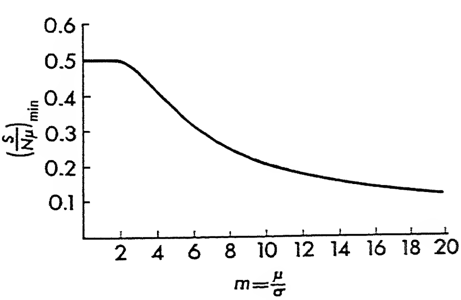

Figure 18–19. ***Minimum values of the scrap function.***

Conclusions

For $m < 2.0$:

1. The amount of scrap is independent of the variability of the process.
2. The amount of scrap is not affected by the mean value obtained by the process. Hence, adjustments of the mean are not required, and as long as there

is no change in the product distribution, no change in the level of scrap should be expected. It is, however, desirable to increase $\bar{x}$ in order to reduce the scrap percentage-wise.

3. The scrap is linearly dependent on the required length of the product.

If, for example, we need bars 10 yards long, and supposing that the standard deviation of the process does not change with the mean (such as in Fig. 18–12a), the scrap function would be as in Fig. 18–20. If the mean length of the rolled bar can be adjusted between 30 and 50 yards, and if $\sigma = 1.0$ yards, it can be seen that minimum scrap occurs at 31.7 and 41.7 yards; of these two points the last is preferable, since the longer the mean rolled bar, the smaller the scrap percentage.

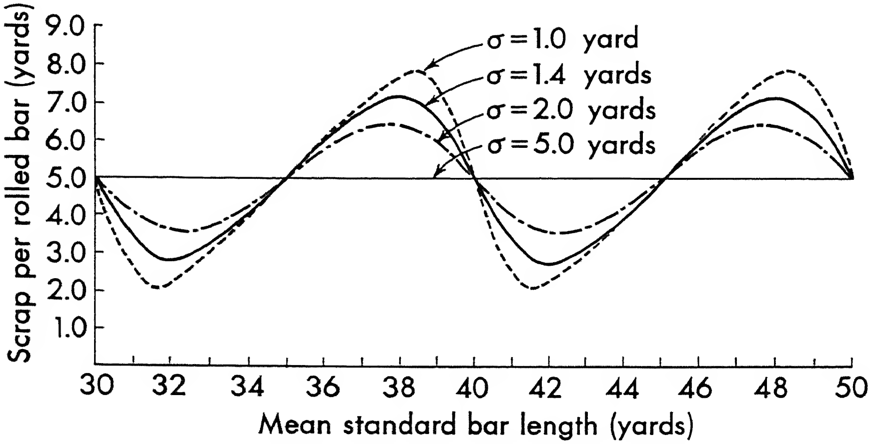

Figure 18–20. ***Scrap function for shearing bars $\mu = 10$ yards long from a bar distribution having a constant σ.***

In this problem we have been concerned with apportioning quantities, and we have assumed throughout that the variability of the small quantities is negligible compared with the variability of the bigger quantities.

There are, in fact, many situations where more than one product is required; for instance, in the case of sugar we may want to stock packages of $\frac{1}{2}$, 1, and 2 pounds, or in the case of bars, we may have a demand for 10, 12, and 18 yards. Furthermore the demand for several products may be specified in given proportions; e.g., 50 per cent of the stock should be in 10-yard bars, 30 per cent in 12 yards, and 20 per cent in 18 yards. Analysis of multiproduct apportioning problems clearly calls for the use of linear programing, since many methods of apportioning can be suggested; of these we seek those that would minimize the total amount of scrap.

The author was laboring under the misconception that minimization of scrap

is a desirable aim, until he was seriously told recently by the works manager of a glass manufacturing plant that he could not allow reduction in scrap because it was one of the main ingredients required for his compounds. These problems, however, are beyond the scope of this book, and the interested reader is referred to the references for further material on the subject.

Summary

Inventory management is always confronted by uncertainties, and since stock replenishment is very often carried out in batches, an analysis of the optimal reorder batch is called for. Two inventory models are analyzed, one for continuous consumption and one for discrete consumption. It is shown that the criterion of minimum costs is a special case of the solution for maximum profit for both models.

Perishable and semiperishable goods are a special case, usually of the discrete consumption type, and it can be shown that determination of optimal sizes of reservoirs and stores is a problem belonging to this class. Apportioning problems deal with division of big quantities into several smaller quantities, and the case when the big quantities are subject to variations according to the normal distribution is analyzed.

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Problems

1. In the case of discrete demand with a single withdrawal per cycle, the probability density function for demand is given as normal, with a mean demand of 1,000 units per cycle and a standard deviation of 100 units. If the penalty for not meeting demand is defined by situation a, and ϵ_1 is given as 0.125,

what reorder quantity should be planned? Notice that the time of withdrawal, t, is not specified. Would wide variations in t make a great difference to the optimal reorder batch? Why?

2. The probability density function for discrete demand was found to be normal with a mean demand of 2,200 units per cycle and a standard deviation of 100 units. The loss per unsold unit is \$2.00 and the penalty for not selling a unit is \$2.50. What reorder quantity should be specified? Should the standard deviation increase twofold, will this quantity be greatly affected?

3. Supposing that in a chain of stores marketing the same product we found the standard deviation to be practically the same for all stores $\sigma = 40$ units, but the mean demand varied appreciably from store to store. If $c_1 = \$5.00$, $c_2 = \$1.00$, plot the ratio $\frac{\text{optimal order quantity}}{\text{mean demand}}$ against mean demand over the range $\bar{D} = 100$ to 600 units per cycle.

4. Find the loss function due to uncertainty in the case of discrete demand, when it is known that a quantity is withdrawn from the stores twice every cycle.

5. In the newspaper problem (Table 18–1), calculate and plot the profit from sales, the loss from left-over papers, the net profit, and the potential gain when the news agent varies his reorder quantity from 60 to 100 papers per day.

6. Given the demand record for a daily newspaper in Table 18–1, and assuming the normal curve in Fig. 18–3 would apply in the future:
   (i) Find the optimal order quantity for the news agent, when he has to pay 7 cents for each copy he orders, sold or unsold.
   (ii) Find the optimal order quantity for the news agent, when he has to pay only 4 cents for each unsold paper.
   (iii) How much should the publisher charge the news agent for each unsold copy, if each copy costs him 5 cents and if he wants to maximize his own profits? Assume that for each policy set by the publisher, the news agents will select the optimal order quantity from his point of view. Are the aims of the publisher and the agent compatible with each other?

7. A theater is to be built for an expected average demand of 400 seats per performance and a standard deviation of 100 seats. There are two kinds of tickets: half the seats sell at \$1.00 each and the remainder at \$1.50 each. The average cost per seat to the management is expected to be \$1.10. Find the optimal seating capacity of the theater. Assumptions are:
   (i) The demand distribution is normal.
   (ii) The demand pattern is as follows: Demand ratio of expensive to cheap tickets is 40:60, until no more cheap tickets are available. Then, each customer who cannot get a cheap ticket is willing to buy an expensive one.

8. A canning plant has a store with a capacity of 10 tons for a certain fruit. Since the working cycle is one week, it was found convenient to arrange for shipments of this fruit to arrive at the plant once a week, when the store would be emptied and cleaned for the next working cycle. Shipments were rather

irregular in quantity: sometimes a shipment would be less than 10 tons, in which case the store would only be partially full, and the cost of not utilizing the space and machine facilities for canning would cost the plant \$200 per ton; sometimes a shipment would exceed 10 tons, and the surplus would be stored outside, thereby causing spoilage of the fruit, which costs \$450 per ton. The average shipment is 12 tons, and in 70 per cent of the cases it is necessary to use the outside storage area.

(i) Assuming the shipments form a normal distribution and that the canning machines can cater for practically any load, so that no fruit is carried over to the next cycle, what is the present cost due to variations in the size of shipments?

(ii) What store capacity would have reduced this cost to a minimum?

9. A certain semiperishable commodity is acquired at \$0.80 a unit and sold at \$1.20 a unit. At the end of the period, any leftovers become second-class goods and are sold at half-price. At the end of the second period, leftovers from the first period are scrapped, and leftovers from the second period become second-class goods in the third period, etc. The demand distribution for first-class goods has 180 units as its mean and a standard deviation of 20 units. The mean demand for second-class goods is 36 units and the standard deviation of this distribution is 12 units. There are two situations: (i) if the demand for one class cannot be satisfied, customers are not prepared to take goods of another class; (ii) if demand for second-class goods is not satisfied, customers are invariably prepared to take first-class goods instead (note that this demand for first-class goods was not included in the demand distribution for first-class goods mentioned above).

   What is the optimal reorder quantity that should be specified for each situation (use simulation)?

10. Sales of dinners at a restaurant were found to be approximately normally distributed with a mean demand of 600 meals per day and a standard deviation of 100. The cost of a meal to the establishment is 75 cents, and if a meal is left over, it has to be thrown away.

    (i) What is the optimal policy regarding the number of meals that should be prepared, when the penalty for turning a customer away is \$4.00?

    (ii) The proprietor successively set the target for satisfying customers (the term *satisfying customers* here is understood to mean "supplying them with a meal," not "making them happy with the cooking, service, or decor of the restaurant") at 99.5, 99, 98, 96, 95, 92, and 90 per cent. What penalties are these figures equivalent to and what is the cost of each policy? Plot the cost C versus the percentage of customer satisfaction.

    (iii) The demand distribution naturally varies from time to time and these variations may be related to (a) the shape of the curve; (b) the mean demand; (c) the standard deviation.
    What could the effect of each be on the results obtained for (i), (ii) above? Illustrate your answer by calculating the effect on the planned number of meals for the cases when the standard deviation does not

change but mean demand fluctuates by ± 20 per cent, and when the mean demand does not change but the standard deviation fluctuates by ± 20 per cent.

11. The demand for a certain semiperishable item at a tea shop is described by the cumulative distribution given in Fig. 18–21. Orders for replenishment are issued in the evening when the tea shop closes, and supplies reach the shop in the morning at opening time. The present inventory policy of the shop is aimed at ensuring that the stock every morning (= left over from previous day + replenishment − quantity of goods to be scrapped) should be equal to the demand during the previous day(= goods sold + demand that could not be satisfied).

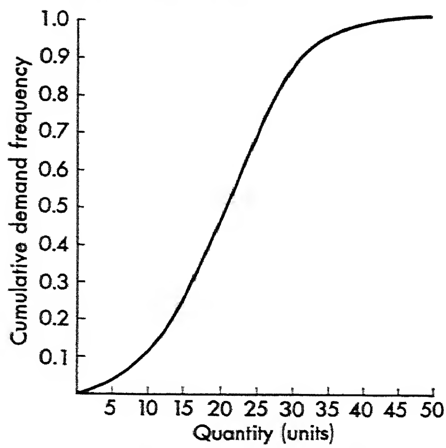

Figure 18–21. ***The cumulative demand distribution for a given commodity.***

The purchase price is \$1.60 per unit, and the sales price is

\$2.40/unit when the goods are fresh.

\$2.00/unit when the goods are one day old.

\$1.20/unit when the goods are two or three days old.

The goods lose all their value after the third day, and the penalty for not meeting demand is \$2.00 per unit.

(i) Comment on the present policy.

(ii) Find the optimal policy and calculate the amount of saving expected if your policy is accepted.

(iii) It has been suggested that the division of old goods into two classes for purposes of sales is too cumbersome and that all goods of one to three days old should be sold at \$1.20 per unit. What effect would this have on the total profit?

12. Expressions 18–51a and 18–51b give the loss ratio for the case when the number of underweight products is not expected to exceed 0.15 per cent. Show that if a higher mean is selected, the loss ratio will increase; i.e., the amount of excess material lost in overweights will be higher than the amount saved through reduction of the number of underweights.

13. A commodity is packaged to customer's specifications, which state that each

product should not be below x_0 in weight. For underweight products there is a heavy penalty c_1 per package, while excess material packed in overweight packages costs the producer c_2 per unit weight. The producer has two policies to choose from:

(a) Adjust the mean weight $\bar{x}$ of the packets until the total cost becomes a minimum.

(b) Introduce an automatic weighing machine that would reject all packages under x_0 in weight. The rejects can be opened and repackaged, and the cost involved is $c_3 = kc_1$, where $k < 1$. The cost of using the automatic inspection and sorting machine is c_4 per package produced.

(i) Find the optimal setting for each of the above policies and state under what conditions (assuming the weight distribution is normal) each becomes preferable.

(ii) Supposing $x_0 = 1$ lb.; $\sigma = 0.2$ lb.; $c_1 =$ \$5.00/package; $c_2 =$ \$0.40/lb.; $c_3 =$ \$0.60/package; $c_4 =$ \$0.10/package. Draw curves to show how the cost changes for each policy, and find the minimum cost for each case.

14. In the stocks of certain perishables, the loss per unsold unit at the end of the cycle is c_1 per unit, the cost of not meeting demand is c_2 per unit. In the analysis in this chapter no account was made for storage costs while the goods are in the store, the assumption being that these costs are constant irrespective of the batch size.

Suppose storage costs involve various operations when the goods are received in the store; show that[3]:

(i) When the storage costs are c_0 per unit received, the optimal batch is given by

$$\Phi = \frac{1 - \epsilon_0}{1 + \epsilon_1}$$

where $\epsilon_0 = c_0/c_2$.

(ii) When the storage costs are c_0 per unit of average stock, the optimal batch is given by

$$\Phi = \frac{1 - \frac{1}{2}\epsilon_0}{1 + \epsilon_1 + \frac{1}{2}\epsilon_0}$$

15. If the difference between the sales price and the acquisition cost is z per unit, and if it is desirable to maximize the profit of perishable goods replenished by the reorder cycle system, show that the profit function is

$$Z = \sigma c_2 [\zeta(\gamma + t - \varphi - t\Phi) - E]$$

where Z is the profit and

$$\zeta = \frac{z}{c_2}$$

(the other symbols are as used in the text; assume a normal distribution for demand), and that the optimal reorder quantity is given by Eq. 18–50a, namely,

$$\Phi = \frac{1 + \zeta}{1 + \zeta + \epsilon}$$

When would you select this solution instead of the one given by Eq. 18–50)?

[3] From "Inventory control: a problem in stocking perishable goods," by S. Eilon, The Production Engineer, April, 1960.

16. In the production of steel bars it was found that for a constant billet input, the rolled bar-length distribution was normal, the mean being 51.0 yards and the standard deviation 1.60 yards. The bars have to be sheared to lengths of 8 yards and 3 yards. It is possible to adjust the mean length of the rolled bars between 30 and 60 yards by suitable selection of the input. If the coefficient of variation is believed to be constant, what mean bar length should be specified for each product?

17. Paper is produced at one mill in rolls 180 inches wide. When narrower rolls are required, the 180-inch roll is cut to the appropriate width. The following order is received at the store:

    (1) supply: 6,000 yd. – 60 in. wide
    4,000 yd. – 84 in. wide
    3,000 yd. – 124 in. wide
    3,000 yd. – 36 in. wide
    2,000 yd. – 24 in. wide
    2,000 yd. – 12 in. wide
    (2) No roll should be shorter in length than 500 yd.

    Formulate the problem as a linear programing one.

18. A quantity Q of semiperishables is held in stock. The goods maintain their full value during the cycle, but any residual stock carried over to the next cycle loses c_1 in value per unit, and if left for a later cycle, it loses a further amount of c_2 in value, and so on. After several cycles we find that we have Q_0 fresh units, Q_1 units one cycle old, Q_2 units two cycles old, and so on. If a quantity D is to be withdrawn from the stock, what sequence of issue should be specified (i.e. from which classes of goods should the D units be taken) to reduce the loss of value to a minimum?[4]

19. A newspaper vendor buys y papers for 3 cents a copy and sells x copies for 5 cents a copy. The news-stand is located at a railway station where 200 customers reach the station every morning and buy one paper each. Another 50 potential customers arrive at the station in a bus, and if it arrives early, each customer buys a paper, but if the bus is late, none buys a paper. Assume that the bus arrives either too early or too late (i.e., never precisely on time) and that the probability of its being early is 50 per cent.

    Find the optimal number of copies y that the vendor should buy in order to maximize his profits. Plot the profit function for the range $y = 150$ to $y = 300$ copies.[5]

[4] Ibid.

[5] From "The inventory problem," by J. Laderman, S. B. Littauer, and L. Weiss, American Statistical Association Journal, December, 1953.

19

QUALITY CONTROL

Quality control is a subject on its own, and several excellent text books are available, covering all the different aspects of this field in great detail. Quality inspection activities, determination of what is acceptable and what should be rejected, evaluation of quality control procedures and techniques—all these important responsibilities are not within the domain of the production planning and control department. Some aspects of quality control must be a separate function, undominated by the interests of executive departments and unbiased, so that objective reports about the level of quality can be forwarded to the production management. It is obvious, however, that quality control is so related to production control procedures, that no treatise on production planning and control can be complete without a brief reference to it.

> The modern conception of the creation and control of quality, by definition, means the development and realization of specifications necessary to produce economically, and in adequate degree, the appearance, efficiency, interchangeability and life which will ensure the product's present and future market.[1]

This broad definition emphasizes the fact that there are two facets to quality: its creation and its measurement. The measurement function, i.e., the responsibility for determining the facts during and after production takes place, is naturally a very important one, but measurement by itself does not constitute control. The facts must be analyzed, the discrepancy between what is desirable and what is actually achieved has to be evaluated, so that some positive conclusions and guiding principles may be derived and applied at all the stages where quality is created, right from the development and design stages to the final assembly lines. Hence, the functions and purposes of quality control are:

- To satisfy the customer by endeavoring to comply with the declared specifications of the product
- To ensure that the work may proceed to the next operation (i.e., to avoid superfluous processing of faulty goods)
- To be instructive, so that recurrence of mistakes could be eliminated

[1] Quality, its creation and control (The Institution of Production Engineers, London, 1958).

- To compare the quality level obtained with the desirable level as a basis for process control, detection of trends and process adjustment procedures
- To determine the optimal quality obtainable for available processes and to provide useful guides for specifications at the design stage
- To reduce scrap to an economic level
- To facilitate a procedure whereby incentive payments will not be made for faulty output
- To make the operators aware of the fact that their work is being inspected and that origin of faults can be traced (this usually has a positive psychological effect in that operators pay more attention to the quality of their work)
- To allow review of the quantity required, so that allowance can be made for faulty work[2]
- To evaluate existing inspection methods and design better and more effective procedures

Some aspects of quality were discussed in Chapter 5 in connection with standards specifications in product development and design. Not all quality attributes are easily definable. Physical dimensions (such as length, area) or physical and chemical properties of materials (strength, weight, composition, etc.) lend themselves to straightforward quantitative definitions. All that is required is:

1. To design comparatively simple and quick testing techniques with which these attributes can be measured.
2. To define what is "good" and what is "bad"; this usually takes the form of a range within which performance is considered to be acceptable, so that results outside the range render the product unsatisfactory.

There are, however, many quality characteristics, which are not easy to define, in the form of tolerances, such as cleanliness, brightness, whiteness, smoothness, characteristics of surface textures, irregularities (spots, stains, blemishes), taste (in food and drink), smell, and softness. Admittedly, some of these characteristics can perhaps be defined and measured by indirect methods. Milk bottles, for instance, have to be clean before filling, and therefore empty bottles move on a conveyor belt and are inspected before they are passed to the filling machines. When the inspector is a human operator, he may find it extremely difficult to define a satisfactory criterion of cleanliness to guide him in passing or rejecting the empty bottles.

Furthermore, we often find that standards for acceptance may vary for different operators or even for the same operator at different times of the day. One indirect way of defining cleanliness would be by the amount of light allowed to

[2] *Example:* 1,000 units are required of a product, which passes three processes. It was found that process 1 incurs 3 per cent defects; process 2, 10 per cent; process 3, 2 per cent. If the quantity to specify is x, after the first process there are only $0.97x$ unit; after the second process, $(0.97x)0.90$; and after the third process, $(0.97 \times 0.90x)0.98$; hence

$$0.97 \times 0.90 \times 0.98x = 1{,}000$$

$$\therefore \quad x = 1{,}170 \text{ units}$$

pass through the bottle; in other words, it is assumed that dirt consists of undesirable material residues that are somewhat more opaque than the clean bottle, so that if the bottle is subjected to a source of light of constant intensity, the amount of light passing to the other side will become a measure of cleanliness. We need in this case an instrument with which the intensity of the light can be measured, and this can be provided by a photoelectric cell located on one side of the conveyor, which in turn can be easily connected to a mechanical sorting device. It is still debatable, however, whether this method is adequate and satisfactory for defining and determining the level of cleanliness, and indeed with some of the other characteristics mentioned above, the problem of definition and measurement becomes even more complicated.

Stages in Quality Control

There are seven stages in quality inspection, each designed for a specific purpose, and each requiring a varying degree of emphasis in the quality control procedural structure, depending on the type of industry, size of plant, type of production, the intricacy of operation sequences, reliability of the processes, etc.

Inspection of incoming materials and bought out components

It is necessary to ensure that the materials and components bought outside are not inferior and are not likely to jeopardize the performance of the final product in any way. The inspection is carried out at two separate points, the purchaser's and the vendor's plants.

Inspection at the purchaser's plant

Before the materials and components are allowed to enter the regular stores, one of three methods are employed.

1. Complete, or 100 per cent inspection: this is tedious, lengthy, and costly.

2. Sample inspection: the sampling procedure is so designed as to ensure effective detection of deviations from the specifications at reasonably low costs (see acceptance sampling).

3. A combination of the two methods is used, the procedure being basically one of sampling but allowing for bigger samples or even 100 per cent inspection to be undertaken should the percentage or number of defects exceed a predetermined level (a provision is often made that extra costs incurred by 100 per cent inspection are to be borne by the vendor).

Inspection on the vendor's premises

Apart from the vendor's own inspection the purchaser's inspectors may check the processes and operation conditions (e.g., temperature and pressure of materials during the operations) to ensure that these conform to methods and procedures previously agreed upon. Inspection at the vendor's end greatly

reduces, although it does not always completely eliminate, inspection at the purchaser's end.

Once the inspection results are known, they can be used to:

1. Authorize admission to the stores and payment to the vendor, or refusal to accept with all the consequences (return to vendor for further processing, legal actions, etc.).
2. Evaluate to what extent the specifications as laid down are realistic.
3. Evaluate vendor's reliability with respect to quality.
4. Study alternative quality inspection procedures.

Inspection of facilities

The next stage is to verify whether the production facilities are up to an acceptable standard of performance, and this responsibility rests with the maintenance department. A record card is kept for each machine so that a complete history of the machine is available at all times, to tell us the frequency and type of breakdowns, details about repairs, etc. Inspection of the machines is carried out at regular intervals (in conjunction with preventive maintenance) and whenever there are complaints from operators or the quality control department about inconsistencies in performance, too frequent settings and adjustments, and so on. These records are very useful when comparing the desirable product specification with the capabilities of the machines and in studying optimal maintenance schedules.

"First-off" inspection

The first component (or several first components) of a batch or a production run must be very carefully inspected in order to determine whether machines are properly set for production. The responsibility for first-off inspection rests with the inspection department, and in shops engaged on job or batch production, it is not uncommon to find "first-off inspectors" whose sole responsibility is to ascertain that the first units coming off the line conform with the specifications laid down. Normally, the operator does not proceed with production, unless he gets the inspector's approval of the first units.

This procedure ensures that no scrap is meanwhile produced and that further alignment of the machines is carried out before full-scale production is embarked upon. However, it implies that the operator is idle from the moment the first-off component is finished until the inspector gives his final decision, and this may become a serious problem in a shop where many first jobs have to be frequently inspected by a comparatively small number of inspectors, so that first-off jobs are made to wait in a queue until an inspector becomes available to inspect them.

Inspection by self-control

The ultimate responsibility for the standard of quality rests with the operator who makes the product and this is why inspection by self-control, whenever

possible, is to be greatly encouraged. The operator is acquainted with the specifications of his work, and where quality characteristics are well defined quantitatively, he can often apply a few simple checks to his finished work, so that he can take corrective action immediately on his own initiative. Self-control is particularly desirable whenever the work cycle involves operator idle time to such an extent that by imposing an independent task on the operator (such as an inspection), the cycle time would not be expected to increase. The advantages of self-control are numerous:

Production, inspection, information, and adjustment follow in rapid succession; there should be no delay in responding to any deviations from the specifications, as this self-regulating mechanism has both the authority to originate information about errors and to take steps to eliminate them.

If self-control is carried out conscientiously, less scrap should be expected.

The operator may be held fully responsible for the quality of his work because he does not have to wait for the inspection department to tell him how well he is doing.

Operator time is better utilized and less inspectors are required.

Inspectors' control after operations

Self-control reduces the amount of control to be exercised by inspectors, but it does not eliminate it, since self-control may be biased, especially in cases of payment by output. Inspection of components should be carried out as close in time to the operation as possible, if the inspection is to be effective and if the risk of having many faulty components as work in process is to be eliminated. An example of delayed inspection was given in Chapter 7 in Fig. 7–13a, where the layout provided for a special store of finished assemblies awaiting inspection. When serious deviations from the specifications occur, a considerable amount of finished assemblies may be faulty before this fact is discovered. In Fig. 7–13b the inspection is integrated into the line and response is appreciably quicker. Figure 7–14 is another example of prompt inspection, the inspector's bench being located right in front of the machine.

Inspectors may either be assigned to check particular operations or be charged with a variety of tasks, depending on the amount of work involved. Static inspectors, assigned to particular jobs, are more common when 100 per cent inspection is required, and then integration into the production line (as suggested by the example in Fig. 7–13b) is highly desirable. When only a certain percentage of the work is inspected, it is common to find patroling inspectors, who wander from one production center to another, either in a regular or a random fashion. In some case studies, it was found undesirable to assign to the same inspectors both first-off tasks and routine inspection responsibility because of the excessive idle time incurred by machines after first-off jobs waiting for inspector's approval. With the introduction of some priority rules for inspection, however, this situation can be somewhat improved.

Final inspection

The main purpose of final inspection is not so much to provide data for immediate action as it is to ensure that only products of a predefined quality standard are allowed to leave the plant and be offered to customers. Its second important function is to provide a basis for evaluation of quality performance on a somewhat broad basis. This evaluation will include the critical study and comparison of processes, reassessment of inspection procedures along the line, calculations of amount of faulty work and scrap, and their effect on the cost analysis.

An example of a final inspection layout was illustrated in Chapter 7 in Fig. 7–15, where inspection stations are located along a conveyor line, each station checks certain characteristics and (if the product does not satisfy the specifications) stops its advance to the next processing station.

Post-sales quality evaluation

The product may have passed the final inspection, but does it render satisfactory service? The study of post-sales quality may follow several lines of investigation:

1. How many rejects were returned by customers? How can the defects be classified? Which defects may be attributed to the storage period after production and final inspection? How could defects pass all the inspection barriers prior to sales?
2. How many claims were there for servicing at the customer's end after sales; what types of repairs were required and what is their significance?
3. What inferences about the quality of the product in use can be made from customers' reports?
4. What conclusions may be drawn from after-sales surveys?

Acceptance Sampling

When we purchase materials or subcontract work, we have to decide upon an inspection procedure by means of which we can check whether the goods conform to the specifications. As mentioned above, we can either resort to 100 per cent inspection or to sampling techniques. Complete, or 100 per cent, inspection is usually considered to be more reliable, although there is ample evidence that it is by no means foolproof. Even competent inspectors err from time to time, passing faulty articles and rejecting good ones, and errors seem to mount when the inspection task is highly repetitive and exceedingly boring. Furthermore, 100 per cent inspection is fairly expensive, since every single unit requires handling, unpacking, and perhaps repacking, and is thus time consuming.

Probability of finding defectives in a sample

In acceptance sampling, decisions about the quality of the goods are made on the basis of a comparatively small sample taken from each consignment or lot.

If the sample conforms to the specifications, the whole lot is accepted; if it does not, the whole lot is rejected. In this way routine and boredom are eliminated, and there is a good chance that inspection will be carried out with great care and that results will be both reliable and fairly quick.

There is, however, a certain amount of risk involved in acceptance sampling: As the sample is taken at random from the lot, it is possible that it will not possess the same qualities as the lot, and if the sample fails to reveal defective components, the purchaser may accept a lot which in fact contains too high a percentage of defectives (this is the purchaser's risk). Alternatively the sample may contain too many defective items, leading to rejection of a lot that in fact has a lower percentage of defectives than a predetermined critical level (this is the vendor's risk).

What are the chances that the sample will reveal defective articles? Supposing we have a lot of 1,000 units, 100 of which are known to be defective. If a sample of n units is taken, and if the sample were to reveal the true character of the lot, we would have obtained after inspection $0.9n$ good units and $0.1n$ defectives. The probability that a unit taken from the lot at random would be a good one is 0.90; that it would be a faulty one, 0.10. The chances that two units taken in succession would be all right are 0.90×0.90; the chances for three good units in

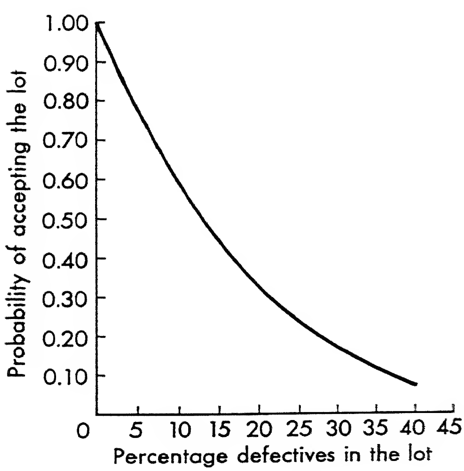

Figure 19–1. ***Probability of accepting a lot when the sample size is 5 and when a lot is accepted, provided no defectives are found.***

a row are 0.90^3, etc.; and the chances that the sample of five units will contain no defectives is $0.90^5 = 0.590$. In short, the probability that a sample of n units will have no defectives is given by q^n, where q is the actual proportion of good articles in the lot. If the sampling plan calls for rejection of the lot, should it reveal even one defective, the chances that it will be accepted when it does contain defective items is shown in Fig. 19–1, which indicates (for $n = 5$) the risk involved to the purchaser by using this particular method.

What are the chances that the sample will contain one defective, or two, or three? The answer is given by the expansion of the binomial expression $(p + q)^n$, where

p = proportion of defective articles in the lot

q = proportion of good articles in the lot (note that $p + q = 1$)

n = number of units in the sample

The chances that r units will be defective in the sample are given by the term $C_n^r p^r q^{n-r}$, or by

$$\frac{n!}{r!(n-r)!} p^r q^{n-r}$$

In the example cited above for a sample of five units, we have $p = 0.10$, $q = 0.90$, $n = 5$, and the probabilities for revealing rejects are given in the accompanying table.

No. of Defectives	*Probability*		
$r = 0$*	$\frac{5!}{0!\ 5!} 0.90^5$	$= 1 \times 0.90^5$	$= 0.590$
1	$\frac{5!}{1!\ 4!} 0.10 \times 0.90^4$	$= 5 \times 0.10 \times 0.90^4$	$= 0.328$
2	$\frac{5!}{2!\ 3!} 0.10^2 \times 0.90^3$	$= 10 \times 0.10^2 \times 0.90^3$	$= 0.073$
3	$\frac{5!}{3!\ 2!} 0.10^3 \times 0.90^2$	$= 10 \times 0.10^3 \times 0.90^2$	$= 0.008$
4	$\frac{5!}{4!\ 1!} 0.10^4 \times 0.90$	$= 5 \times 0.10^4 \times 0.90$	$= 0.0005$
5	$\frac{5!}{5!\ 0!} 0.10^5$	$= 1 \times 0.10^5$	$= 0.00001$
		Total	$= 1.000$

* By definition, $0! = 1$.

Some sampling plans specify acceptance if no more than a certain number of defectives is found in the sample. If in the above example no more than one defective per sample is allowed, the probability of acceptance is the sum of probabilities for $r = 0$ and $r = 1$, or $0.590 + 0.328 = 0.918$. Thus the risk for the purchaser increases accordingly, as suggested in Fig. 19–2, where the lower curve stands for the probability of acceptance when $r = 0$ (as in Fig. 19–1), the the second curve is for $r = 1$, and the top curve is the sum of the other two.

The purchaser is naturally interested in reducing his risk as much as possible. The producer, too, wants to minimize the risk of his products being turned down on poor evidence. But both sides realize that absolute certainty can be attained only with 100 per cent inspection (assuming no errors in inspection), which is far too costly. This, in short, is the purpose of acceptance sampling: to provide

economic procedures that have a fairly good chance of revealing the true character of the lot, and thus become a part of a formal agreement between vendor and purchaser as to the definition of quality and determination of whether the lot complies with it.

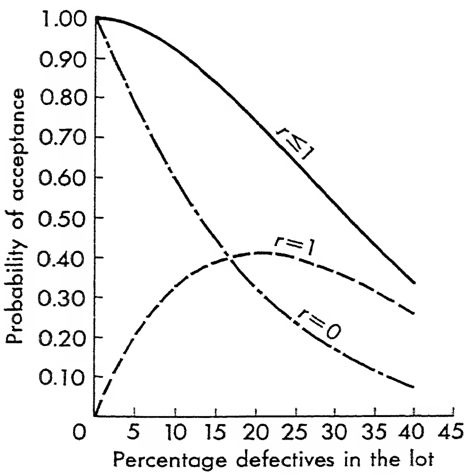

Figure 19–2. ***Probability of accepting a lot when the sample may contain no more than the defective sample size, $n = 5$.***

Sampling plans

The two main factors that characterize acceptance sampling procedures are the sample size and the sampling plan, or the number of successive samples that need be taken in the process of arriving at a final decision about the quality of the lot. Determining the sample size as a certain percentage of the lot is common when the lot is comparatively small; otherwise the sampling plans usually call for a certain number of units to be inspected, irrespective of the lot size. As for sampling plans, they may be classified into three broad groups (see Fig. 19–3).

Single sampling plans

One sample consisting of a certain number of units n is inspected. If the number of defectives found is below or equal to a certain predetermined acceptance limit, r, the lot is accepted; if the number of defectives exceeds this limit, the lot is rejected.

Double sampling plans

A sample of n units is inspected. If the number of defects is below r_1, the lot is accepted; if it is above a second limit, r_2 (where $r_2 > r_1$), the lot is rejected. If the number of defectives falls between r_1 and r_2, the result is inconclusive and a second sample is taken. The rule now becomes similar to that of a single sampling plan: If the total number of defectives of the two samples is below a predetermined limit r_3, the lot is accepted; otherwise it is rejected.

Multiple or sequential sampling plans

This is similar to the double sampling plan except that with the second sample we have again an inconclusive range between r_3 and r_4. Below r_3, the lot is accepted; above r_4, it is rejected; and if the total number of defectives is between r_3 and r_4, a third sample is taken; and so on. Eventually, after a number of samples, the inspector must come to a final decision, and one critical limit is set

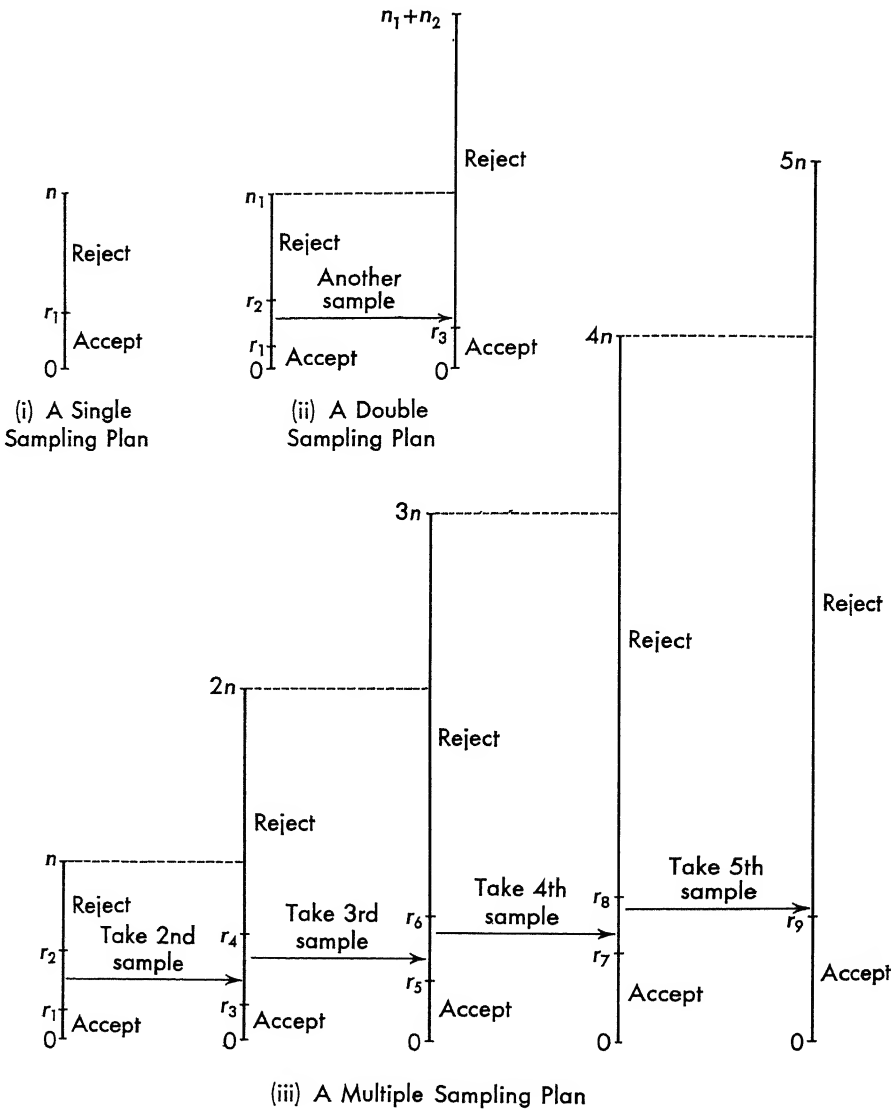

Figure 19–3. ***Single, double, and multiple sampling plans.***

(as in the single sampling plan), which determines whether the lot is accepted or rejected. An example for a multiple sampling plan is shown in Table 19–1 and presented graphically in Fig. 19–4.

Table 19–1

An Example for a Multiple Sampling Plan

(the total number of defectives is r)

Sample No.	*Size*	*Cumulative Number Inspected*	*If r ⩽*	*If r ⩾*
1	10	10	0	4
2	10	20	1	5
3	10	30	3	7
4	10	40	4	8
5	10	50	5	9
6	10	60	7	10
7	10	70	10	11
			↓ *Accept*	↓ *Reject*

The sample size and the sampling plan are obviously interdependent. In double and multiple sampling plans the sample sizes are smaller than those in

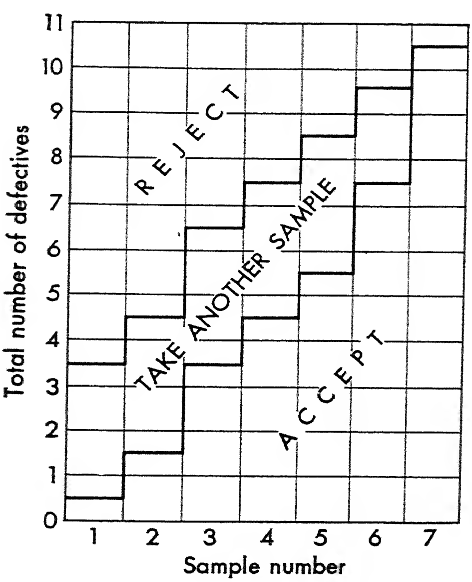

Figure 19–4. ***An example for a multiple sampling plan.*** (*See* Table 19–1)

the single plans, the main purpose being to save labor in case the lot in question is either very good or very bad. If the issue is clear cut, there is no need to take further samples, but when the lot is of moderate quality level, the full procedure

of the multiple sampling scheme has to be used. The advantage of the single sampling plan is that it is simple to use and to explain, while the multiple schemes are usually more economic to use.

Acceptance sampling is one of the factors that figures prominently in assessing the reliability of vendors or subcontractors. The other main factors are compliance with delivery dates and supply of the quantities specified in the orders. An example of a vendor's card record is shown in Fig. 19–5.

VENDOR:

ADDRESS:

COVERING PERIOD FROM .... TO ......

ORDER NO.	PART NO.	SCHED. DELIVERY	ACTUAL DELIVERY	QUANTITY ORDERED	QUANTITY RECEIVED	SAMPLING PLAN	NO. INSPECTED	DEFECTIVES IN SAMPLE	% DEFECTIVES	* DECISION	% DEFECTIVES 0 1 2 3 4 5 6 7
209	J/191	4/1	4/1	6,000	6,000	D-1 (1)	50	1	2.0	A	
218	ME/52	4/20	4/22 !	6,000	6,000	D-1	150	4	2.7	A	
231	J/180	5/1	4/25	5,000	5,000	D-1	150	5	3.3	A	
233	J/180	5/5	5/4	4,000	4,000	D-1	150	7	4.7	S (2)	*
236	J/191	5/10	5/9	4,800	4,800	D-1	150	9	6.0	R	

REMARKS: (1) Double sampling plan: Accept Reject
1st sample 50 – 2 7
2nd " 100 – 6 7
(2) Actual defective found · 5.0%
! Late delivery

* A = ACCEPT
R = REJECT
S = 100% INSPECTION AT VENDOR'S EXPENSE
SX = 100% INSPECTION AT OUR EXPENSE

Figure 19–5. ***Vendor's record card.***

Process Capability

Every production engineer knows that repetitive production cycles (seemingly identical in their conditions and governing parameters) result in products that are not identical in their characteristics. The reason for this is simply that the production cycles are not really identical in every respect. First, the raw materials vary slightly in their properties from batch to batch, and these variations may affect the properties of the final product. Secondly, operators are different from each other in their skill, performance, aptitude, and interest in their work. Thirdly, the parameters of the operations may change from cycle to cycle, as may the speeds, the motions of the operators, and the characteristics of the tools and machines. Fourthly, external conditions such as temperature, pressure, light, and humidity may vary. These causes for variations in the production process are summarized in Fig. 19–6.

Inherent process variability

The inherent variabilities of production processes have long been recognized; hence the specifications of tolerances. In specifying physical dimensions, strength, density, composition, etc., an ideal quantity is indicated, but at the same time it is pointed out that deviations from this ideal figure will be tolerated, provided they remain within predetermined limits. One of the purposes of quality control is to decide whether these tolerances are compatible with the inherent variabilities of the processes.

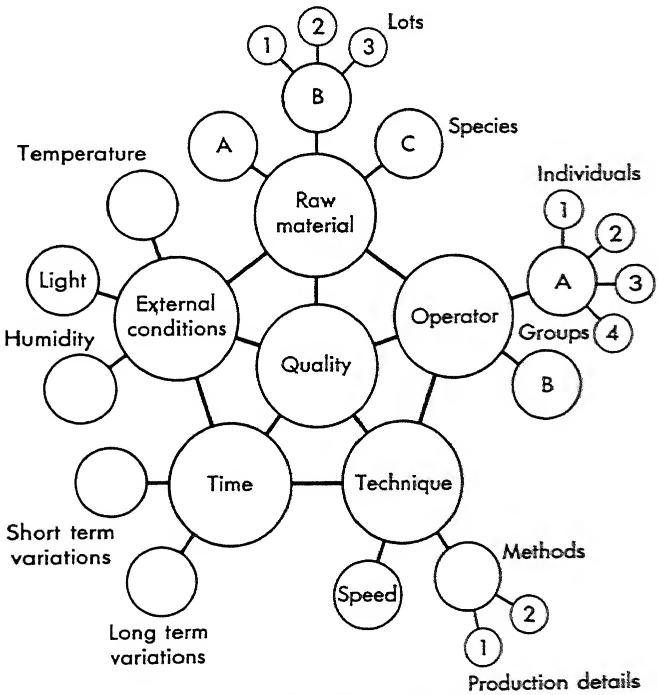

Figure 19–6. ***Causes of variation in the production process according to A. Hald.*** (Courtesy EPA, Paris; reproduced from the OEEC pamphlet, "Statistical Quality Control," 1956)

Process variability is often demonstrated by a pinball device, as shown in Fig. 19–7. Steel balls or beads are let from a store through a hopper onto an array of pins. The balls get knocked about by the pins as they fall; sometimes they are pushed to the right, sometimes to the left, until eventually they are collected in the grooves at the bottom part of the device. The distribution of balls in the grooves (see bottom of Fig. 19–7) is said to be analogous to the frequency distribution of a batch of components when they are classified, say, according to their outer dimension. The peak of the distribution (or the groove

that has the largest number of balls) corresponds to the dimension that occurs most often, the grooves on the right representing larger dimensions, and the ones on the left, smaller dimensions.

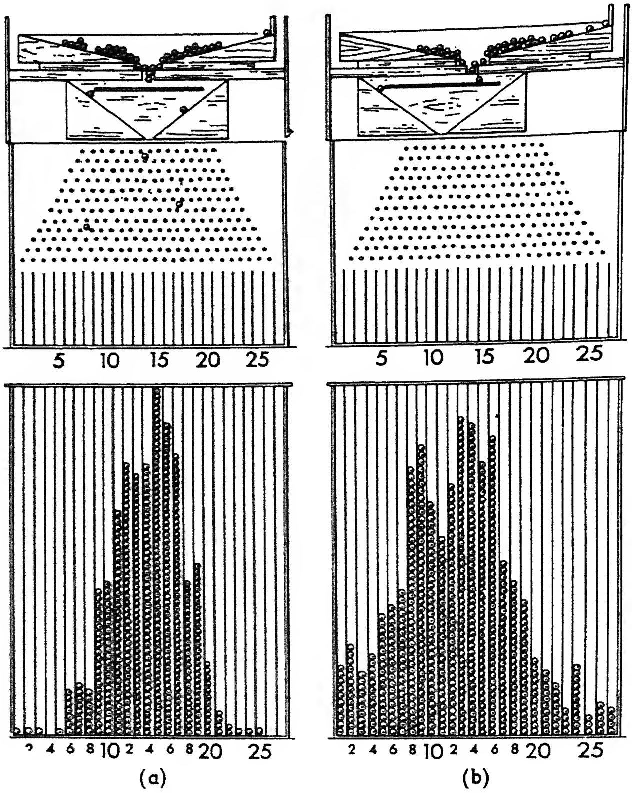

Figure 19–7. ***A pinball device to illustrate process variability.*** (Courtesy EPA, Paris; reproduced for the OEEC pamphlet, "Statistical Quality Control," 1956)

If a large enough number of balls are fed through the hopper, or if a large enough number of components are gaged, the results would usually be represented by the normal (or Gaussian) distribution, as shown in Fig. 19–8. Because it is so common, the statistical procedures in quality control are based on the characteristics of this distribution. A table for the standard normal distribution is given as in the Appendix to this book, from which the reader will observe that (see Fig. 19–8):

Within the limits $\pm 0.67\sigma$ of the center line there will be 50% of the cases
Within the limits $\pm\sigma$ of the center line there will be 68.3% of the cases
Within the limits $\pm 2\sigma$ of the center line there will be 95.5% of the cases
Within the limits $\pm 3\sigma$ of the center line there will be 99.7% of the cases

where σ is the standard deviation of the distribution. This, in fact, is the basis for control charts in quality control, in which $\pm 3\sigma$ limits are usually adopted; in other words, we expect 99.7 per cent of the results to fall within the $\pm 3\sigma$ limits,

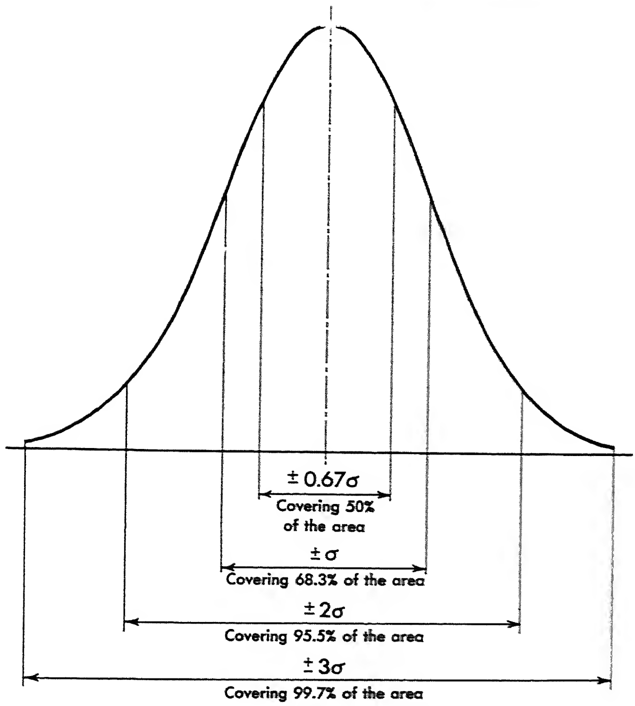

Figure 19–8. ***The normal curve.***

and if results begin to appear beyond those limits, we begin to feel uncomfortable.

In using statistical methods for process control, we attempt to distinguish between two sources of variability:

1. Inherent process variability, caused by the factors shown in Fig. 19–6.
2. Additional, so to speak, "external" causes that result in consistent trends or shifts of the mean value of the measured attribute.

The inherent process variability cannot be removed unless some basic change in the conditions is introduced, such as an over-all repair or a change of machines. As long as the process variations are confined to the inherent process variability, the process is said to be in statistical control, and the study of this inherent variability reveals whether the process is capable of producing work within the specifications. Figure 19–9 shows the process variabilities of two

machines A and B. Machine A has a far narrower spread than that of machine B, the spread being determined by 6σ of the corresponding frequency distribution (i.e., the machine is expected to produce work within this range in 99.7 per cent of the time). Suppose the specified dimensions of the component are $x \pm a$. If the tolerances are narrower than the process variability of machine B but wider than that of machine A, i.e.,

$$6\sigma_A < 2a < 6\sigma_B$$

as shown in Fig. 19–9, and provided the mean dimension of the products can be made to coincide with x, then we may conclude that:

1. Machine A is more suitable than machine B; even small shifts in the mean dimension will not result in any appreciable amount of scrap.
2. Machine B will produce a certain amount of defectives and may be used if 100 per cent inspection is undertaken to sort out the good from the bad after production.

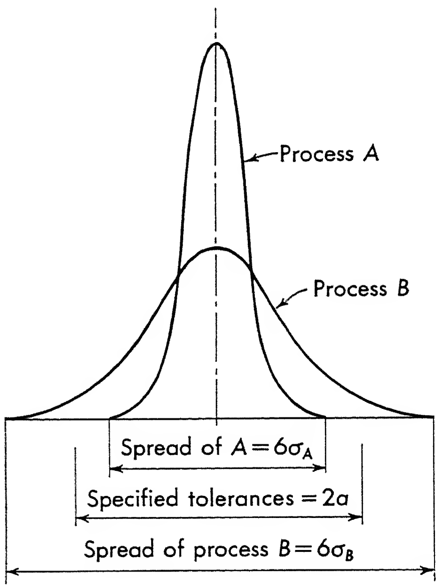

Figure 19–9. ***Process capability.***

These remarks should not be interpreted to mean that the narrower the process variability, the better. If in the above example $2a > 6\sigma_B$, machine B is perfectly capable of handling the job, and we may find that by assigning it to machine A, we are using a machine that is far too precise, and too costly. The study of process capability is, therefore, an economic problem that has to take into account the chances and effects of shifts in the mean, the cost of operating the machines, the cost of sorting when 100 per cent inspection becomes necessary, and the loss due to scrap.

Analysis of capability

Process capability studies are carried out by three methods:

1. *Analysis of control charts of past performances*, from which one can find the $\pm 3\sigma$ limits of the process for different types of work (control charts are described later).

2. *Conducting a trial run* for 100 pieces of a representative component and recording the dimensions in a form of a frequency distribution, from which the mean dimension $\bar{X}$ and the standard deviation σ are calculated by

$$\bar{X} = \frac{1}{n} \sum f X \tag{19–1}$$

$$\sigma = \sqrt{\frac{\sum f X^2}{n} - \bar{X}^2} \tag{19–2}$$

where X = actual dimensions
f = frequency of occurrence of each dimension
n = total number of measurements (in this case 100)

An example is shown in Table 19–2.

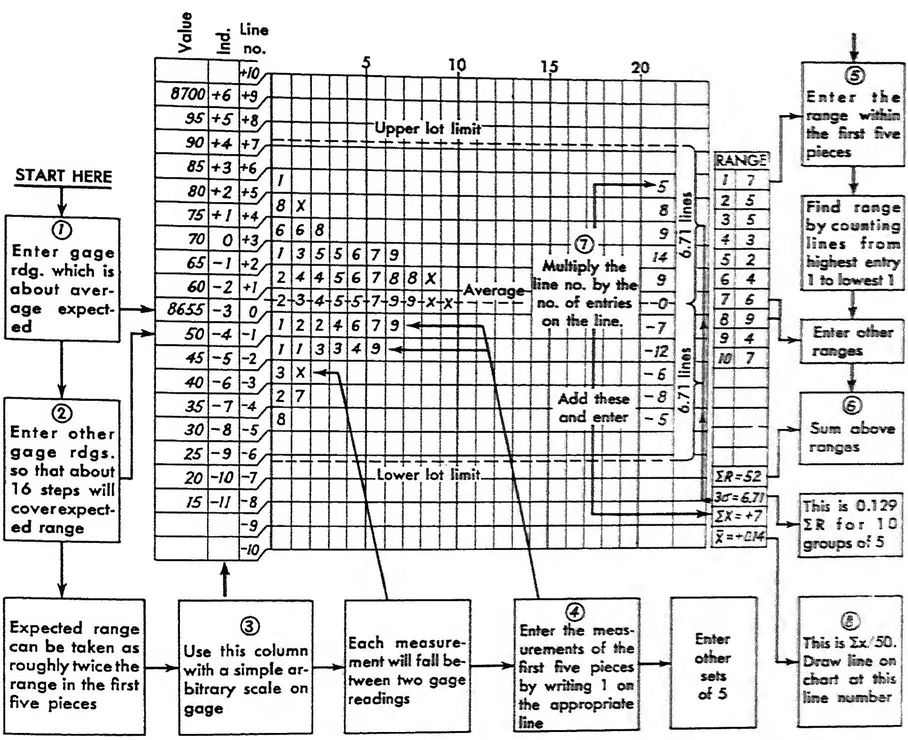

Figure 19–10. ***Example for a provisional determination of machine capability.*** (From "Inspection in Industry," a Productivity Report, courtesy British Productivity Council, 1953)

3. *Calculation of the standard deviation* through sampling, by taking, say, ten samples of five units each. An example is shown in Fig. 19–10, in which instructions for use have been incorporated.

Table 19–2

Determining Process Variability by a Trial Run of 100 Units

Dimension X (*inch*)	*Deviation* x *from* 0.500 ($\times$ 10^{-3} inch)	*Number Found*	*Frequency* f	fx	fx^2
0.490	−10	/	1	−10	100
0.491	− 9		0	0	0
0.492	− 8	/	1	− 8	64
0.493	− 7		0	0	0
0.494	− 6	/	1	− 6	36
0.495	− 5	/	1	− 5	25
0.496	− 4	//	2	− 8	32
0.497	− 3	///	3	− 9	27
0.498	− 2	//	2	− 4	8
0.499	− 1	~~////~~ /	6	− 6	6
0.500	0	~~////~~ ///	8	0	0
0.501	+ 1	~~////~~ ~~////~~ ~~////~~	15	15	15
0.502	+ 2	~~////~~ ~~////~~ ~~////~~ ~~////~~	20	40	80
0.503	+ 3	~~////~~ ~~////~~ ~~////~~	15	30	135
0.504	+ 4	~~////~~ ///	8	32	128
0.505	+ 5	~~////~~ /	6	30	150
0.506	+ 6	////	4	24	144
0.507	+ 7	///	3	21	147
0.508	+ 8	//	2	16	128
0.509	+ 9	/	1	9	81
0.510	+10	/	1	10	100
Total			100	$\Sigma fx = 171$	$\Sigma fx^2 = 1{,}406$

$$x = \frac{171}{100} = 1.71; \qquad \bar{X} = 0.500 + 0.0017 \simeq 0.502$$

$$\sigma = \sqrt{\frac{1{,}406}{100} - 1.71^2} = 3.32$$

hence process variability is $6\sigma = 19.9$ thousandths of an inch.

Process Control

One of the difficult tasks in quality control is to determine how and at what stages in the production sequence to apply process control. Yet, this is very important for obvious reasons: Exercising quality control where it is likely to yield little benefit is a waste of time and money; furthermore, quality control often involves the collection of great masses of data, some of which may be absolutely vital for analysis, some of which may later prove to be utterly superfluous. How much data should be collected, bearing in mind that data collection and record keeping is rather an expensive business?

Quantity and quality of data

Naturally, no clear-cut answer can be given to this problem, except that the amount of data collected should be geared to the amount of variations that occur. The more stable the conditions that govern an operation, the less observations are required to provide adequate information about its parameters. But precisely how much information? That depends on the level of quality of the final product, on the company's policy regarding quality, on whether processes demand more urgent attention; in short, it depends on an *ad hoc* evaluation of the quality situation in the plant.

Process control is greatly aided by continuous measurement of relevant parameters that govern the process. Fairly common in industry are measurement and recording of temperature, pressure, humidity, velocity, forces, etc., and operators and machines are often guided by time devices to tell them when to start and when to stop an operation. Figure 19–11 is an example of a record of torque measurement in the mixing of dough in the food industry. The curve shows how the consistency of the dough increases with time during the "development" period, maximum consistency is maintained during the "stability" period, after which the consistency begins to fall. The change in the consistency characteristics of the dough with mixing time, as shown by the graph, facilitates a process control through which dough with desirable physical properties can be obtained.

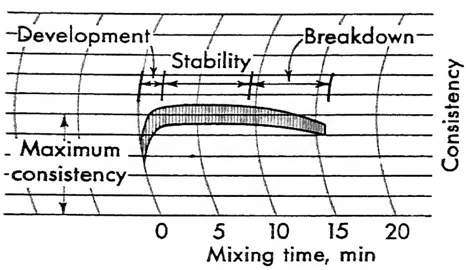

Figure 19–11. ***An example of process control—mixing of dough.*** (From "Quality Control in the Food Industry," by J. H. Bushill, Journal of the Institution of Production Engineers, December, 1951).

Another example is shown in Fig. 19–12, which is a similar record of pressure measuren e :ts in the mixing of rubber in a mill prior to tire making. The raw material in this case was manually fed into a rolling mill, and as the rollers ran at different speeds, the effect of the rolling was to spread the material on one roller as a thick sheet. The operator's task was to peel off this sheet repeatedly and refeed the material to the mill in order to obtain rubber of even consistency. In feeding the material, pressure was applied to the rollers, and this is shown in Fig. 19–12 by the fluctuating curve. The total time of the operation for mixing one 200-pound load of rubber is given by one cycle of the graph.

Records of this kind may be very useful and instructive in studying optimal process conditions. The rolling mill for the rubber, for instance, was run 24 hours a day by three operators. The mixing time per load was specified in the instruc-

tion sheet as 14.5 minutes (including the initial loading operation and final unloading), but the total output figures suggested that the cycle-time was in fact significantly different and that, on the average, a load was not kept in the mill as long as it should have been.

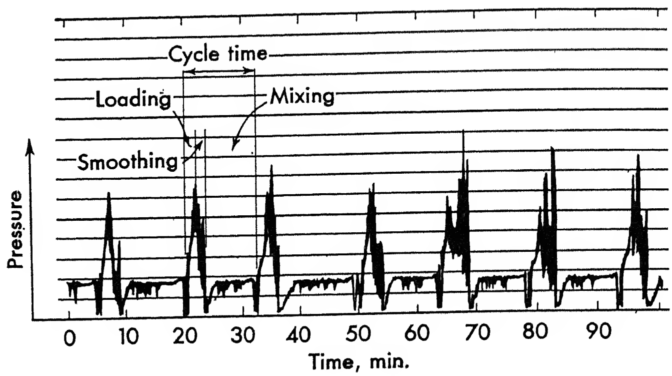

Figure 19–12. ***An example of process control—mixing time of rubber in a mill in the manufacture of tires.***

Analysis of the records for mixing time per load showed great variations between the operators and also that the variations in performance for each operator were rather wide, as suggested by Fig. 19–13. Since the beginning and end of each cycle was solely determined by the operator and since he had no

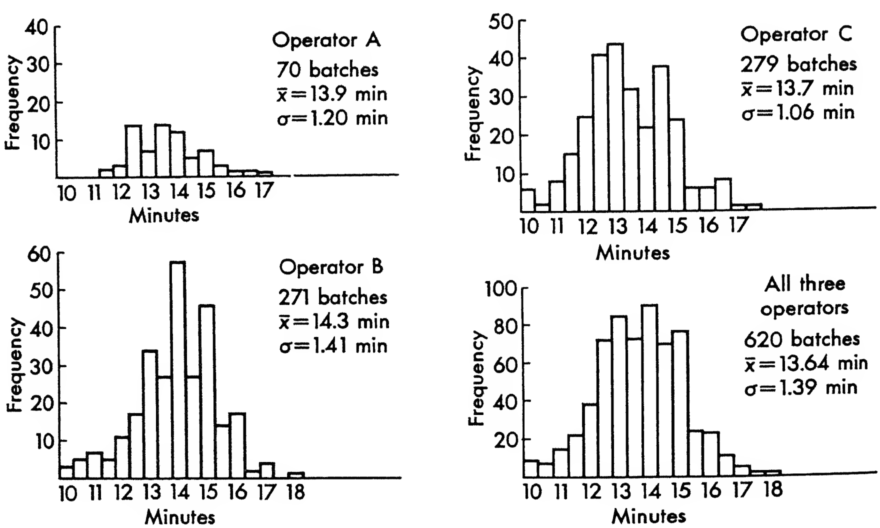

Figure 19–13. ***Cycle-time variations of three operators working on a mill for mixing rubber—before training.*** (Cycle-time specified in the job card = 14.5 minutes.)

timing device to tell him how long he had been mixing any particular load, the results were not really surprising. After these findings had been explained to the operators, their performance greatly improved (as suggested by Fig. 19–14), but the need for training and introduction of timing devices (using a bell or a system of lights) to guide the operators was definitely indicated. This example shows how process control can be instructive and can lead to improved performance.

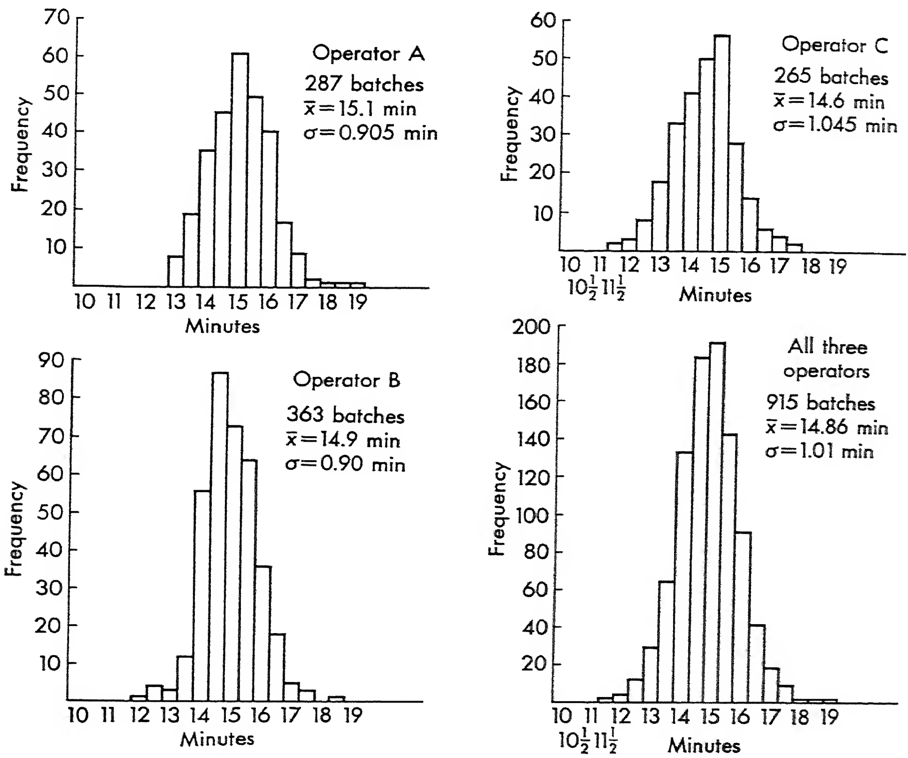

Figure 19–14. ***Cycle-time variations of the three operators—after some training.*** (*See* Figure 19–13).

Control Charts in Statistical Quality Control

Three charts are commonly used in statistical quality control:

1. $\bar{X}$ and R chart, for process control
2. p chart, for analysis of fraction defectives
3. c chart, for control of number of defects per unit

The descriptions that follow must of necessity be brief, and for a thorough study of these tools and the statistical theory on which they are based, the reader is

advised to consult special treatises on quality control, some of which are mentioned in the reference list at the end of the chapter.

The $\bar{X}$ and R chart

The purpose of this chart is to:

Establish whether the process is in statistical control, in which case the variations are attributed to chance. The variability that is inherent in the process cannot be removed, unless there is a change in the basic conditions under which the process is operating.

Guide the production engineer in determining whether the process capability is compatible with the specifications.

Detect trends in the process, so as to assist in planning adjustment and resetting of the process, or to show when the process is out of control, in which case an effort must be made to trace the causes for this phenomenon.

The procedure is as follows: A number of samples of components coming out of the process are taken over a period of time, each sample consisting of a number of units taken at random. For each sample the average value $\bar{X}$ of all the measurements and the range R (= the difference between the highest and the lowest reading) are calculated. The grand average $\bar{\bar{X}}$ (= the average value of all the averages $\bar{X}$) and the average range $\bar{R}$ (= the average of all the ranges R) are then found, and from these we can calculate the control limits of the $\bar{X}$ and R charts, in which UCL is the abbreviation for upper control limit and LCL for lower control limit.

$$\left.\begin{array}{ll} \bar{X} \textit{ chart:} & \text{UCL } \bar{X} = \bar{\bar{X}} + A_2\bar{R} \\ & \text{LCL } \bar{X} = \bar{\bar{X}} - A_2\bar{R} \\ R \textit{ chart:} & \text{UCL } R = D_4\bar{R} \\ & \text{LCL } R = D_3\bar{R} \end{array}\right\} \quad (19\text{–}3)$$

where the factors A_2, D_4, and D_3 naturally depend on the number of items per sample; the larger this number, the closer the limits. Table 19–3 gives values for these factors for various sample sizes (the symbols being those conventionally used in statistical quality control), based on the assumption that the distribution is normal. As long as the $\bar{X}$ and R values for each sample are within the control limits, the process is said to be in statistical control.

Table 19–3

Factors Used in the $\bar{X}$ and R Quality Control Charts

(based on normal distribution)

No. of Items in Sample	A_2	D_3	D_4
2	1.88	0	3.27
3	1.02	0	2.57
4	0.73	0	2.28
5	0.58	0	2.11

Table 19–3 (*continued*)

Factors Used in the $\bar{X}$ and R Quality Control Charts

(*based on normal distribution*)

No. of Items in Sample	A_2	D_3	D_4
6	0.48	0	2.00
7	0.42	0.08	1.92
8	0.37	0.14	1.86
9	0.34	0.18	1.82
10	0.31	0.22	1.78
11	0.29	0.26	1.74
12	0.27	0.28	1.72
13	0.25	0.31	1.69
14	0.24	0.33	1.67
15	0.22	0.35	1.65

Control chart for $\bar{X}$: UCL $\bar{X} = \bar{\bar{X}} + A_2\bar{R}$
LCL $\bar{X} = \bar{\bar{X}} - A_2\bar{R}$

Control chart for R: UCL $R = D_4\bar{R}$
LCL $R = D_3\bar{R}$

Example

In the production of an aluminium strip by a rolling process, the specified thickness being 0.250 ($\pm$0.010) inch, samples were taken every 30 minutes for thickness measurements, which are given in Table 19–4. Five measurements were taken per sample; the average $\bar{X}$ per sample and the range R are shown in the last two columns. The grand average is

$$\bar{\bar{X}} = \frac{\Sigma\bar{X}}{N} = \frac{6{,}300}{25}10^{-3} = 0.252 \text{ in.}$$

where N is the number of samples taken and the average range is

$$\bar{R} = \frac{\Sigma R}{N} = \frac{560}{25}10^{-3} = 0.0224 \text{ in.}$$

From Table 19–3 we find that for sample size of five items. $A_2 = 0.58$; $D_3 = 0$; $D_4 = 2.11$. Hence

Table 19–4

Samples of Aluminium Strip Thickness Taken Every 30 Minutes

Sample No.	*Five Measurements Per Sample (in thousandths of one inch)*					*Average* ($\bar{X}$)	*Range* (R)
1	248	264	250	262	256	256	20
2	236	269	258	240	252	251	33
3	251	235	258	247	239	246	23
4	255	263	259	256	257	258	8
5	262	250	266	269	238	257	31
6	258	263	244	243	242	250	21
7	247	251	249	240	253	248	24

Table 19–4 (*continued*)

Samples of Aluminium Strip Thickness Taken Every 30 Minutes

Sample No.	*Five Measurements Per Sample (in thousandths of one inch)*					*Average* ($\bar{X}$)	*Range* (R)
8	242	237	236	251	249	243	25
9	270	254	266	258	262	262	16
10	242	239	238	260	246	245	34
11	250	264	252	251	253	254	14
12	265	248	251	247	249	252	18
13	255	266	264	258	262	261	11
14	258	243	250	257	267	255	24
15	248	267	261	250	264	258	19
16	235	250	236	237	237	239	15
17	251	258	236	270	260	255	34
18	245	261	240	245	249	248	36
19	250	257	256	253	254	254	7
20	235	235	255	235	235	239	20
21	255	255	241	245	254	250	40
22	270	244	269	262	265	262	26
23	269	246	258	250	257	256	23
24	253	246	251	260	245	251	15
25	240	263	257	241	249	250	23
Total						$\Sigma\bar{X} = 6{,}300$	$\Sigma R = 560$

$$\text{UCL } \bar{X} = \bar{\bar{X}} + A_2\bar{R} = 0.252 + 0.58 \times 0.0224 = 0.265$$
$$\text{LCL } \bar{X} = \bar{\bar{X}} - A_2\bar{R} = 0.252 - 0.58 \times 0.0224 = 0.239$$
$$\text{UCL } R = D_4\bar{R} = 2.11 \times 0.0224 = 0.0473$$
$$\text{LCL } R = D_3\bar{R} = 0$$

Figure 19–15 consists of three charts:

1. Individual measurements; this chart gives a general picture of the dispersion of results.
2. $\bar{X}$ chart
3. R chart

$\left.\right\}$ from these we see that the process is in statistical control.

The control procedure associated with the $\bar{X}$ and R charts is summarized in Fig. 19–16. Having drawn the charts and the control limits, it is necessary to determine whether the process is in statistical control. If it is not, there must be an external cause that throws the process out of control, a cause that is beyond the "natural" inherent variability of the process. This cause must be traced and eliminated so that the process may return to operation under stable statistical conditions. Reasons for the process being out of control vary from faulty tools, a sudden significant change in properties of new materials in a new consignment, breakdown of the lubrication system, faults in timing or speed mechanisms, etc. Tracing these causes is sometimes simple and straightforward, but in some cases it may be a rather lengthy and complicated business, especially when the process is subject to the combined effect of several external causes simultaneously.

If the process is found to be in statistical control, a comparison between the required specifications and the process capability may be carried out to determine whether the two are compatible. Should the specified tolerances prove to be too tight for the process capability, there are three possible alternatives:

1. Re-evaluate the specifications: Are the tight tolerances really necessary for effective performance, or could they, perhaps, be relaxed with no detriment to the quality of the product?
2. If relaxation of the specifications is not acceptable, perhaps a more accurate process should be selected for the purpose?
3. If both the previous alternatives are out of the question, a 100 per cent inspection must be undertaken, to sort out the defective products.

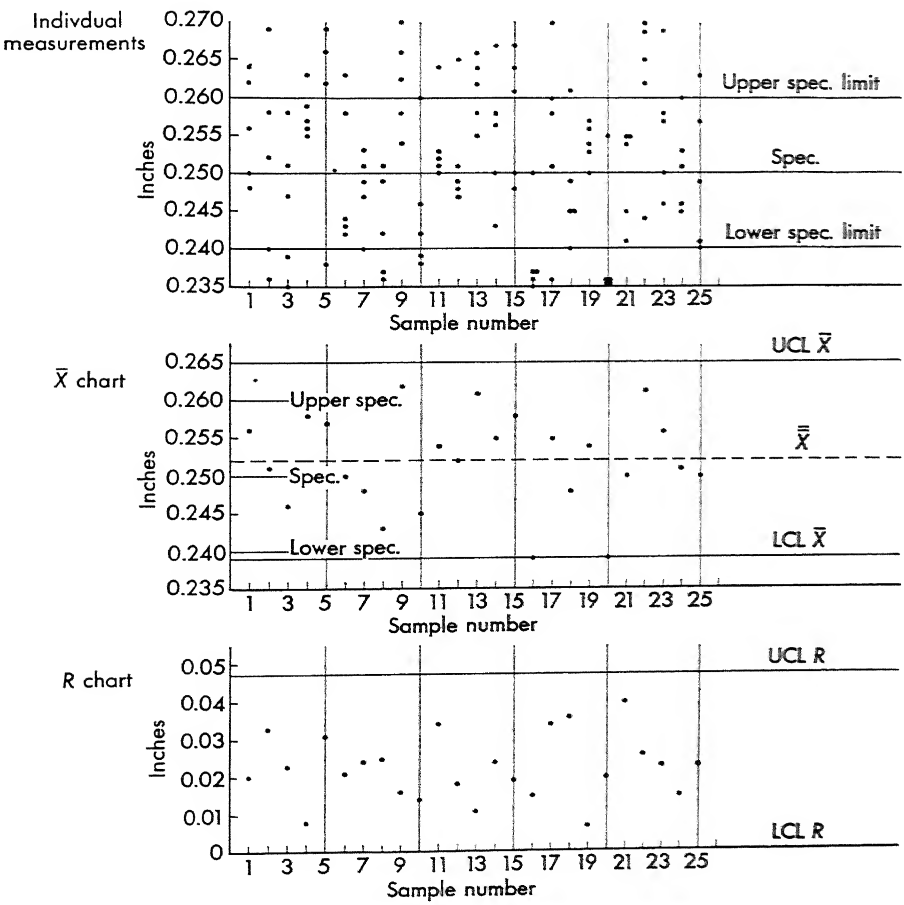

Figure 19–15. ***$\bar{X}$ and R control charts.*** (*See* Table 4)

The $\bar{X}$ and R charts are also useful for the purpose of detecting trends in production, causing a consistent shift in the mean $\bar{\bar{X}}$. Tool wear and the need for resetting machines often account for such a shift, and it is essential to determine when machine resetting becomes desirable, bearing in mind that too frequent adjustments are a serious setback to production output.

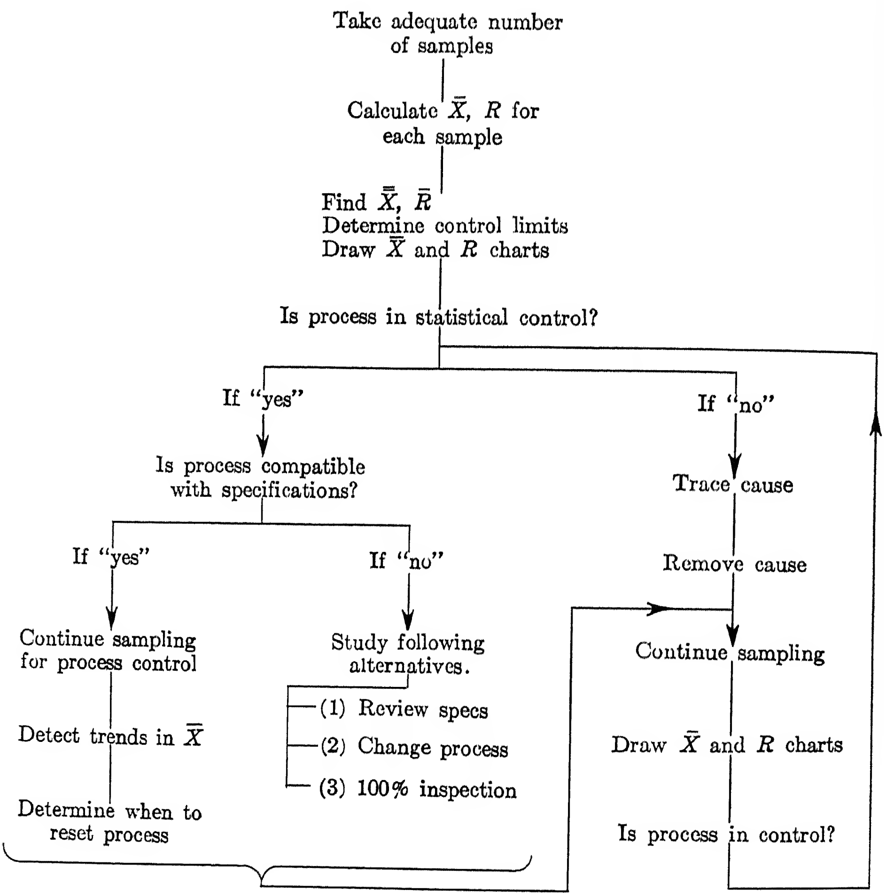

Figure 19–16. ***Control procedure for $\bar{X}$ and R charts.***

Figure 19–17 shows four examples of $\bar{X}$ charts. The first three illustrate the relationship of process variability to the specified tolerances; the fourth chart is an example of an adequate process, from the point of view of the specifications, but there is a consistent shift in $\bar{X}$, which makes it necessary to reset the machine periodically in order to bring down the value of $\bar{X}$ to a desirable level. This shift in $\bar{X}$ is caused by some basic change in the process conditions. Using the pinball

device in Fig. 19–7 as an illustration, there is a change in the distribution of balls collected in the grooves when the hopper feeds the balls from a different position (as shown in Fig. 19–7b). Resetting of the hopper's position is required,

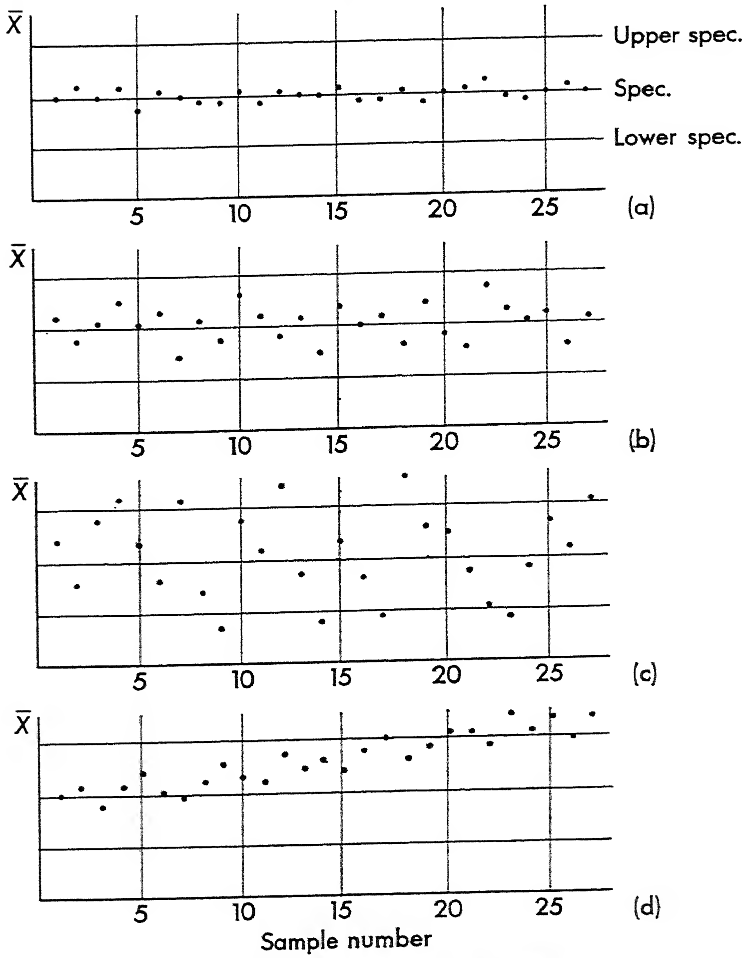

Figure 19–17. ***Examples of $\bar{X}$ charts.***

(a) Process variability comparatively small.
(b) Spread is wider, but still adequate.
(c) Process not compatible with specifications.
(d) Periodic resetting required due to upward trend in $\bar{X}$.

if the original conditions are to be regained. Evidently the problem of resetting is closely associated with the relationship between process capability and the specifications. Case (a) in Fig. 19–17 would require a smaller number of machine resets than case (b). This is further illustrated in Fig. 19–18. In case (a), the

mean value $\bar{\bar{X}}$ can shift a great deal without causing a noticeable increase in the amount of defective items. In case (c), the process $\pm 3\sigma$ spread is slightly wider than the specified tolerance, so that the amount of defectives becomes quite sensitive to the level of $\bar{X}$; even a comparatively small shift results in an appreciable increase of oversized or undersized items, and frequent process adjustment becomes inevitable.

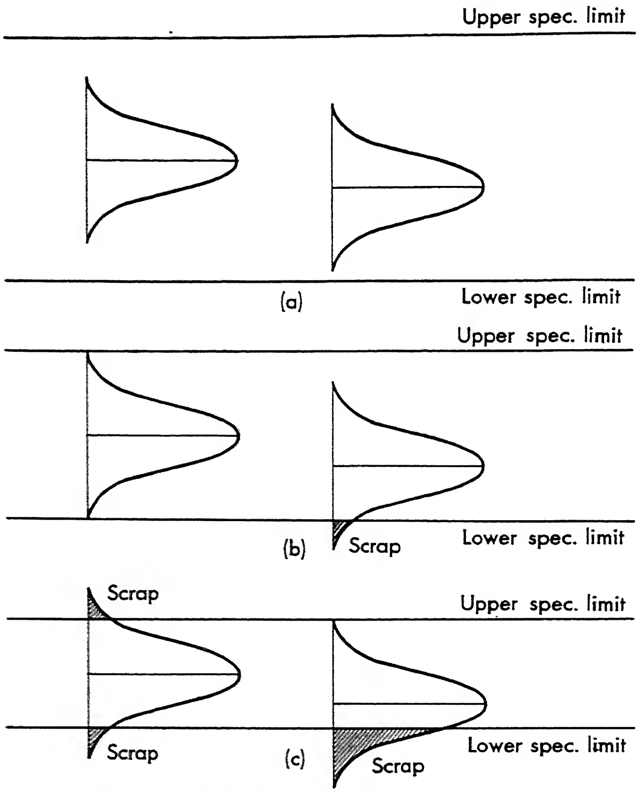

Figure 19–18. ***The effect of a shift in the mean on the amount of scrap.***

(a) Wide tolerances compared with process capability.
(b) Tolerances comparable to process capability.
(c) Tolerances are comparatively tight, resulting in excessive scrap, especially in case of a shift in the mean.

Left: The mean is halfway between the lower and upper spec. limits.
Right: A downward shift of the mean.

The p chart

The p (or "fraction defective") chart is employed to control the general quality of the product and, in particular, to ascertain whether fluctuations of the quality level of the product are due to chance alone. Again, the procedure is to

determine control limits, and if the defective fraction is within these limits, the quality level is considered to be in statistical control; if it is not, there is a strong indication that one of the processes or operations is out of control and that further investigation is necessary.

The basic difference between the $\bar{X}$ and R chart and the p chart is that the former is based on control by attributes; in other words, by some specific measurable quality characteristics. We examine an operation and decide what are the main features that ought to be measured and proceed to construct an $\bar{X}$ and R control chart for each feature or *attribute* (this is the formal term used in statistical quality control). First, however, we often find these attributes difficult to define and to measure quantitatively, and secondly (even if we could) the number of charts that would be required would be prohibitive. The $\bar{X}$ and R chart, though an effective tool, cannot be used in excess, and sometimes cannot be used at all. The p chart, on the other hand, keeps a record of the percentage of defective items, irrespective of the cause of the defects, and thereby provides an over-all picture of quality level. It can sometimes replace the $\bar{X}$ and R chart if, say, "go–no go" gages are used, without actual measurements of the attribute being recorded. It may suggest that some processes should be subjected to $\bar{X}$ and R control charts, and in this way initiate and coordinate further investigation and control.

Fraction defective p is simply defined as the ratio of the number of defective items to the total number of items inspected. The control limits of the charts are given by:

$$\begin{aligned} \text{UCL } p &= p' + 3\sigma_{p'} \\ \text{LCL } p &= p' - 3\sigma_{p'} \end{aligned} \tag{19–4}$$

where p' is the desirable ratio of the total number of defectives found to the total number inspected; $\sigma_{p'}$ is the standard deviation of the desirable distribution of fraction defectives, and from statistical theory it is known that

$$\sigma_{p'} = \sqrt{\frac{p'(1 - p')}{n}} \tag{19–5}$$

where n is the number of units inspected. To start with, the value of p' is estimated by $\bar{p}$, which is the ratio of the total number of defectives actually found in all samples to the total number inspected in all samples. Hence the control limits of the p chart, when p' is substituted by $\bar{p}$ (called *trial control limits* for p), are

$$\begin{aligned} \text{UCL } p &= \bar{p} + 3\sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} \\ \text{LCL } p &= \bar{p} - 3\sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} \end{aligned} \tag{19–6}$$

Example

In the production of tires, the output of a given size was inspected every day prior to the tires being delivered to the finished goods stores. The number of defectives found every day was summarized in Table 19–5, from which the mean fraction defective $\bar{p}$ and the control limits were calculated. The control chart for fraction defective for this example is shown in Fig. 19–19.

Table 19–5

Inspection Results for Percentage Defectives of Tires

Date	*Number Inspected*	*Defectives*	*Fraction Defective* (p)	3σ	$\bar{p} + 3\sigma$	$\bar{p} - 3\sigma$
March 1	600	77	0.128	0.040	0.164	0.084
2	500	78	0.156	0.044	0.168	0.080
3	540	66	0.122	0.043	0.167	0.081
4	620	93	0.150	0.040	0.164	0.084
5	680	99	0.146	0.038	0.162	0.086
6	660	112	0.170	0.039	0.163	0.085
8	660	79	0.120	0.039	0.163	0.085
9	720	89	0.124	0.037	0.161	0.087
10	750	80	0.107	0.036	0.160	0.088
11	710	85	0.120	0.037	0.161	0.087
12	680	73	0.107	0.038	0.162	0.086
13	660	74	0.112	0.039	0.163	0.085
15	660	83	0.126	0.039	0.163	0.085
16	500	68	0.136	0.044	0.168	0.080
17	540	54	0.100	0.043	0.167	0.081
18	580	61	0.105	0.041	0.165	0.083
19	620	60	0.097	0.040	0.164	0.084
20	660	112	0.170	0.039	0.163	0.085
22	700	83	0.119	0.037	0.161	0.087
23	750	60	0.080	0.036	0.160	0.088
Total	12,790	1,586	$\bar{p} = \frac{1,586}{12,790} = 0.124$			

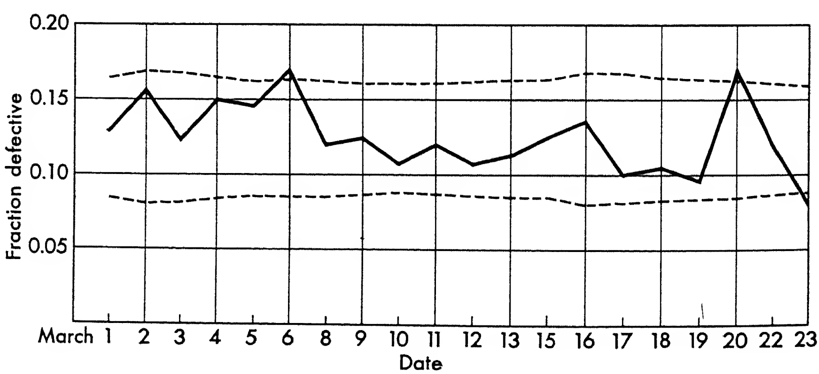

Figure 19–19. ***A fraction defective control chart.*** (*See* Table 19–5)

The c chart

The c chart is used for the control of the number of defects observed per unit. The difference between the p chart and the c chart is that the former takes into account the number of items found defective in a given sample size (each defective item may have one or more defects in it), while the latter records the number of defects found in a given sample size. Although the application of the c chart is somewhat limited, compared with the p chart, there are many instances in industry where it is very useful; e.g., in the control of the number of defects in textile material, the number of stains or blemishes on a surface, the number of defects on soiled packages in a given consignment, etc.

The construction of the control chart is similar to that of the p chart except that here the control limits are based on the Poisson distribution, which has often been found fit to describe distributions of defects. The control limits for the chart are

$$\begin{aligned} \text{UCL } c &= c' + 3\sqrt{c'} \\ \text{LCL } c &= c' - 3\sqrt{c'} \end{aligned} \tag{19–7}$$

where c' is the desirable level of the number of defects per unit and $\sqrt{c'}$ is the standard deviation of the Poisson distribution, and to start with c' is estimated by $\bar{c}$, which is the average number of observed defects per unit (being the ratio of the total number of defects found in all units or samples to the total number of units or samples inspected), so that the estimated control limits are

$$\begin{aligned} \text{UCL } c &= \bar{c} + 3\sqrt{\bar{c}} \\ \text{LCL } c &= \bar{c} - 3\sqrt{\bar{c}} \end{aligned} \tag{19–8}$$

Whenever $\bar{c} < 3\sqrt{\bar{c}}$, so that the LCL is negative, it is taken as being 0.

Summary

Problems in quality control may be classified as follows:

1. How to define quality and how to measure it? Some attributes are easily measurable; some characteristics are somewhat complicated to define.

2. At what stages to inspect and after what operations? The various stages of inspection include incoming materials, production facilities, "first-off" inspection, process control, final product inspection, and post-sales evaluation.

3. How to inspect? This would include selection of measuring devices, deciding how much to leave to self-inspection by the operators, determination of inspection methods (100 per cent or sampling plans).

4. Where to inspect? Inspection can be centralized, located along the production line as a sorting station, or it may be carried out by patroling inspectors.

5. Can and should statistical quality control be employed? The two main

tools used in statistical quality control are acceptance sampling and control charts:

(i) Acceptance sampling plans prescribe the sample size on which decision to accept or reject the whole lot will be made; there are single, double, or multiple sampling plans.

(ii) Control charts are used in order to ascertain the behavior of the production processes:

- (a) The $\bar{X}$ and R charts are used for control of specific measurable attributes. The $\bar{X}$ chart reveals trends in the mean value of the attribute and provides useful guidance for resetting processes. The R chart controls the uniformity of the products.
- (b) The p charts for fraction defectives control the percentage of defectives, irrespective of specific attributes. They are useful for indication of the general quality level, for initiating $\bar{X}$ and R charts for selected operations, and for analyzing the discrepancy between the desired level of quality to that actually obtained.
- (c) The c charts are used for control of defects. These charts often lead to classification of defects and to studies of methods to reduce them.

6. How to evaluate quality? This includes studies of process capabilities, critical assessment of specifications and tolerances, and finally, evaluation of the effectiveness of the quality control methods employed in the plant.

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Problems

1. The acceptance sampling procedure in one plant specifies that 5 per cent of the incoming lot should be inspected, and if no defectives are found, the lot is accepted. Show that the probability of acceptance varies with the lot size. Plot the probability of acceptance against the actual percentage of defectives when the lot size is 2,000, 1,000, 500, 200, 100 units.

2. For the following double sampling plan, find the probability of acceptance of the lot, when it is known to contain: (i) 2% defective items; (ii) 10% defective items.

Sample No.	*Quantity*	*Acceptance Number*	*Rejection Number*
1	10	1	4
2	20	3	4

3. For the following multiple sampling plan, plot the probability of acceptance of a large lot, when the percentage defectives varies from 0 to 10 per cent.

Sample No.	*Quantity*	*Acceptance Number*	*Rejection Number*
1	4	*	2
2	4	0	2
3	4	1	3
4	4	1	3
5	4	3	4

* If number of defectives remains 0, the lot is accepted after second sample.

4. A manufacturer produces 10,000 electric bulbs, the average life of each being 1,000 hours. It was found after 600 hours that 375 bulbs have failed.
   (i) Assuming the life of the bulbs follows a normal distribution curve, find the standard deviation, σ.
   (ii) How many bulbs failed in the first 400 hours and how many would you expect to work after 2,000 hours?
   (iii) What percentage of the batch could be classed as "life 1,000 hours $\pm 5\%$"?
   (iv) To ensure continuous light in a square, it was suggested that twin bulbs be fitted on each lamp, so that when one bulb failed, the second would be automatically switched on. The maintenance schedule was to provide for all bulbs to be changed after 1,800 hours. If there were 200 lamps in the square, how many would you expect to be still functioning on the changeover day? In how many of these would the second bulb not have been switched on yet?

   (*Note:* The standard deviation of the twin bulb arrangement is given by

$$\sigma^2_{\text{twin}} = 2\sigma^2\ )$$

5. Bars emerging from a rolling mill have a diameter of 1.000 ± 0.024 inch in 99.7 per cent of the cases, and it was found that these diameters form a normal distribution.
   (i) Find the standard deviation of the process.
   (ii) In a batch of 1,000 bars, how many would you expect to have a diameter smaller than 1.000 inch?
   (iii) How many would have a diameter of 1.000 (± 0.008) inch? How many would have a diameter from 1.004 to 1.016 inch?
   (iv) Sixty per cent of the bars have a diameter of 1.000 ($\pm x$). Find x.

6. A sample of four components is taken from a very large lot, which is known to contain 5 per cent defectives.

   (i) What is the probability of obtaining 0, 1, 2, 3, 4 defectives in the first sample?

   (ii) What is the probability of obtaining 0, 1, 2, 3, 4 defectives in the second sample?

   (iii) Five successive samples are taken. What is the probability that the 20 components inspected would contain 5 per cent defectives?

7. Figure 19.7 exhibits the pinball device for illustrating the statistical distribution of steel ball diameters. The device consists of three parts: (i) a reservoir, being analogous to the process; (ii) a pin board, being analogous to a combined diameter measuring apparatus and sorting mechanism; (iii) columns for collecting the balls, being analogous to a recording instrument. Discuss whether this device proves that steel balls coming out of a manufacturing process are normally distributed with respect to their diameter.

8. In the manufacture of brass bars the diameter was specified as $1.000 \begin{pmatrix} +0.020 \\ -0.010 \end{pmatrix}$ inch. Samples taken from the production line showed the diameter measurements given in the accompanying table. (a) Is the process in statistical control? (b) Comment on the specification.

Sample No.	*Measurements of Five Articles*					*Average* ($\bar{X}$)	*Range* (R)
1	1.010	1.020	1.015	1.008	1.000	1.011	0.020
2	1.001	1.023	0.990	0.995	0.990	1.000	0.023
3	1.010	1.030	1.030	1.010	1.000	1.016	0.030
4	1.010	1.005	0.990	0.985	0.990	0.996	0.025
5	0.990	0.990	0.995	0.998	1.002	0.995	0.012
6	1.005	1.008	0.990	1.015	1.020	1.008	0.030
7	1.010	1.015	1.002	0.980	0.998	1.001	0.035
8	1.000	1.005	1.008	0.998	0.992	1.001	0.016
9	1.000	1.006	1.006	0.990	0.998	1.000	0.016
10	1.010	1.015	1.020	1.025	1.010	1.016	0.015
11	1.020	1.020	1.010	1.015	1.000	1.013	0.020
12	1.001	0.998	0.995	0.980	0.985	0.992	0.021
13	0.980	0.985	0.995	0.990	1.010	0.992	0.030
14	1.001	1.005	0.990	1.010	1.010	1.003	0.020
15	1.015	1.018	1.020	1.005	1.000	1.012	0.020
16	1.005	0.995	0.995	0.990	1.003	0.998	0.015
17	1.005	1.005	1.010	1.010	1.020	1.010	0.015
18	1.020	1.020	1.020	1.010	1.000	1.014	0.020
19	1.000	1.000	1.001	0.990	0.985	0.995	0.016
20	0.990	0.995	1.000	1.010	1.000	0.999	0.020
Total						20.072	0.419

9. Circular brass disks were inspected before reaching the assembly line. The thickness specification of the disk was 0.110 ($\pm$0.020) inch. Sampling every hour gave results as shown in the accompanying table.

Sample No.	*Four Measurements per Sample (in thousandths of an inch)*			
1	87	105	97	95
2	100	83	109	101
3	113	92	112	98
4	113	95	94	93
5	101	107	88	87
6	92	95	111	106
7	90	102	115	90
8	99	109	102	102
9	82	109	105	116
10	89	99	105	90
11	103	110	106	86
12	86	115	96	89
13	109	92	98	105
14	75	113	103	110
15	105	84	84	100
16	115	92	101	108
17	124	115	114	89
18	97	103	98	93

Wide variations in the thickness of the disks were noticed at the assembly line and led to the assertion that raw material was faulty.

Construct an $\bar{X}$ and R chart and express your views on this matter. The design office considered the question of specifications and is prepared to alter them to 0.108 ($\pm$0.024) inch. Will this modification resolve the difficulties?

10. Inspection of glass vessels coming off a production line is summarized in the accompanying table. Draw a p control chart. What conclusions would you draw?

Date	*Number Inspected*	*Number Defectives*
April 4	100	10
5	100	8
6	100	7
7	100	5
8	100	12
9	150	16
11	200	14
12	150	13
13	100	8
14	100	5
15	80	4
16	80	3
18	75	5
19	70	6
20	60	12
21	100	15
22	120	12

11. In the inspection of cloth material, a sample of 20 yards was taken several times every day and the number of defects recorded in the accompanying table. Draw a control chart for c.

	Sample No.	*Number of Defects*
1st day	1	5
	2	4
	3	3
	4	5
	5	6
2nd day	1	2
	2	1
	3	6
	4	8
	5	3
3rd day	1	2
	2	4
	3	8
	4	2
	5	5
4th day	1	4
	2	10
	3	12
	4	8
	5	6
	6	12
5th day	1	6
	2	6
	3	2
	4	3

12. A manufacturer makes one million bushes per annum. The specifications of the internal diameter are 0.600 ± 0.010 inch. The process capability may be defined by 0.600 $\pm x$, and the cost of producing and inspecting one bush is $1/(5x)$ dollars, when $x \leqslant 0.100$. A 100 per cent inspection is carried out after production to sort out the defectives, each involving a loss of $0.16/x$ dollars. Assuming that the process can be easily adjusted to produce a mean internal diameter of 0.600 inch, and that the final results conform to the normal distribution,

    (i) Plot the cost of production when x is varied from 0.100 to 0.010.

    (ii) Plot the loss due to the number of defectives for the same range of values of x.

    (iii) Combine the two curves to find the total cost and determine the value of x for minimum costs.

20

PRODUCTION COST CONTROL

Like quality control, cost control is not, strictly speaking, a production planning and control function, but since it is used as a criterion for measuring the effectiveness of the plant, this treatise will not be complete without a brief reference to the subject. Both quality control and cost control tell us how well the plant is doing; both feed vital information to the production departments, information on which corrective actions can be based in order to improve performance.

As stated in Chapter 15, plant efficiency is indicated by comparison of actual with planned performance. In quality control we compare the quality level of components or products with the desirable level as stated by specifications. Similarly the purpose of cost control is to compare the actual costs incurred in production with predetermined cost factors. The procedure is as indicated by the control elements enumerated in Chapter 15, namely: recording data while operations are performed, analysis of the data in the form of cost computations and comparison with cost estimates, feeding information for immediate corrective action by the appropriate departments and for final evaluation of performance efficiency.

For cost control to be effective, it has been recognized in recent years that its procedure must follow this basic pattern, which is common to all control functions. The development of standard cost accounting has been very helpful in realizing this aim and in making cost control a useful managerial control tool, and it has been successfully applied in many industries.

Briefly, standard cost accounting begins by setting standard costs to the various sources at which costs are incurred. This is a planning function (= estimating), which tries to analyze the production operations, the planned utilization of materials and facilities, the services provided by various departments, and the managerial effort involved, and to interpret all these factors into cost terms, the sum total of which will give the cost of production. Through cost control we try to find whether these standards, or "targets," can be attained —and if not, why not? Is it because there is a waste of materials, too much scrap, or wasteful use of facilities? And how can we reduce this waste to improve our performance, so that we can be more competitive? This, in short, is what cost control is about.

Standard Cost Accounting

Control is facilitated by assignment of costs to cost centers. A cost center is defined on a functional basis to include machines and operators, whose control is clearly directed by one supervisor or manager. In this way, costs can be traced, investigated, and analyzed, and responsibility for them is unquestionably definable. Naturally the demarcation between cost centers must be clearly determined, with no overlap, and there may be several cost centers in one department, depending on their functional responsibility. The production department, for example, may have the following centers: Press shop, machine shop, tool room, painting, maintenance, etc.

Within each center, costs are attributed to specific activities, depending on the types of machines or skills employed. The machine shop, for instance, may have the following operations: turning, milling, shaping, grinding, etc. The expenses at each center are classified according to expense items, or accounts, and a typical monthly summary of cost breakdown into accounts is shown in Table 20–1.

Table 20–1

Account	*Cost Center*								Total
	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	
0. Direct supervision									
1. Indirect supervision									
2. Direct labor									
3. Indirect labor									
4. Direct materials									
5. Indirect materials									
6. Maintenance labor									
7. Maintenance materials									
8. Power and water									
9. Other services									

These costs must be related to a common denominator for the purpose of comparison and analysis, and in standard cost accounting the "productive standard hour" (PSH for short) is commonly adopted for this purpose. In this way the costs are related to the productive capacity of the plant and provide a measure of the variable costs incurred. For reasons explained in Chapter 5 in connection with the break-even chart, cost control based on unit of output will not provide useful information about the performance of the plant because the

total costs consist of a fixed cost and a variable cost component. It is by segregation of the two that analysis becomes possible, and an example of a standard cost statement based on the productive standard hour is shown in Table 20-2. Each operation is related to the number of PSH required to complete it (taking into account the methods prescribed by production planning), and if the fixed costs and variable costs per PSH for this type of operation are known, the standard cost of the operation can be readily computed. By adding the standard costs of materials required and making permissible allowances for swarf and scrap, the total standard cost of each component can be found. These standard values are then compared with actual cost figures, and whenever the discrepancy is so pronounced that further investigation is required, the breakdown figures can be compared, account for account and cost center for cost center, until the causes are finally traced and studied in more detail.

Table 20-2

Standard Cost Statement of a Component (in dollars)

Cost Center	*Operation*	*PSH*	*Cost per PSH*			*Operation Cost*
			Fixed	*Variable*	*Total*	
B	1. heat treatment	0.022	4.50	2.50	7.00	0.154[1]
	2. turning	0.045	6.00	3.00	9.00	0.405
	2. waste material	—	—	—	—	—0.150
	3. milling	0.060	6.60	4.00	10.60	0.636
	3. waste material	—	—	—	—	—0.120
C						
D						
etc.						

In setting standard costs and in recording cost items during the operations, it is important to have clear-cut definitions of the various accounts; otherwise the whole basis for comparison of actual with standard figures may become rather shaky. Some common definitions of accounts are given below:

Direct supervision: supervisors' time employed in directing the activities of the production centers embodied in the cost center

Indirect supervision: including staff relationships and clerical personnel

Direct labor: operators' time closely related to PSH

Indirect labor: time that cannot be related to PSH, so that costs can be computed only as an average over a number of PSH

Direct materials: materials required to make one unit of the component or product, including permissible swarf and scrap

[1] Derived by multiplying the total cost per PSH of the operation: $7.00 \times 0.022 = 0.154$
PSH = productive standard hours.

Indirect materials: materials that are not incorporated in the product but are required in the process of production and supervision, such as sandpaper, oil, rags, stationery, etc.

Maintenance materials: materials and parts used for servicing of equipment.

Details on how these accounts are administered can be found in most of the standard textbooks on cost accounting.

Further Considerations of Break-even Charts—Step-wise Cost Function[2]

Cost control and the analysis of the break-even chart in Chapter 5 are normally based on the assumption that the cost function is linear and continuous and is clearly defined by two components: A fixed component, F, and a variable one, aQ, where Q is the quantity produced. In practice, however, it is often found that the fixed costs increase in a step-wise fashion as the volume of production grows (Fig. 20–1), and this phenomenon is attributed to the fact that additional

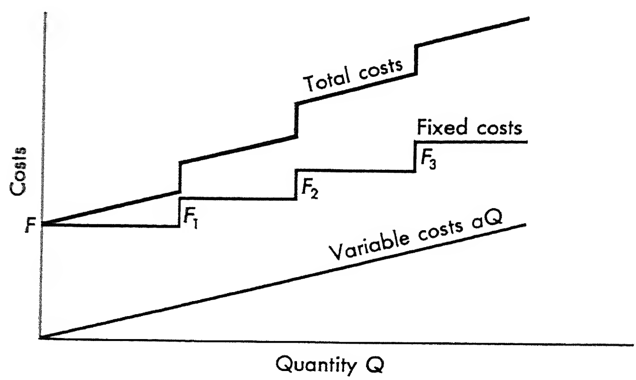

Figure 20–1. *A step-wise cost function.*

personnel, equipment, and even whole departments have to be added when plant activity attains certain levels. Had this increase been a gradual one, it could have been included in the variable costs component. But in their very nature these increases to the cost function are similar to the fixed costs, F, except that they occur at different stages of the firm's growth when the need arises. The BEP is obtained at

$$F + \sum_{1}^{k} F_i + aQ = bQ \tag{20–1}$$

when F_i is the increase in fixed costs at the ith step and when the BEP occurs after k steps.

[2] This section may be omitted at first reading.

Hence
$$Q = \frac{F + \Sigma F}{b - a} = Q_1 + \frac{\Sigma F}{\varphi} \qquad (20\text{–}2)$$

where Q_1 is the BEP for the case of a continuous function ($F_i = 0$) and φ is the P/V ratio (see Chapter 5).

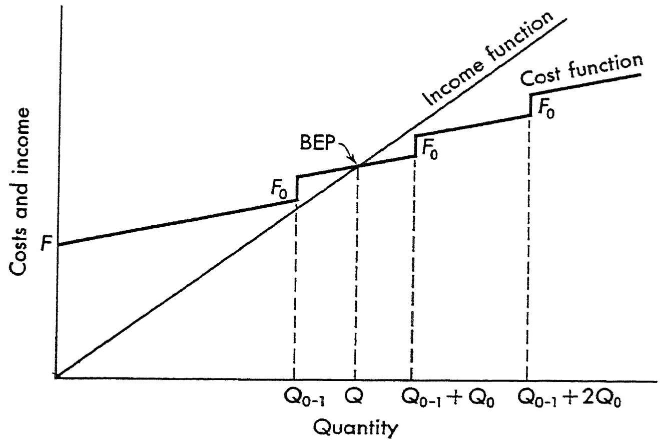

Figure 20–2. ***Break-even analysis with a step-wise cost function.***

For the special case, when a first increase of F_0 is expected after Q_{0-1} and subsequently at constant intervals of activity Q_0 (Fig. 20–2), the BEP

$$Q = Q_1 + k\frac{F_0}{\varphi} \qquad (20\text{–}3)$$

and we know that

$$Q_{0-1} + (k - 1)Q_0 \leqslant Q \leqslant Q_{0-1} + kQ_0$$

and from these two expressions Q can be evaluated. First let us assume that the solution lies at $Q = Q_{0-1} + (k - 1)Q_0$. Hence

$$Q = Q_1 + \left(\frac{Q - Q_{0-1}}{Q_0} + 1\right)\frac{F_0}{\varphi}$$

or
$$Q = \frac{Q_1 - [(Q_{0-1}/Q_0) - 1](F_0/\varphi)}{1 - (1/Q_0)(F_0/\varphi)} \qquad (20\text{–}4)$$

which provides a first approximation for Q. k can then be evaluated by

$$k = \frac{Q - Q_{0-1}}{Q_0} + 1$$

and the solution for the BEP may be sought through Eq. 20–3 by trial and error.

It should be noted that in some situations, more than one solution for the BEP is possible, as illustrated in Fig. 20–3 where break-even occurs both at Q' and Q'':

$$\begin{aligned} &\text{BEP at } Q' \text{ when } F + kF_0 + aQ' = bQ' \\ &\text{BEP at } Q'' \text{ when } F + (k+1)F_0 + aQ'' = bQ'' \end{aligned} \tag{20–5}$$

In the interval between Q' and Q'' the plant is first operating with a very small profit; then at $Q = Q_{0-1} + kQ_0$, it operates at a loss due to an increase in the fixed costs by F_0, and beyond Q'' the profit constantly increases. This interval between Q' and Q'' may be called *the break-even range* (which can be easily shown to be $Q'' - Q' = F_0/\varphi$), and its extreme value, Q'', should be considered the decisive factor for calculation of the safety factor.

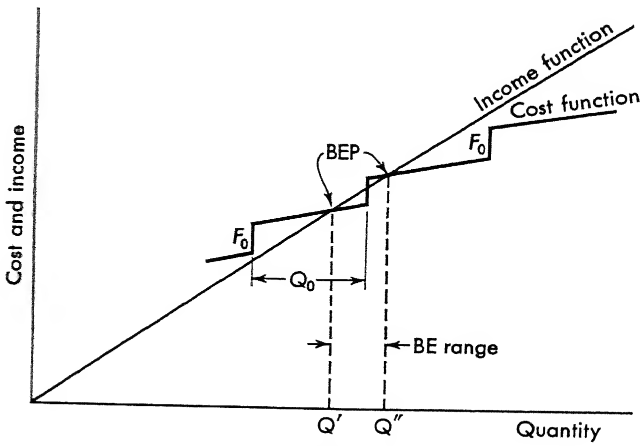

Figure 20–3. ***The break-even range.***

It is obvious from Fig. 20–2 that at any interval of Q_0, it is best to operate near the "end of the step," so to speak, since an increment of F_0 adversely affects the final profit. If the plant operates at $Q = jQ_0$, where $j > k$, the profit is

$$Z_j = bQ - [F + jF_0 + aQ] = \varphi Q - [F + jF_0] \tag{20–6}$$

and this profit is reduced to

$$Z'_j = Z_j - F_0$$

when F_0 fixed costs are added.

Example

The annual fixed costs of a firm is now $4,200,000, and it was found in practice that these fixed costs increase by $200,000 for every increase of 15,000 units in the production volume. If the P/V ratio is known to be $41 per unit and the

annual production volume is 123,000 units, find (a) the BEP and the break-even range, if any; (b) the margin of safety; (c) the profit.

Solution

Since an increment of $F_0 = \$200{,}000$ occurs every $Q_0 = 15{,}000$ units (in this case $Q_{0-1} = Q_0$), the firm has experienced $j = 123{,}000/15{,}000 \simeq 8$ such increments. Given

$$F + jF_0 = 4{,}200{,}000$$

$$\therefore \qquad F = 4{,}200{,}000 - 8 \times 200{,}000 = \$2{,}600{,}000$$

Hence
$$Q_1 = \frac{F}{\varphi} = \frac{2{,}600{,}000}{41} = 63{,}500 \text{ units}$$

(a) First evaluation of Q by Eq. 20–4:

$$Q = \frac{63{,}500}{1 - \dfrac{200{,}000}{15{,}000 \times 41}} = 94{,}100 \text{ units}$$

An approximation for k:

$$k = \frac{Q}{Q_0} = \frac{94{,}100}{15{,}000} \simeq 6$$

Try $k = 5$ and 6. By Eq. 20–3,

$$\text{For } k = 5: \qquad Q' = Q_1 + k\frac{F_0}{\varphi}$$

$$= 63{,}500 + 5\left(\frac{200{,}000}{41}\right) = 87{,}900 \text{ units}$$

$$\text{For } k = 6: \qquad Q'' = 63{,}500 + 6\left(\frac{200{,}000}{41}\right) = 92{,}700 \text{ units}$$

Check for k:

$$k = \frac{Q'}{Q_0} = \frac{87{,}900}{15{,}000} > 5 \quad \text{but below 6} \quad \text{(O.K.)}$$

$$k = \frac{Q''}{Q_0} = \frac{92{,}700}{15{,}000} > 6 \quad \text{but below 7} \quad \text{(O.K.)}$$

It can be easily verified that other values of k do not qualify under this test. Hence the break-even points are at Q' and Q'', as calculated above.

(b) The margin of safety is

$$\Delta = \frac{Q}{Q''} - 1 = \frac{123{,}000}{92{,}700} - 1 = 0.325$$

(c) The profit is calculated by Eq. 20–6 as

$$Z = \varphi Q - \underbrace{[F + jF_0]}_{=4{,}200{,}000} = 41 \times 123{,}000 - 4{,}200{,}000 = \$840{,}000$$

The Effect of the Learning Curve on Unit Costs[3]

So far, the break-even analysis was based on a linear cost function, which assumes that for each additional unit produced, the total costs increase by a constant amount, a, irrespective of the production volume. The coefficient a in the variable costs term aQ of the total costs function is the constant costs per unit in this analysis. Costs per unit can be described by

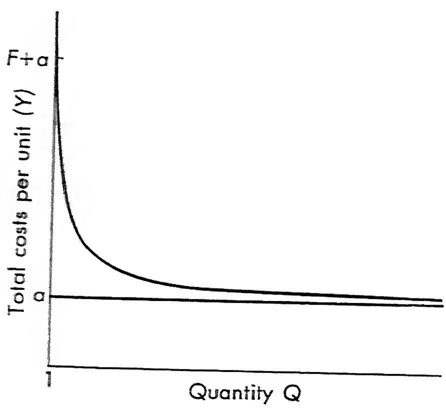

Figure 20–4. ***Effect of quantity on unit costs (as given by $Y = \frac{F}{Q} + a$).***

$$Y = \frac{\text{total costs for } Q \text{ units}}{Q \text{ units}} = \frac{F}{Q} + a \tag{20–7}$$

This is a hyperbola, with $Y = a$ as its asymptote, when $Q \to \infty$ (see Fig. 20–4). The reduction of costs per unit with the increase of production volume is due to the fact that the fixed costs can be assigned to a larger number of units, while the direct costs are assumed to remain constant.

However, ample evidence is available to show that the so-called constant direct costs per piece, a, are in fact a function of Q. For one thing, the labor content per unit is known to be greatly affected by the learning curve. The

[3] This section may be omitted at first reading.

direct labor costs function was investigated during World War II in the airframe industry, and it was found that direct labor costs were reduced by 20 per cent when the number of frames was doubled, the function being described by

$$l = CQ^n \tag{20-8}$$

where l = direct labor

Q = number of units producted

C,n = parameters ($-1 < n < 0$)

The function is shown in Fig. 20–5. Here, C represents the cost of the first unit, as $l = C$ for $Q = 1$. The cost function implies an assumption that as the number of units produced increases, the costs diminish, and for an infinite number of units the direct labor costs are zero. This assumption is based on two facts: (1) due to the learning curve, workers become more skilled and efficient, and (2) the increased degree of mechanization of the plant expands production. For the purposes of this analysis this model is adequate.

Figure 20–5. ***Direct labour cost function*** ($l = CQ^n; -1 < n < 0$).

A survey of several firms from various industries (aircraft instruments, laminated aircraft plastic assemblies, electronic data processing equipment, producers of electronic and mechanical units, aircraft models, World War II liberty ships, a semiautomatic machine producer, a textile machine, etc.)[4] seemed to suggest that although the initial cost greatly differed for different products, there was very little variation in the percentage of cost reduction. If at any production volume the direct labor costs were considered as 100 per cent,

[4] Reno R. Cole: Increasing utilization of the cost quantity relationship in manufacturing (J. Industrial Engineering, May–June, 1958).

they were reduced on the average to 79.6 per cent (say, 80 per cent) when the volume doubled. Thus, for the first volume,

$$l_1 = CQ^n{}_1$$

subsequently

$$l_2 = CQ^n{}_2$$

$$\therefore \qquad \frac{l_2}{l_1} = \left(\frac{Q_2}{Q_1}\right)^n$$

but $l_2/l_1 = 0.8$ and $Q_2 = 2Q_1$; hence $0.8 = 2^n$, or

$$n = -0.322 \simeq -\tfrac{1}{3}$$

Provided the findings of the survey mentioned above prove to be characteristic for all industry, this result for n is independent of the type of product manufactured or the values of C and Q. The direct-labor cost function then becomes

$$l = CQ^{-0.322} \simeq \frac{C}{Q^{\frac{1}{3}}} \tag{20–9}$$

and this is a very useful tool when direct labor costs have to be estimated for certain predetermined production volumes.

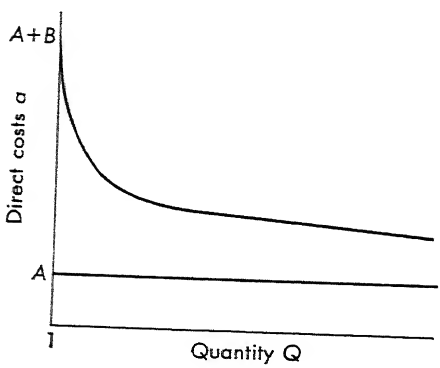

Figure 20–6. ***Direct cost function*** $(a = A + BQ^n: -1 < n < 0)$.

The direct costs per unit do not consist of labor alone, but also materials, cost of running and maintaining machines, and other expenses directly associated with the production process. Although little research is available to indicate what form the direct cost function follows, it is reasonable to assume that it is somewhat similar to the direct labor cost function, so that

$$a = A + BQ^n \tag{20–10}$$

as shown in Fig. 20–6, where a is the total direct costs per unit and $-1<n<0$ and the function is valid only from $Q \geqslant 1$. When only one unit is produced ($Q = 1$), the direct costs are $a = A + B$, while for $Q \to \infty$ we get $a = A$. The

factor A, therefore, indicates the cost of materials and other direct costs when the production volume is so high that hardly any direct labor costs are incurred. To find n, the method shown above can be used:

For $Q = Q_1$: $$a_1 = A + BQ^n{}_1$$

For $Q = 2Q_1$: $$a_2 = A + B(2Q_1)^n$$

or $$\frac{a_2 - A}{a_1 - A} = W = 2^n$$

and $$n = \frac{\log W}{\log 2}$$

Further research is required to establish values for n and compare it with that obtained for the direct labor cost function.

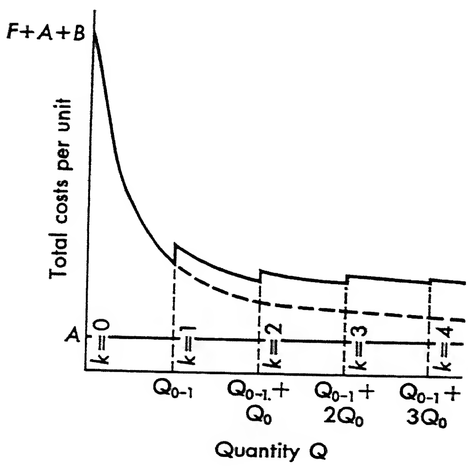

Figure 20–7. ***Modified total costs per unit.***

Application to the break-even analysis

In the break-even analysis, the total costs per Q units were taken as $F + aQ$, when it was assumed that (a) fixed costs remain unchanged, irrespective of the volume Q, and (b) direct costs per unit remain constant. If we modify these assumptions in the light of the above discussion, the total costs function may be restated as:

$$y = \underbrace{F + \sum_{1}^{k} F_i}_{\text{Fixed costs after } k \text{ increments}} + \underbrace{[A + BQ^n]Q}_{\text{Variable costs}} \qquad (20\text{–}11)$$

or, when it is assumed that each fixed costs increment is equal to F_0, the first occurring after Q_{0-1} and the others every Q_0, we may conclude that

$$y = \underbrace{F + kF_0}_{\substack{\text{Fixed costs}\\ \text{after } k\\ \text{increments}}} + \underbrace{AQ + BQ^{n+1}}_{\substack{\text{Variable}\\ \text{costs}}} \tag{20–12}$$

Figures 20–7 and 20–8 show the modified total unit costs and the modified break-even chart (taking Eq. 20–12 into account), and these should be compared with Figs. 20–4 and 5–9, respectively. The profit at point Q (see Fig. 20–8) is

$$Z = bQ - y$$

or

$$Z = (b - A)Q - BQ^{n+1} - (F + kF_0) \tag{20–13}$$

and the BEP is found when $Z = 0$.

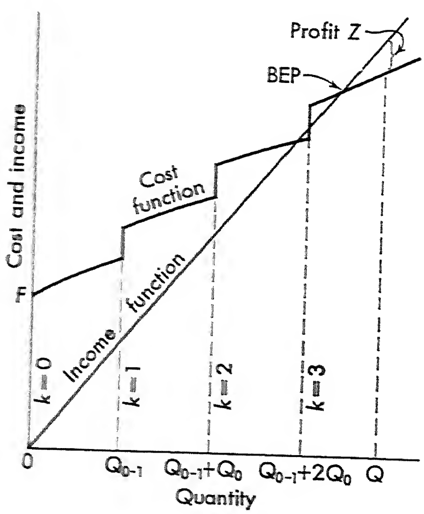

Figure 20–8. ***Break-even analysis using the modified cost function.***

It is clear from Fig. 20–8 that when we stay on the same "step" on the cost function, it always pays to increase the production volume. What happens, however, when an increase of ΔQ in the production volume brings us one step higher and the fixed costs increase by an increment F_0? The increase in profit, by Eq. 20–13, is

$$\Delta Z = (b - A)\Delta Q - B[(Q + \Delta Q)^{n+1} - Q^{n+1}] - F_0$$

When $\Delta Z > 0$, the increase is worth while; i.e., when

$$(b - A)\Delta Q - B[(Q + \Delta Q)^{n+1} - Q^{n+1}] > F_0$$

or

$$\frac{\Delta Q}{Q} - \frac{B}{b - A}\left[\left(1 + \frac{\Delta Q}{Q}\right)^{n+1} - 1\right]Q^n > \frac{F_0}{b - A}\frac{1}{Q}$$

Using the following notation

$$\left.\begin{aligned} \frac{\Delta Q}{Q} &= \nu \\ \frac{B}{b - A} &= g \\ \frac{F_0}{b - A} &= h \end{aligned}\right\} \qquad (20\text{–}14)$$

The last expression may be rewritten as

$$\nu - g\left[(1 + \nu)^{n+1} - 1\right]Q^n > \frac{h}{Q}$$

An approximation to substitute the square brackets can be found through the binomial expansion (using the first two terms):

$$(1 + \nu)^{n+1} - 1 \simeq 1 + (n + 1)\nu - 1 = (n + 1)\nu$$

$$\therefore \qquad \nu - g(n + 1)\nu\, Q^n > \frac{h}{Q}$$

or

$$\nu\left[1 - g(n + 1)Q^n\right] > \frac{h}{Q} \qquad (20\text{–}15)$$

When investigations of the cost function characteristics justify using

$$n = -\tfrac{1}{3}$$

then

$$\nu(1 - \tfrac{2}{3}\, gQ^{-\frac{1}{3}}) > \frac{h}{Q} \qquad (20\text{–}16)$$

Obviously, the larger the ν (= the contemplated expansion in output) and the smaller the Q (= the present output), the better is the chance that expansion will increase profit, in spite of the increase in fixed costs. The issue depends, however, on the values of the parameters g and h. Furthermore we can see that if g is sufficiently small and when Q is large, the effect of the learning curve is negligible, and the simplified linear analysis is obtained, since then

$$\nu > \frac{h}{Q}$$

or, by substituting Eq. 20–14,

$$\frac{\Delta Q}{Q} > \frac{F_0}{b - A}\frac{1}{Q}$$

$$\therefore \qquad \Delta Q > \frac{F_0}{b - A}$$

In this case $b - A$ is the P/V ratio; hence the condition that positive profit will result through expansion in output becomes

$$\Delta Q > \frac{F_0}{\varphi}$$

and this is similar to the analysis included in Chapter 5.

Summary

Cost control is a management tool that assists in the evaluation of plant performance by comparing actual cost factors with the corresponding figures determined at the planning stage. Cost analysis is usually based on the linear break-even chart, and if a cost accounting system is to be effective in cost control, it must involve recording procedures that would facilitate the segregation of cost factors into fixed and variable parameters. Sometimes the simplified linear break-even chart is too crude a method for cost analysis and several modifications have to be considered, such as the significance of stepwise changes in the fixed costs and the effect of the learning curve on the total costs function.

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Problems

1. Consult a textbook on cost accounting and compare any conventional cost accounting with the standard cost accounting method. What features does the latter have to make it potentially an effective control tool?
2. A small firm employing 75 people is engaged on job production where only 25 per cent of the orders are repeated. As a production engineer, you are asked to outline a cost control procedure that would help you in planning and control production. Prepare a memorandum with your suggestions for the plant manager.
3. Costs records for a packing and shipping department are shown in the accompanying table (all figures in dollars):

Account	*Jan.*	*Feb.*	*Mar.*	*Apr.*	*May*	*June*
Supervision and clerical	1,100	1,200	1,100	1,270	1,300	1,300
Direct labor	2,800	3,820	5,630	6,000	6,910	8,400
Indirect labor	2,900	3,010	3,580	3,810	4,000	4,080
Direct materials	2,100	3,000	4,250	5,000	5,600	6,730
Indirect materials	810	830	910	980	1,020	1,050
Maintenance labor	510	530	610	600	540	600
Maintenance materials	420	440	430	400	400	440
Power and water	310	290	330	280	360	330
PSH	500	650	950	1,100	1,250	1,500

Account	*July*	*Aug.*	*Sept.*	*Oct.*	*Nov.*	*Dec.*
Supervision and clerical	1,700	1,700	1,600	1,600	1,410	1,380
Direct labor	9,000	8,800	7,850	7,060	6,030	5,020
Indirect labor	4,900	4,900	4,100	4,000	3,500	3,400
Direct materials	7,800	7,660	6,300	5,600	4,800	3,830
Indirect materials	1,050	1,040	1,050	1,030	1,000	950
Maintenance labor	520	500	580	590	570	550
Maintenance materials	460	390	400	450	430	460
Power and water	340	280	310	330	320	350
PSH	1,750	1,700	1,400	1,250	1,050	850

(i) Find the cost items per productive standard hour and plot their variations against time.

(ii) Can you suggest any correlation between the various accounts?

4. A plant is operating at a margin of safety of 100 per cent. Show by the linear break-even chart analysis that the profit is equal to the fixed costs. Would this mean that the profit could be increased by increasing the fixed costs?

5. (a) When the cost function assumes a stepwise shape, as shown in Fig. 20–3, show that the break-even region is

$$Q'' - Q' = \frac{F_0}{\varphi} < Q_0$$

(b) Under what circumstances can the break-even occur at three points?

6. The fixed costs for a certain product amount to \$120,000. When the annual output reaches 25,000 pieces, it is estimated that the fixed costs would increase as a stepwise function by F_0 = \$25,000 for every 10,000 more pieces per annum. The variable costs are \$2.04 per unit and the sales income is \$6.60 per unit.

(i) Find the break-even point and the break-even range, if any.

(ii) The margin of safety and the profit when output reaches 50,000 per annum.

(iii) Plot the change in profit when the output changes from 10,000 to 100,000 pieces per annum.

7. Show that when the effect of the learning curve is neglected, the BEP is higher than the one suggested by Eq. 20–13.

8. (a) Formula 20–15 was obtained by using only the first two terms in the expansion of the binomial $(1 + \nu)^{n+1}$. Modify Eq. 20–15 by including the third term in the expansion, and discuss to what extent this would add to the accuracy of your expression.

   (b) Show that if $B \longrightarrow 0$, there is no learning curve effect on the cost function and hence on Eq. 20–15.

9. A plant is operating at an output of 10,000 units per month, and it is known that (symbols as used in the text):

$$b = \$25/\text{unit}$$
$$A = \$13/\text{unit}$$
$$B = \$15/\text{unit}$$

   Discuss the effect on profit of increasing the output by 20 per cent, if the fixed costs may thereby increase by $20,000.

10. A component can be produced by three methods in a plant, the cost estimates being as follows:

Process	*Setup Costs Including Dies*	*Variable Costs Coefficients ($/unit)*	
		A	*B*
Forging	$40,000	0.30	0.20
Machining	$3,000	1.10	0.40
Casting	$8,000	0.70	0.30

   A fourth alternative is to buy the component outside. One vendor's offer is given by:

   Up to 20,000 pieces: $1.60/piece
   20,000–40,000 pieces: $1.10/piece
   More than 40,000 pieces: $0.80/piece

   If the learning curve is taken into account (take $n = -\frac{1}{3}$), plot the cost function of the four alternatives (by Eq. 20–11), and state under what condition is each preferable.

TABLES

The Normal Cumulative Distribution

Area between 0 and t (= Φ − 0.5000)

$t = (x - \bar{x})/\sigma$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.00000	0.00399	0.00798	0.01197	0.01595	0.01994	0.02392	0.02790	0.03188	0.03586
0.1	0.03983	0.04380	0.04776	0.05172	0.05567	0.05962	0.06356	0.06749	0.07142	0.07535
0.2	0.07926	0.08317	0.08706	0.09095	0.09483	0.09871	0.10257	0.10642	0.11026	0.11409
0.3	0.11791	0.12172	0.12552	0.12930	0.13307	0.13683	0.14058	0.14431	0.14803	0.15173
0.4	0.15554	0.15910	0.16276	0.16640	0.17003	0.17364	0.17724	0.18082	0.18439	0.18793
0.5	0.19146	0.19497	0.19847	0.20194	0.20450	0.20884	0.21226	0.21566	0.21904	0.22240
0.6	0.22575	0.22907	0.23237	0.23565	0.23891	0.24215	0.24537	0.24857	0.25175	0.25490
0.7	0.25804	0.26115	0.26424	0.26730	0.27035	0.27337	0.27637	0.27935	0.28230	0.28524
0.8	0.28814	0.29103	0.29389	0.29673	0.29955	0.30234	0.30511	0.30785	0.31057	0.31327
0.9	0.31594	0.31859	0.32121	0.32381	0.32639	0.32894	0.33147	0.33398	0.33646	0.33891
1.0	0.34134	0.34375	0.34614	0.34850	0.35083	0.35313	0.35543	0.35769	0.35993	0.36214
1.1	0.36433	0.36650	0.36864	0.37076	0.37286	0.37493	0.37698	0.37900	0.38100	0.38298
1.2	0.38493	0.38686	0.38877	0.39065	0.39251	0.39435	0.39617	0.39796	0.39973	0.40147
1.3	0.40320	0.40490	0.40658	0.40824	0.40988	0.41149	0.41308	0.41466	0.41621	0.41774
1.4	0.41924	0.42073	0.42220	0.42364	0.42507	0.42647	0.42786	0.42922	0.43056	0.43189
1.5	0.43319	0.43448	0.43574	0.43699	0.43822	0.43943	0.44062	0.44179	0.44295	0.44408
1.6	0.44520	0.44630	0.44738	0.44845	0.44950	0.45053	0.45154	0.45254	0.45352	0.45449
1.7	0.45543	0.45637	0.45728	0.45818	0.45907	0.45994	0.46080	0.46164	0.46246	0.46327
1.8	0.46407	0.46485	0.46562	0.46638	0.46712	0.46784	0.46856	0.46926	0.46995	0.47062
1.9	0.47128	0.47193	0.47257	0.47320	0.47381	0.47441	0.47500	0.47558	0.47615	0.47670
2.0	0.47725	0.47778	0.47831	0.47882	0.47932	0.47982	0.48030	0.48077	0.48124	0.48169
2.1	0.48214	0.48257	0.48300	0.48341	0.48382	0.48422	0.48461	0.48500	0.48537	0.48574
2.2	0.48610	0.48645	0.48679	0.48713	0.48745	0.48778	0.48809	0.48840	0.48870	0.48899
2.3	0.48928	0.48956	0.48983	0.49010	0.49036	0.49061	0.49086	0.49111	0.49134	0.49158
2.4	0.49180	0.49202	0.49224	0.49245	0.49266	0.49286	0.49305	0.49324	0.49343	0.49361
2.5	0.49379	0.49396	0.49413	0.49430	0.49446	0.49461	0.49477	0.49492	0.49506	0.49520
2.6	0.49534	0.49547	0.49560	0.49573	0.49585	0.49598	0.49609	0.49621	0.49632	0.49643
2.7	0.49653	0.49664	0.49674	0.49683	0.49693	0.49702	0.49711	0.49720	0.49728	0.49736
2.8	0.49744	0.49752	0.49760	0.49767	0.49774	0.49781	0.49788	0.49795	0.49801	0.49807
2.9	0.49813	0.49819	0.49825	0.49831	0.49836	0.49841	0.49846	0.49851	0.49856	0.49861

3.0	0.49865	0.49869	0.49874	0.49878	0.49882	0.49886	0.49889	0.49893	0.49896	0.49900
3.1	0.49903	0.49906	0.49910	0.49913	0.49915	0.49918	0.49921	0.49924	0.49926	0.49929
3.2	0.49931	0.49934	0.49936	0.49938	0.49940	0.49942	0.49944	0.49946	0.49948	0.49950
3.3	0.49952	0.49953	0.49955	0.49957	0.49958	0.49960	0.49961	0.49962	0.49964	0.49965
3.4	0.49966	0.49967	0.49969	0.49970	0.49971	0.49972	0.49973	0.49974	0.49975	0.49976
3.5	0.49977	0.49978	0.49978	0.49979	0.49980	0.49981	0.49981	0.49982	0.49983	0.49983
4.0	0.4999683									
4.5	0.4999966									
5.0	0.4999997133									

The normal density function $\varphi = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} = 0.3989\, e^{-\frac{1}{2}t^2}$

The normal cumulative distribution $\Phi = \int_{-\infty}^{t} \varphi\, dt$

To find the area Φ: Find in the table the area between 0 and t, and add 0.5000.
Example: for $t = 1.02$, the area between 0 and t is 0.34614. Therefore

$$\Phi = 0.84614$$

If t is negative, the area between 0 and t is negative.
Example: for $t = -1.02$, the area between 0 and t is -0.34614. Therefore

$$\Phi = 0.15386$$

The Normal Density Function φ

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.3989	0.3989	0.3989	0.3988	0.3986	0.3984	0.3982	0.3980	0.3977	0.3973
0.1	0.3970	0.3965	0.3961	0.3956	0.3951	0.3945	0.3939	0.3932	0.3925	0.3918
0.2	0.3910	0.3902	0.3894	0.3885	0.3876	0.3867	0.3857	0.3847	0.3836	0.3825
0.3	0.3814	0.3802	0.3790	0.3778	0.3765	0.3752	0.3739	0.3725	0.3712	0.3697
0.4	0.3683	0.3668	0.3653	0.3637	0.3621	0.3605	0.3589	0.3572	0.3555	0.3538
0.5	0.3521	0.3503	0.3485	0.3467	0.3448	0.3429	0.3410	0.3391	0.3372	0.3352
0.6	0.3332	0.3312	0.3292	0.3271	0.3251	0.3230	0.3209	0.3187	0.3166	0.3144
0.7	0.3123	0.3101	0.3079	0.3056	0.3034	0.3011	0.2989	0.2966	0.2943	0.2920
0.8	0.2897	0.2874	0.2850	0.2827	0.2803	0.2780	0.2756	0.2732	0.2709	0.2685
0.9	0.2661	0.2637	0.2613	0.2589	0.2565	0.2541	0.2516	0.2492	0.2468	0.2444
1.0	0.2420	0.2396	0.2371	0.2347	0.2323	0.2299	0.2275	0.2251	0.2227	0.2203
1.1	0.2179	0.2155	0.2131	0.2107	0.2083	0.2059	0.2036	0.2012	0.1989	0.1965
1.2	0.1942	0.1919	0.1895	0.1872	0.1849	0.1826	0.1804	0.1781	0.1758	0.1736
1.3	0.1714	0.1691	0.1669	0.1647	0.1626	0.1604	0.1582	0.1561	0.1539	0.1518
1.4	0.1497	0.1476	0.1456	0.1435	0.1415	0.1394	0.1374	0.1354	0.1334	0.1315
1.5	0.1295	0.1276	0.1257	0.1238	0.1219	0.1200	0.1182	0.1163	0.1145	0.1127
1.6	0.1109	0.1092	0.1074	0.1057	0.1040	0.1023	0.1006	0.09893	0.09728	0.09566
1.7	0.09405	0.09246	0.09089	0.08933	0.08780	0.08628	0.08478	0.08329	0.08183	0.08038
1.8	0.07895	0.07754	0.07614	0.07477	0.07341	0.07206	0.07074	0.06943	0.06814	0.06687
1.9	0.06562	0.06438	0.06316	0.06195	0.06077	0.05959	0.05844	0.05730	0.05618	0.05508
2.0	0.05399	0.05292	0.05186	0.05082	0.04980	0.04879	0.04780	0.04682	0.04586	0.04491
2.1	0.04398	0.04307	0.04217	0.04128	0.04041	0.03955	0.03871	0.03788	0.03706	0.03626
2.2	0.03547	0.03470	0.03394	0.03319	0.03246	0.03174	0.03103	0.03034	0.02965	0.02898
2.3	0.02833	0.02768	0.02705	0.02643	0.02582	0.02522	0.02463	0.02406	0.02349	0.02294
2.4	0.02239	0.02186	0.02134	0.02083	0.02033	0.01984	0.01936	0.01888	0.01842	0.01797
2.5	0.01753	0.01709	0.01667	0.01625	0.01585	0.01545	0.01506	0.01468	0.01431	0.01394
2.6	0.01358	0.01323	0.01289	0.01256	0.01223	0.01191	0.01160	0.01130	0.01100	0.01071
2.7	0.01042	0.01014	$0.0^{2}9871$	$0.0^{2}9606$	$0.0^{2}9347$	$0.0^{2}9094$	$0.0^{2}8846$	$0.0^{2}8605$	$0.0^{2}8370$	$0.0^{2}8140$
2.8	$0.0^{2}7915$	$0.0^{2}7697$	$0.0^{2}7483$	$0.0^{2}7274$	$0.0^{2}7071$	$0.0^{2}6873$	$0.0^{2}6679$	$0.0^{2}6491$	$0.0^{2}6307$	$0.0^{2}6127$
2.9	$0.0^{2}5953$	$0.0^{2}5782$	$0.0^{2}5616$	$0.0^{2}5454$	$0.0^{2}5296$	$0.0^{2}5143$	$0.0^{2}4993$	$0.0^{2}4847$	$0.0^{2}4705$	$0.0^{2}4567$

3.0	$0.0^{2}4432$	$0.0^{2}4301$	$0.0^{2}4173$	$0.0^{2}4049$	$0.0^{2}3928$	$0.0^{2}3810$	$0.0^{2}3695$	$0.0^{2}3584$	$0.0^{2}3475$	$0.0^{2}3370$
3.1	$0.0^{2}3267$	$0.0^{2}3167$	$0.0^{2}3070$	$0.0^{2}2975$	$0.0^{2}2884$	$0.0^{2}2794$	$0.0^{2}2707$	$0.0^{2}2623$	$0.0^{2}2541$	$0.0^{2}2461$
3.2	$0.0^{2}2384$	$0.0^{2}2309$	$0.0^{2}2236$	$0.0^{2}2165$	$0.0^{2}2096$	$0.0^{2}2029$	$0.0^{2}1964$	$0.0^{2}1901$	$0.0^{2}1840$	$0.0^{2}1780$
3.3	$0.0^{2}1723$	$0.0^{2}1667$	$0.0^{2}1612$	$0.0^{2}1560$	$0.0^{2}1508$	$0.0^{2}1459$	$0.0^{2}1411$	$0.0^{2}1364$	$0.0^{2}1319$	$0.0^{2}1275$
3.4	$0.0^{2}1232$	$0.0^{2}1191$	$0.0^{2}1151$	$0.0^{2}1112$	$0.0^{2}1075$	$0.0^{2}1038$	$0.0^{2}1003$	$0.0^{3}9689$	$0.0^{3}9358$	$0.0^{3}9037$
3.5	$0.0^{3}8727$	$0.0^{3}8426$	$0.0^{3}8135$	$0.0^{3}7853$	$0.0^{3}7581$	$0.0^{3}7317$	$0.0^{3}7061$	$0.0^{3}6814$	$0.0^{3}6575$	$0.0^{3}6343$
4.0	$0.0^{3}1338$	$0.0^{3}1286$	$0.0^{3}1235$	$0.0^{3}1186$	$0.0^{3}1140$	$0.0^{3}1094$	$0.0^{3}1051$	$0.0^{3}1009$	$0.0^{4}9687$	$0.0^{4}9299$
4.5	$0.0^{4}1598$	$0.0^{4}1528$	$0.0^{4}1461$	$0.0^{4}1396$	$0.0^{4}1334$	$0.0^{4}1275$	$0.0^{4}1218$	$0.0^{4}1164$	$0.0^{4}1112$	$0.0^{4}1062$
4.9	$0.0^{5}2439$	$0.0^{5}2322$	$0.0^{5}2211$	$0.0^{5}2105$	$0.0^{5}2003$	$0.0^{5}1907$	$0.0^{5}1814$	$0.0^{5}1727$	$0.0^{5}1643$	$0.0^{5}1563$

*Random Sampling Numbers**

20 17	42 28	23 17	59 66	38 61	02 10	86 10	51 55	92 52	44 25
74 49	04 49	03 04	10 33	53 70	11 54	48 63	94 60	94 49	57 38
94 70	49 31	38 67	23 42	29 65	40 88	78 71	37 18	48 64	06 57
22 15	78 15	69 84	32 52	32 54	15 12	54 02	01 37	38 37	12 93
93 29	12 18	27 30	30 55	91 87	50 57	58 51	49 36	12 53	96 40
45 04	77 97	36 14	99 45	52 95	69 85	03 83	51 87	85 56	22 37
44 91	99 49	89 39	94 60	48 49	06 77	64 72	59 26	08 51	25 57
16 23	91 02	19 96	47 59	89 65	27 84	30 92	63 37	26 24	23 66
04 50	65 04	65 65	82 42	70 51	55 04	61 47	88 83	99 34	82 37
32 70	17 72	03 61	66 26	24 71	22 77	88 33	17 78	08 92	73 49
03 64	59 07	42 95	81 39	06 41	20 81	92 34	51 90	39 08	21 42
62 49	00 90	67 86	93 48	31 83	19 07	67 68	49 03	27 47	52 03
61 00	95 86	98 36	14 03	48 88	51 07	33 40	06 86	33 76	68 57
89 03	90 49	28 74	21 04	09 96	60 45	22 03	52 80	01 79	33 81
01 72	33 85	52 40	60 07	06 71	89 27	14 29	55 24	85 79	31 96
27 56	49 79	34 34	32 22	60 53	91 17	33 26	44 70	93 14	99 70
49 05	74 48	10 55	35 25	24 28	20 22	35 66	66 34	26 35	91 23
49 74	37 25	97 26	33 94	42 23	01 28	59 58	92 69	03 66	73 82
20 26	22 43	88 08	19 85	08 12	47 65	65 63	56 07	97 85	56 79
48 87	77 96	43 39	76 93	08 79	22 18	54 55	93 75	97 26	90 77
08 72	87 46	75 73	00 11	27 07	05 20	30 85	22 21	04 67	19 13
95 97	98 62	17 27	31 42	64 71	46 22	32 75	19 32	20 99	94 85
37 99	57 31	70 40	46 55	46 12	24 32	36 74	69 20	72 10	95 93
05 79	58 37	85 33	75 18	88 71	23 44	54 28	00 48	96 23	66 45
55 85	63 42	00 79	91 22	29 01	41 39	51 40	36 65	26 11	78 32
67 28	96 25	68 36	24 72	03 85	49 24	05 69	64 86	08 19	91 21
85 86	94 78	32 59	51 82	86 43	73 84	45 60	89 57	06 87	08 15
40 10	60 09	05 88	78 44	63 13	58 25	37 11	18 47	75 62	52 21
94 55	89 48	90 80	77 80	26 89	87 44	23 74	66 20	20 19	26 52
11 63	77 77	23 20	33 62	62 19	29 03	94 15	56 37	14 09	47 16
64 00	26 04	54 55	38 57	94 62	68 40	26 04	24 25	03 61	01 20
50 94	13 23	78 41	60 58	10 60	88 46	30 21	45 98	70 96	36 89
66 98	37 96	44 13	45 05	34 59	75 85	48 97	27 19	17 85	48 51
66 91	42 83	60 77	90 91	60 90	79 62	57 66	72 28	08 70	96 03
33 58	12 18	02 07	19 40	21 29	39 45	90 42	58 84	85 43	95 67
52 49	40 16	72 40	73 05	50 90	02 04	98 24	05 30	27 25	20 88
74 98	93 99	78 30	79 47	96 92	45 58	40 37	89 76	84 41	74 68
50 26	54 30	01 88	69 57	54 45	69 88	23 21	05 69	93 44	05 32
49 46	61 89	33 79	96 84	28 34	19 35	28 73	39 59	56 34	97 07
19 65	13 44	78 39	73 88	62 03	36 00	25 96	86 76	67 90	21 68
64 17	47 67	87 59	81 40	72 61	14 00	28 28	55 86	23 38	16 15
18 43	97 37	68 97	56 56	57 95	01 88	11 89	48 07	42 60	11 92
65 58	60 87	51 09	96 61	15 53	66 81	66 88	44 75	37 01	28 88
79 90	31 00	91 14	85 65	31 75	43 15	45 93	64 78	34 53	88 02
07 23	00 15	59 05	16 09	94 42	20 40	63 76	65 67	34 11	94 10

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90 08	14 24	01 51	95 46	30 32	33 19	00 14	19 28	40 51	92 69
53 82	62 02	21 82	34 13	41 03	12 85	65 30	00 97	56 30	15 48
98 17	26 15	04 50	76 25	20 33	54 84	39 31	23 33	59 64	96 27
08 91	12 44	82 40	30 62	45 50	64 54	65 17	89 25	59 44	99 95
37 21	46 77	84 87	67 39	85 54	97 37	33 41	11 74	90 50	29 62
16 16	57 04	81 71	17 46	53 29	73 46	42 73	77 63	62 58	60 59
98 63	89 52	77 23	61 08	63 90	80 38	42 71	85 70	04 81	05 50
01 03	09 35	02 54	51 96	92 75	58 29	24 23	25 19	89 97	91 29
29 07	16 34	49 22	52 96	89 34	17 11	06 91	24 38	55 06	83 59
72 61	80 54	70 99	24 64	11 38	83 65	27 23	40 37	84 58	48 53
71 11	41 82	79 37	00 45	98 54	52 89	26 34	40 13	60 38	08 86
61 05	66 18	76 82	11 18	61 90	90 63	78 57	32 06	39 95	75 94
81 89	42 34	00 49	97 53	33 16	26 91	57 58	42 48	51 05	48 27
10 24	90 84	22 16	26 96	54 11	01 96	58 81	37 97	80 98	72 81
14 28	33 43	01 32	58 39	19 54	56 57	23 58	24 87	77 36	20 97
35 41	17 89	87 04	28 32	13 45	59 03	91 08	69 24	84 44	42 83
07 89	36 87	98 73	77 64	75 19	05 61	11 64	31 75	49 38	96 60
27 59	15 58	19 68	95 47	25 69	11 90	26 19	07 40	83 59	90 95
95 98	45 52	27 35	86 81	16 29	37 60	39 35	05 24	49 00	29 07
12 95	72 72	81 84	36 58	05 10	70 50	31 04	12 67	74 01	72 90
35 23	06 68	52 50	39 55	92 28	28 89	64 87	80 00	84 53	97 97
86 33	95 73	80 92	26 49	54 50	41 21	06 62	73 91	35 05	21 37
02 82	96 23	16 46	15 51	60 31	55 27	84 14	71 58	94 71	48 35
44 46	34 96	32 68	48 22	40 17	43 25	33 31	26 26	59 34	99 00
08 77	07 19	94 46	17 51	03 73	99 89	28 44	16 87	56 16	56 09
61 59	37 08	08 46	56 76	29 48	33 87	70 79	03 80	96 81	79 68
67 70	18 01	67 19	29 49	58 67	08 56	27 24	20 70	46 31	04 32
23 09	08 79	18 78	00 32	86 74	78 55	55 72	58 54	76 07	53 73
89 40	26 39	74 58	59 55	87 11	74 06	49 46	31 94	86 66	66 97
84 95	66 42	90 74	13 71	00 71	24 41	67 62	38 92	39 26	30 29
52 14	49 02	19 31	28 15	51 01	19 09	97 94	52 43	22 21	17 66
89 56	31 41	37 87	28 16	62 48	01 84	46 06	04 39	94 10	76 21
65 94	05 93	06 68	34 72	73 17	65 34	00 65	75 78	23 97	13 04
13 08	15 75	02 83	48 26	53 77	62 96	56 52	28 26	12 15	75 53
03 18	33 57	16 71	60 27	15 18	39 32	37 01	05 86	25 14	35 41
10 04	00 95	85 04	32 80	19 01	85 03	29 29	80 04	21 52	14 76
23 94	97 28	60 43	42 25	26 48	48 13	34 68	39 22	74 85	03 25
35 63	42 90	90 74	33 17	58 77	83 36	76 22	00 89	61 55	13 17
42 86	03 36	45 33	60 77	72 92	10 76	22 55	11 00	37 60	47 73
67 26	92 87	09 96	85 37	82 61	39 01	70 05	12 66	17 39	99 34
91 93	88 56	35 76	97 35	19 37	14 66	07 57	24 41	06 90	07 72
37 14	73 35	32 01	07 94	78 28	90 33	71 56	63 77	89 24	24 28
07 46	50 58	08 73	42 97	20 42	64 68	48 35	04 38	28 28	36 94
92 18	09 46	94 99	17 41	28 60	67 94	26 54	63 70	84 73	76 61
00 49	98 43	39 67	68 40	41 31	92 28	49 57	15 55	11 81	41 89
08 59	41 41	33 59	43 28	14 51	02 71	24 45	41 57	22 11	79 79
67 05	19 54	32 33	34 68	27 93	39 35	62 51	35 55	40 99	46 19
24 99	48 06	96 41	21 25	29 03	57 71	96 49	94 74	98 90	21 52
65 86	27 46	70 93	27 39	64 37	01 63	21 03	43 78	18 74	77 07
52 70	03 20	84 96	14 37	51 05	63 99	81 02	84 56	17 78	48 45
32 88	29 93	58 21	71 05	68 58	79 08	86 37	98 76	70 45	66 23
54 16	39 40	98 57	02 05	65 15	73 23	51 51	75 06	38 13	51 68
95 22	18 59	54 57	44 22	72 35	81 24	14 94	24 04	42 26	92 14
93 10	27 94	90 45	39 33	50 26	88 46	90 57	40 47	71 63	62 59
19 20	85 20	15 67	78 03	32 23	50 59	24 83	64 99	18 00	78 50

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